THE GROVND OF ARTES, TEACHING the perfect worke and practise of Arithmaticke, both in whole numbers and Fractions, after a more easie and exact sort, than hitherto hath beene set foorth.

Made by M. ROBERT RECORD, *D. in Phisicke.*

And now lately diligently corrected and beautified with sundry new Rules and necessary Additions: And further endowed with a third part, of Rules of Practise, abridged into a briefer method than hitherto hath bene published: with diuerse such necessarie Rules as are incident to the trade of Merchandise.

Whereunto are also added diuerse Tables and Instructions that will bring great profite and delight vnto Merchants, Gentlemen, and others, as by the Contents of this Treatise shall appeare.

By IOHN MELLIS.

AT LONDON,
Imprinted by T. D. for Iohn *Harison, at the Greyhound in* Paules Churchyard.
1594.

To the Reader.

THat which my friend hath well begun
For verie loue to common weale,
Of good will hath bene ouer runne
Correcting each part euery deale.
And beautifull for thy behoofe
With briefer wayes for practiſe lore,
As by the triall and the proofe
Was neuer yet in print before.
Of numbers vſe the endleſſe might
No Wit nor language can expreſſe:
Apply, and trie, both day and night,
And then this trueth thou wilt confeſſe.

The Bookes verdict.

TO pleaſe or diſpleaſe ſure I am,
But not of one ſort to euerie man.
To pleaſe the beſt ſort would I faine,
The froward diſpleaſe ſhall I certaine.
Yet wiſh I will, though not with hope,
All eares and mouthes to pleaſe or ſtop.

TO THE RIGHT WORshipful M. Robert Forth Doctor of Law, and one of the Masters of the Queenes Maiesties high Court of Chauncerie.



Hereas heretofore (Right worshipful Sir) I trauelled to the enlarging of this Author, wherein I abbreuiated some Rules to a more plainer method, as also enlarged him with sundry rules & questions for the better vnderstanding of such as delight in this excellent Art. And seeing that within this 8. yeares, two impressions of these my labors dedicated to your Worship are already worne out. And for asmuch as my former labors to your Worship dedicated, were right curteously accepted of you: I am still bold, being my first patron, to present you with this a third Impression: the rather that it may appeare both to your Worship and other that fauour knowledge, how this noble Art now florisheth in this noble Realme by the imparting of so many of these bookes in so short a time. Which simple talent of like pure affection as afore time, (as one of the greatest Iewels that God hath bestowed vpon me) I againe dedicate to your Worship as a testimonie of the dutifull zeale & good will I beare you: not doubting but that you will accept the giuers mind more thā the gift, which I confesse is small. But being throughly ac-

quainted with the great fauour your Worship beareth to such as delight in any good exercise, emboldneth me to put forth this simple Additiõ vnder your Worships defence. The entire loue & exercise of this excellent Art, with drawing of Proportions, Maps, Cards, Buildings, Plats, &c. were the only studies whereunto I euermore haue bin enclined. Touching Drawing, it was only Dei beneficio, naturally giuen me from my youth without instructiõ of any man, more than Loue thereof, delectation, desire and practise. In this Art also hauing great delight, I had no other instruction at my first beginning, but onely this good Authors booke, but afterwardes I greatly increased the same during the time I serued your Worship in Cambridge, in going to the Arithmeticke Lecture at the common Schole: And more furthered since the time I left your Worships seruice, which is about eight and twentie yeares past, by continuall exercise therein (the mother & nurse of Science) during which time my onely vocation hath bin (thinking it a meet exercise for a common wealth) in trayning vp of youth to write and drawe, With teaching of them the infallible principles and briefe practises of this worthie Science, hauing (I praise God for it) brought vp a number to become faithfull and seruiceable to their maisters in great affaires, and many of them good mẽbers of a common wealth, which is no small comfort to mee in Christ. Amongst which number a

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countriman of mine, a Norwich man, a Booke-seller here in London, was the first that became an earnest suter vnto me, to doe a deed of charitie vpon the ground of Artes, viz. to peruse and amend it of the imperfections and faults that haue crept into it through negligence of often printing. Which earnest request of his & others, bred two strifes in me: The one was, I was loth to do it, knowing my selfe inferior to a great number that might better do it than I. And on the other side, cõsidering it is a booke hath done many a thousand good, which when a young beginner commeth to a confused or mistaken figure, bringeth him into a wonderfull discouragement and maze: which thing considered, for meere loue to a common wealth, and to the booke, being my first Author, I willingly graunted to do my good will. And passing vnder the file of correction, I here and there increast it with such necessarie Additions as I knew might encourage a yonger learner, and more would haue done, but for feare the booke would rise too thicke or grow too deere. And being thus entred into the vaine thereof, and knowing that this Author was the onely light and the chiefe Lodestone vnto the vulgar sort of English men in this worthy Science, that euer writ in our naturall tongue, I haue (according to my simple knowledge) yeelded againe some part of my receiued talent with aduantage: and endowed him to the further increase of his memory, with Rules of breuitie and

 practise,

practise, abridged into a briefer method than hitherto hath bene published in our English tonge, With other right necessary Additions, Rules and Tables, which I trust will do my Countrie good, and be right commodious to all sortes of men: all which I cõmit to the fauourable censure of your Worship, & all such as loue knowledge, desiring their fauourable correctiõ herein, if ought be amisse. The which with greater affection than I am able to vtter, I dedicate to your Worship, as a meete Patrone, both for learning, godlines, and loue of the same. Which cõming from your Worship into the hands of many, shall (I doubt not) do many good, as heretofore it hath alreadie done. So shall you (as the best benefactor of these labours) be partaker of all their prayers that shall reape profite or knowledge by this worthie Art: in commendation whereof if I should write, I should rather blemish than adorne it. For the Authors Epistle vnto that famous Prince of worthie memorie King Edward the sixt, and his Preface to the Reader, are sufficient.

Thus crauing fauorable acceptation of this my homely and duetifull present, I humbly leaue you to the conduction of the Almightie, whom I beseech long to preserue you in continuall health, with daily increase of Worship, to the glory of his name, and to the ioy of all such as vnfeignedly loue you.

At my house in Southwarke this 12 of August,
Your Worships most bounden to cõmaund,

Iohn Mellis.

TO THE MOST MIGHty Prince Edward the sixt, by the grace of God king of England, *Fraunce, and Ireland, &c.*

King of England

THE excellencie of mãs nature, being such, as it is by GODS diuine fauor (most mightie Prince) not onely created in highnes of degree farre aboue all other corporall thinges, but by perfectiõ of reason, and search of wit, much approching toward the image of God, as not only the holy Scriptures do testifie, but also those naturall Philosophers, which exactly did cõsider the nature of man, and namely the farre reach and infinite compasse of the workes of the mind, were inforced to confesse, that man scarcely was able to know him selfe. And if he would duely ponder the nature of himselfe; he would find it so strange, that it might seeme vnto him a verie miracle. And thereof sprang that saying: *Magnum miraculum est homo, maximum miraculum sapiens homo*. For vndoubtedly as man is one of the greatest miracles that euer God wrought, so a wise man is plainely the greatest.

And therefore was it that ſome did account the head of a man the greateſt miracle in the world, becauſe not onely of the ſtraũge workemanſhip that is in it, but much more of the efficacie of reaſon, wit, memory, imagination, and ſuch other powers and workes of the mind, which can more eaſily conceiue any thing in a maner, than vnderſtand it ſelfe. And amongſt al the creatures of God it findeth none more difficult to be perceiued than the ſame powers it ſelfe, whereby it doth conceaue & iudge: as it may be well coniectured by the diuerſitie of opinions, that the wiſeſt Philoſophers did vtter touching the ſpirit of man & the ſubſtance of it: whereof at this preſent I intend to make no rehearſall, but who ſo liſteth to read therof, may find it largely ſet forth, not onely in Ariſtotle his bookes *de Anima*, but alſo in Galene his booke called *Hiſtoria Philoſophica*, and againe in Plutarch his worke *De Philoſophorum placitis*, whoſe words are alſo repeated of Euſebius in the 15 booke, Εὐαγγελικὴς προπαρασκεύης, vnto whom I remit them that haue deſired to vnderſtand the intricate difficultie of knowing our owne ſelues, as touching our beſt parte, and that parte whereby we deſerue to beare the name of men.

This matter ſeemed ſo obſcure and difficult, in knowledge, that Galene, who for his excellent wiſedome and iudgement in naturall workes, is called of many men a Miracle in nature, yet in ſearching the nature & ſubſtance of the ſpirit of man, he not onely confeſſeth him ſelfe ignorant, but counteth it plaine temeritie to attempt to finde it: ſo farre aboue the hope of mans knowledge is that part, whereby man doth know and iudge of things. And although the ignorant ſort (which hate all things that they know not) do litle eſteeme the profoũdneſſe of mans ſpirit and of reaſon, the chiefe power and faculty of it, yet as there is a kind of feare and obedience of vnreaſonable beaſts vnto mã by the working power of God,

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ſo is there in thoſe ſmall reaſoned perſons a certaine kind of reuerence toward wiſdome & reaſons, which they do ſhew oftentimes, and by power of perſwaſion are enforced to obey reaſon, will they nill they.

And hereby came it to paſſe, that the rudeneſſe of the firſt age of man was brought vnto ſome more ciuill trade, as it is well declared by Cicero in the beginning of his firſt booke *De inuentione Rhetorica*, where he ſaith thus.

Nam fuit quoddam tempus quod in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant, nec ratione animi quicquam, ſed ploraque viribus corporis adminiſtrabant. Nondum diuinæ religionis, non humani officij ratio colebatur: Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque inſcitiam, cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam viribus corporis abutebatur, pernitioſiſſimis ſatellitibus.

Quo tempore quidam, magnus videlicet vir & ſapiens, cognouit quę materia eſſet, & quanta ad maximas res opportunitas in animis ineſſet hominum, ſi quis eam poſſet elicere, & præcipiendo meliorem reddere: Qui diſperſos homines in agris, & in tectis ſyluestribus additos ratione quadam compulit in vnum locum, & congreganit: & eos in vnamquamque rem inducens vtilem atque honeſtam primo propter inſolentiam reclamantes, deinde propter rationem atque orationem ſtudioſius audientes, ex feris & immanibus mites reddidit, & manſuetos.

This long repetition of Tullies wordes will ſeeme tediouſe to them which loue but litle, and care much leſſe for the knowledge of reaſon, but vnto your Maieſtie (I dare ſay) it is a delectable remembraunce, & vnto me it ſeemeth ſo pleaſaunt, that I could ſcarce ſtay my penne from writing all that mine eyes did ſo greedily read.

This ſentéce of Cicero am I loath to tranſlate into Engliſh, partly for that vnto your Maieſtie it needeth

deth no translatiõ, but especially knowing how farre that grace of Tullies eloquence doth excell any English mans tongue, and much more exceedeth the baseneſſe of my barbarous ſtile, yet for the fruict of the ſentence, I had rather vnto my meere Engliſh Countreymen vtter the rudeneſſe of my tranſlation, than to defraude them of the benefite of ſo good a leſſon, truſting they will alſo gladly and greedily imbrace all good ſciences that may help to the iuſt furniture of the ſame, when they cõſider that informed reaſon was the only inſtrument, or at leaſt the chiefe meane to bring men vnto Ciuill gouernement, from barbarous maners and beaſtly conditions.

For the time was (ſaith Tullie) that men wãdered abrode in the fields vp and downe like beaſts, & vſed no better order in feeding than they, ſo that by reaſons rule they wrought nothing, but moſt of their doing did they atchieue by force of ſtrength. At this time there was no iuſt regard of religiõ toward God, nor of dutie toward mã. No man had ſeene right vſe of mariage, neither did any man know their owne children frõ other, nor no man had felt the cõmoditie of iuſt Lawes: ſo that through errour & ignorãce, wilfull luſt, like a blind & headie ruler, abuſed bodily ſtrength as a moſt mortall miniſter for the ſatisfying of his deſire. At that time was there one, which not onely in power, but alſo in wiſedome was great: & he conſidered, how that in the mindes of men was both apt inſtruments, and great occaſion to the due accompliſhment of moſt waightie affaires, if a man could applie them to vſe, and by teaching of rules frame them to better trade. This man with perſwaſion of reaſon gathered into one place the people that were wandring about the fields, and lay lurking in wild cottages and woods: And bringing them into one cõmon ſocietie, did trade them to al ſuch things, as eyther were profitable or honeſt, although not without repining at the firſt, by reaſon that they had

not bene ſo accuſtomed before. Yet at lẽgth through reaſon and perſwaſion of wordes, they obeyed him more diligently, and ſo of a wild and cruell people, he made them courteous and gentle.

Thus did Tullie ſet forth the efficacie of reaſon and perſwaſion, how it was able to conuert wild people to a mildnes, & to change their furious cruelnes into gentle curteſie. Were it not now a great reproch in our this time (when knowledge raigneth ſo large) that men ſhould ſhew thẽſelues leſſe obſequious to reaſon? vnleſſe it may be thought, that now euery man hauing ſufficient knowledge of himſelfe, needeth not to hearken to the perſwaſion of others.

In deede he that thinketh him ſelfe wiſe, will not eſteeme the reaſon of any other, be he neuer ſo wiſe, ſo that of ſuch one it may be well ſayd, He that thinketh him ſelfe wiſer than he is, may iuſtly be counted a double foole: wherefore ſuch men are not to be permitted in open audience to talke, but muſt be put to ſilence, and made to giue eare to reaſon, which reaſon conſiſteth not in a multitude of wordes, heaped raſhly together, and applied for one purpoſe: but reaſon is the expreſſing of a iuſt mater with wittie perſwaſions, furniſhed with learned knowledge. Such knowledge had *Moſes* being expert in all learning of the *Ægyptians*, as the Scriptures declare, and therefore was able to perſwade the ſtubburne people of the *Iewes*, although not without great payne. Such knowledge and ſuch reaſon did *Druys* ſhewe, which was the firſt Law-maker of all the Weſt parts of *Europe*. Like reaſon and wiſedome did *Xamolxis* vſe amongſt the *Gothes*: *Lycurgus* vnto the *Lacedæmonians*, *Zeleucus* to the *Locrians*, *Solon* to the *Athenienſes*, and *Donwallo Molmitius* two thouſand yeares paſt, amongſt the old *Brittaines* of this realme. And thereby it came to paſſe, that their Lawes continued long, till more perfect reaſon altered many of them, and wilfull power oppreſſed moſt of them.

Druys was ſonne to king Sarron, and ſucceeded him in his kingdome.

At the beginning when these wise men perceiued how harde it was to bring the rude people to vnderstand reason, they iudged the best meanes to attaine this honest purpose, to depend of learning in euery kind: for by learning, as Ouid saith: *Pectora mollescũt, asperitasque fugit*: Stout stomaches do waxe mild, and sharp fierceneſſe is exiled. Therfore as Berosus doth testifie, Sarron that was the third King ouer all this West parte of Europe, for to bring the people from beastly rage to manly reason, did erect schooles of liberall Artes, which tooke so good succeſſe, that his name cõtinued in that sort famouse aboue two thousand yeares after: for Diodorus Siculus which was in the time of Iulius Cæsar maketh mẽtion of the learned men of the Gothes, & named them Sarronides, that is to say, Sarron his scholers and followers.

Among these Artes that then were taught, some did informe the tongue, and make men able both to vtter aptly their mind, and also to perswade, as Grãmer, Logicke, and Rhetoricke, although not so curiously as in this time: some other did appertaine to the iust order of partition of Lands, the true vsing of waights, measures, and reckenings in al sortes of bargaines, and for order of building and sundry other vses, those were Arithmeticke & Geometrie. Againe, to encourage mẽ to the honour of God, they taught Astronomie, whereby the wonderfull workes of God were so manifestly set foorth, that no mans tongue nor pẽne can in like sort expresse his infinite power, his vnspeakable wisedome, and his exceeding goodneſſe toward man, whereby he doth bountifully prouide for man all neceſſaries, not onely to liue, but also to liue pleasantly. And so was their confidence in Gods prouidence strongly stayed, knowing his goodneſſe to be such, that he would help mã as he could, and his power to be so great, that he could do what he would: and thirdly, his wisdome to be so pure, that he would do nothing but that was best. Beside these

Sciences,

Sciences, they taught alſo Muſicke, which moſt commonly they did applie partly to religious Sciences, to draw men to delight therein, and partly to ſongs made of the maners of men in praiſe of Vertue and diſcommendation of Vice, whereby it came to paſſe, that no man would diſpleaſe them, nor do any thing euill that might come to their hearing: for their Songs did make euill men more abhorred in that time, than any excommunication doth in this time. The poſteritie of theſe Muſitians continue yet both in Wales & Ireland called *Bardes* vnto this day, by the auncient name of *Bardus*, their firſt founder.

This Bardus Druidus the fifth king of the Celtes, raigned 69. yeares, and 1832 yeares before Chriſt.

And as theſe Sciences did increaſe, ſo did vertue increaſe therby. Againe, as theſe Sciences did decay, ſo vertue loſt her eſtimation, and conſequently was litle in vſe: whereof to make a full declaration, were a thing meete for a Prince to heare, but it would require a peculiar treatiſe. Wherfore at this preſent I count it ſufficient, lightly to haue touched this matter in generall words, and to ſay no more of the particularitie thereof, but onely touching one of thoſe ſciences, that is Arithmetike, by which not onely iuſt partition of lands was made, but alſo touching buying and ſelling, al aſſiſes, weights, and meaſures were deuiſed, and all reckenings and accounts driuen: yea by proportion of it were the true orders of Iuſtice limited, (as Ariſtotle in his Ethiks doth declare, and the degrees of eſtates in the common wealth eſtabliſhed Although that proportion be called Geometricall, and not Arithmeticall, yet doth that proportion appertaine to the Art of Arithmeticke: & in Arithmetike is taught the Progreſſion of ſuch proportions, and all things thereto belonging. Wherefore I may well ſay, that ſeeing Arithmeticke is ſo many wayes needfull vnto the firſt planting of a common wealth, it muſt needes be as much required to the preſeruation of it alſo: for by the ſame meanes is any common wealth continued, by which it was erected

and eſtabliſhed. And if I ſhall in ſmall matters in appearance, but in deede verie waightie, put one example or two, what ſhall wee ſay for the Statutes of this Realme, which be the onely ſtay of good order in manner now? As touching the meaſuring of ground by length and breadth, there is a good and auncient Statute made by the Arte of Arithmeticke, and now it ſhall be to little vſe, if by the ſame Art it be not practiſed and tried. For the aſſiſe of bread & drinke, the two moſt common and moſt neceſſarie things for the ſuſtentation of man, there was a goodly ordinance in the Lawe made, which by ignorance hath ſo growen out of knowledge and vſe, that fewe men doe vnderſtand it, and therefore the Statute bookes are wonderfully corrupted, & the Commons cruelly oppreſſed: notwithſtanding ſome men haue written, that it is too doubtfull a matter to execute thoſe Aſſiſes by thoſe ſtatutes, by reaſon they depēd of the ſtandard of the Coine, which is much changed from the ſtate of that time when thoſe ſtatutes were made. Thus ſhall euery man read that liſteth in the Abridgement of Statutes, in the title of Waightes & Meaſures, in the ſeuenth number of the Engliſh booke, where he ſhould haue tranſlated a good ordinaunce, which is ſet foorth in the French booke: but no maruell if the Abridgement doth omit it, ſeeing the great booke of ſtatutes doeth omit the ſame ſtatute, as it hath done diuerſe other very good Lawes. And this is the fruite of ignorance, to reiect all that it vnderſtandeth not, although they vſe ſome clokes for it: but ſuch clokes, as being allowed, might ſerue to repell all good lawes: which God forbid.

Againe there is an ancient order for Aſſiſe of firewood, and coales, which was renued not many yeares paſt, and now how Auarice & Ignorance doth canuas the ſtatute, it is too pitifull to talke of, and more miſerable to feele.

Furthermore, for the ſtatute of Coynage, and

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the Standerd therof, if the people vnderſtood rightly the Statute, they ſhould not, nor would not (as they often doe) gather an excuſe for their follie thereby: but as I ſayd, theſe Statutes by wiſedome and good knowledge of Arithmeticke were made, and by the ſame muſt they be continued. And let ignoraunce no more meddle with the vſe of them, than it did with the making of them. Oh in how miſerable caſe is that Realme, where the Miniſters and Interpreters of the Lawes are deſtitute of all good Sciéces, which be the keyes of the Lawes? How can they eyther make good Lawes or maintayne thé, that lacke that true knowledge whereby to iudge them? And happie may that Realme be accounted, where the Prince him ſelfe is ſtudious of learning, & deſireth to vnderſtand equitie in all Lawes. Therfore moſt happie are wee the louing ſubiects of your Maieſtie, which may ſee in your Highneſſe not onely ſuch towardneſſe, but alſo ſuch knowledge of diuerſe Artes, as ſeldome hath bene ſeene in any Prince of ſuch yeares, whereby we are enforced to cõceiue this hope: Certainely, that he which in thoſe yeares ſeeketh knowledge when knowledge is leaſt eſteemed, and of ſuch an age can diſcerne them to be enemies both to his royall perſon, and to his Realme, which labour to withdraw him from knowledge to exceſſiue paſtime, and from reaſonable ſtudie to idle or noiſome pleaſures, he muſt needes when he commeth to more mature yeares, bee a moſt prudent Prince, a moſt iuſt Gouernour, and a right Iudge, not onely of his ſubiectes commonly, but alſo of the miniſters of his Lawes, yea, and of the Lawes théſelues, and to be able to conceiue the true equity and exact vnderſtanding of all his Lawes & ſtatutes, to the cõfort of his good ſubiectes, and the confuſion and reproch of them which labour to obſcure or peruert the equitie of the ſame Lawes & ſtatutes. How ſome of thoſe Statutes may bee applied to vſe, as well in

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our time, as in any other time, I haue particularly declared in this booke, & ſome other I haue omitted for iuſt conſiderations, till I may offer them firſt vnto your Maieſty, to weigh them, as to your Highneſſe ſhall ſeeme good: for many things in them are not to be publiſhed without your Highneſſe knowledge and approbation, namely becauſe in them is declared all the rates of alloyes for all ſtanderds from one ownce vpward, with other myſteries of Mynt matters, and alſo moſt part of the varieties of coynes that haue bene currant in this your Maieſties Realme by the ſpace of ſixe hundred yeares laſt paſt, and many of thē that were currant in the time that the Romanes ruled here.

All which, with the ancient deſcription of Englād & Ireland, and my ſimple cenſure of the ſame, I haue almoſt cōpleted to be exhibeted to your Highneſſe.

In the meane ſeaſon moſt humbly beſeching your Maieſtie to accept this ſimple treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Maieſtie is, that this amongſt other of my bookes may paſſe vnder the protection of your Highneſſe, whom I beſeech God moſt earneſtly and daily, according to my duetie, to aduaunce in all honour and princely regalitie, and to increaſe in all knowledge, iuſtice, and godly pollicie.

Amen.

Your Maieſties moſt obedient Subiect and Seruant.

ROBERT RECORDE.

TO THE LOVING READER.

The Preface of Maister R. Recorde.



Ore oftétimes haue I lamented with my selfe the infortunate condition of *England*, seeing so many great Clerks to arise in sundry other parts of the worlde, and so fewe to appeare in this our natiō: whereas for pregnancie of naturall witte (I thinke) fewe nation doe exceede Englishmen: But I cannot impute the cause to any other thing then to the contempt and misregard of learning. For as Englishmen are inferiour to no men in mother wit, so they passe all men in vayne pleasures, to which they may attaine without great paine or labour: and are as slacke to any neuer so great cōmoditie, if there hang of it any painfull studie or trauelsome labour.

Howbeit, yet all men are not of that sorte, though the most part be, the more pitie it is: but of them that are so glad, not onely with painefull studie and studious paine to attaine learning, but also with as great studie and paine to communicate their learning to others, and make all England, if it might be, partakers of the same, the most parte are such, that vnneth they can supporte their owne necessarie charges, so that

they are not able to beare any charges in doing of that good, that elſe they deſire to do.

But a greater cauſe of lamentation is this: that when learned men haue taken paines to do things for the aid of the vnlearned, ſcarce they ſhall be allowed for their well doing, but derided and ſcorned, and ſo vtterly diſcouraged to take in hand any like enterpriſe againe. So that if any be found (as there are ſome) that do fauour learning and learned wits, and can be content to further knowledge, yea onely with their word, ſuch perſons, though they be rare, yet ſhall they incourage learned men to enterpriſe ſome things, at the leaſt, that *England* may reioyce of. And I haue good hope that *England* will (after ſhe hath taken ſure taſte of learning) not onely bring foorth more fauourers of it, but alſo ſuch learned men, that ſhe ſhall be able to compare with any Realme in the world. But in the meane ſeaſon, where ſo fewe regarders of learning are, how greatly they are to be eſteemed that do fauour and further it, my penne will not ſuffice at full to declare.

Therefore gentle Reader, whereas I do vpon moſt iuſt occaſion iudge, yea and know aſſuredly, that there be ſome men in this Realme, which both loue, and alſo much deſire to further good learning, and yet am not well able to write their condigne praiſe for the ſame, I thinke it better with ſilence to ouerpaſſe it, than either to ſay too litle of it, or to prouoke againſt them the malice of ſuch other, which doo nothing themſelues that is prayſe worthie, and therefore cannot abide to heare the prayſe of any other mans good deed.

And conſidering their great fauour vnto learning, though I my ſelf be not worthie to be reckoned in the number of great learned men, yet am I bold to put my ſelfe in preace with ſuch abilitie as God hath lent me, though not with ſo great cunning as many men, yet with as great affection as any man, to helpe my countrimen,

trimen, and will not ceaſſe daily (as much as my ſmall ability will ſuffer me) to indite ſome ſuch thing, that ſhall be to the inſtruction, though not of learned men, yet at the leaſt of the vulgar ſort, whoſe argument alwaies ſhalbe ſuch, that it ſhall delight all learned wits, though they do not learne any great things out of it.

But to ſpeake of this preſent booke of Arithmetike, I dare not, nor will not ſet it forth with any words, but remit it to the iudgement of all gentle Readers, and namely ſuch as loue good learning, beſeeching them ſo to eſteeme it, as it doth ſeeme worthie. And ſo either to accept the thing for it ſelfe, either at the leaſt to allow my good indeuour. But I perceiue I need not to vſe any perſwaſiõs vnto them, whoſe gentle nature and fauourable minde is readie to receiue thankfully, & interpret to the beſt, of all ſuch enterpriſes attempted for ſo good an end, though the thing do not alwaies ſatisfie mens expectation.

This conſidered, did bolden me to publiſh abroad this litle booke of the Art of numbring, which if you ſhal receiue fauourably, you ſhal incourage me to gratifie you hereafter with ſome greater thing.

And as I iudge ſome mẽ of ſo louing a mind to their natiue Countrey that they would much reioyce to ſee it to proſper in good learning and wittie Artes, ſo I hope well of all the reſt of Engliſh men, that they will not be vnmindfull of his due prayſe, by whoſe meanes they are helped and furthered in any thing. Neither ought to eſteeme this thing of ſo little value, as many men of litle diſcretion oftentimes do: For who ſo ſetteth ſmall price by the wittie deuiſe and knowledge of numbring, he litle cõſidereth it to be the chiefe point (in maner) whereby men differ from all brute beaſts: for as in all other things, (almoſt) beaſts are partakers with vs, ſo in numbring wee differ cleane from them, and in maner peculiarly, ſith that in many things they excell vs againe.

The Foxe in craftie Wit exceedeth most men,
A Dogge in smelling hath no man his peere,
To foresight of weather if you looke then,
Many beasts excell man, this is cleere.
The wittinesse of Elephants doth letters attaine,
But what cũning doth there in the Bee remaine?
The Emmet foreseeing the hardinesse of winter,
Prouideth victuals in the time of sommer.
The Nightingale, the Linet, the thrush, the Lark
In Musicall harmonie passe many a Clarke.
The Hedgehog of Astronomie seemeth to know,
And stoppeth his caue where the Wind doth blow.
The Spider in weauing such arte doth show,
No man can him mend, nor follow I trow,
When a house will fall, the Mise right quicke,
Flee thence before. Can man do the like?

Many things else of the wittinesse of beasts & birds might I here say, saue that another time I intende to write wherin they excell in maner al men, as it is daily seene: but in number was there neuer beast found so cunning, that could know or discerne one thing from many, as by daily experience you may well consider, when a Bitch hath many whelpes, or a Hen many Chickens: and likewise of other whatsoeuer they be, take from them all their young, sauing onely one, & ye shall perceiue plainely that they misse none, though they will resist you in taking them away, and will seeke them againe if they may know where they be, but else they will neuer misse thẽ truly, but take away that one that is left, and then will they crie and complaine: and restore to them that one, then are they pleased again: so that of number this may I iustly say, It is the onely thing (almost) that separateth man from beastes. He therefore

therefore that ſhall contemne nnmber, he declareth himſelfe as brutiſh as a beaſt, and vnwoorthy to be counted in fellowſhip of men. But I truſt there is no man ſo fowlie ouerſeene, though many right ſmally do it regard.

Therefore will I now ſtay to write againſt ſuch, and returne againe to this booke, which I haue written in the forme of a Dialogue, becauſe I iudge that to bee the eaſieſt way of inſtruction, when the Scholer may aske euery doubt orderly, & the Maſter may anſwere to his queſtion plainely.

Howbeit I thinke not the contrary, but as it is eaſier to blame another mans worke than to make the like, ſo there wilbe ſome that will find fault, becauſe I write in a Dialogue: but as I coniecture, thoſe ſhall be ſuch, as do not, cannot, either will not perceiue the reaſon of right teaching, and therfore are vnmeete to be anſwered vnto, for ſuch men with no reaſon will be ſatisfied.

And if any man obiect, that other bookes haue bene written of Arithmetike alreadie ſo ſufficiently, that I needed not now to put penne to the booke, except I wil cõdemne other mens writings: to them I anſwere. That as I cõdemne no mans diligence, ſo I know that no one man can ſatisfie euery man, and therefore like as many do eſteeme greatly other bookes, ſo I doubt not but ſome will like this my booke aboue any other Engliſh Arithmatike hitherto written, & namely ſuch as ſhal lacke inſtructers, for whoſe ſake I haue ſo plainly ſet forth the exãples, as no book (that I haue ſeene) hath done hitherto: which thing ſhall be great eaſe to the rude readers.

Therefore gentle Reader, though this booke can be ſmall aid to the learned ſort, yet vnto the ſimple ignorant (which needeth moſt helpe) it may be a good furtherance and meane vnto knowledge. And though vnto the king his Maieſtie priuately I do it dedicate,

yet I doubt not (such is his clemencie) but that he can be content, yea and much desirous, that all his louing subiects shall take the vse of it, and employ the same to their most profite. Which thing if I perceiue that they thankfully do, and receiue with as good will as it was written, then will I shortly with no lesse kindnesse set forth such Introductions into Geometrie and Cosmographie, as I haue at other times promised, and as hitherto in English hath not bene enterprised, wherewith I dare say all honest hearts will be pleased, and all studious wits greatly delighted.

I will say no more, but let euery man iudge as he shall see cause. And thus for this time I will stay my penne, committing you all to that true fountaine of perfect number, which wrought the whole world by number and measure: he is Trinitie in Vnitie, and Vnitie in Trinitie: To whom be all prayse, honor, and glorie.

Amen.

HERE FOLLOWETH
a Table of all the Contents of this Booke.

The Contents of the first Dialogue containeth the Declaration of the profit of Arithmetike.

- Numeration with an easie & large Table.
- Addition, Subtraction, Multiplication, Diuision. } with diuerse examples, and all their proofes, and some new formes of workings, &c.
- Reduction with diuerse Declarations of Coines, Waights and Measures of sundrie formes newly added, with a new Table, containing most part of the gold coines throughout Christendome, with the true waight & valuation of them in currant money English, &c.

Progression both Arithmeticall and Geometricall, with diuerse sundrie questions touching the same.

The Golden Rule of three: and the Backer Rule of three: with diuerse questions thereunto belonging, newly added and augmented.

The double rule of Proportion.

The Rule of three comyoned of 5 numbers:

The Rule of Felowship, both with time, and without time.

Unto all these are added their proofes.

The second Dialogue containeth

The first 5 kindes of Arithmetike wrought by Counters:

The common kindes of casting of accounts after the Merchants fashion, & Auditors also.

The Contents of the second part, touching Fractions.

What a Fraction is.
Numeration in fractiõs.
The order of working fractiõs.
Multiplicatiõ.
Diuision.

} with diuers familiar questions proponed for the perfite vnderstanding, and proofe of eth of thẽ.

Reduction of
- diuers fractions into one denomination in three varieties.
- Fractions of Fractions.
- Improper Fractions.
- Fractions to the smallest denomination, with easie rules how to conuert them thereunto.
- Fractions in other partes of things, with a table demõstratiue of their proportions.
- Fraction, and how it may be turned into any other Fractiõ, or into what denomination you list.

Againe of { Multiplication, Duplation, Diuision, Mediation, Addition, Substraction } in Fractions.

The Golden Rule with diuerse questions, and their proofes.

The Backer rule.

A question of Loane.

The Statute of assise of Bread and Ale recognised and applied to this time, with newe tables thereunto annexed.

The Statute of measuring of ground, with a Table thereof faithfully calculated and corrected.

Questions of societie, with the reasons of the Rules and proofes of their workes.

To find three numbers in any proportion.

The Rule of Alligation, with diuerse questions and the proofes of their worke, with many varieties of such solutions.

The Rule of Falshoode, or false Position, with diuerse questions and their proofes.

The Contents of the third Addition to this Booke.

The first Chapter entreateth of Rules of Breuitie and Practise, after a briefer Methode than hitherto hath beene published in

the

the English tongue.

The second chapter treateth of briefe Reduction of diuers measures, as Elles, Yardes, Braces, &c. by rules of Practise.

The third Chapter intreateth of the Rule of three in broken Numbers after the trade of Merchants, something differing from Master Records order, which is comprehended in three rules.

The fourth chapter intreateth of losse & gayne in the trade of merchandise.

The fifth chapter intreateth of losse and gaine in the trade of merchandise vppon time &c. with necessarie questions therein wrought by the double Rule of three, or the Rule of three composed.

The sixth chapter intreateth of Rules of payment, and one of the necessariest rules that appertaineth to buying and selling, &c.

The seuenth Chapter intreateth of buying and selling in the Trade of Merchaundise, wherein is taken parte readie money, and diuerse dayes of payment giuen for the rest, and what is woon or lost in the 100 pound forbearance for the twelue moneths, &c.

The eight Chapter intreateth of Rates and allowances in the trade of merchandise, sold by waight, and of their losses and gaynes therein, &c.

The ninth chapter intreateth of Lengths and breadths

breadths of Arras, and other Clothes, with diuers questions incident thereunto.

The tenth Chapter entreateth of reducing of Paunes of Geanes into English yards.

The eleuen Chapter entreateth of Rules of Loane and Interest with diuers questions incident thereunto.

The twelft Chapter entreateth of the making of Factors.

The thirteenth Chapter entreateth of Rules of Barter or Exchange of Merchaundise, wherein is taken part ware, and part readie money, with their proofes, and diuerse other necessarie questions thereunto belonging.

The fourteenth Chapter entreateth of exchanging of money from one place to an other, with diuerse necessarie questions incident thereunto.

The fifteenth Chapter intreateth of sixe sundry formes of practise for Reduction of English, Flemmish, and French money, and how each of them may easily be brought to others money sterling.

The sixteenth Chapter containeth a briefe note of the ordinarie Coynes of most places of Christendome for traffike, and the manner of their exchanging frō one Citie or Towne to an other, which knowen the Italians call Pary: whereby they find the Gaine or Losse vpon the Exchange.

The

The seuenteenth Chapter contayneth also a declaration of the diuersitie of the waights and measures of most places of Christendome for traffike, proportionated in equalitie one to another, as also vnto our English measure and weight, whereby the ingenious practitioner may easily reduce the weight and measure of each Countrey into other.

The eighteenth Chapter intreateth of diuerse sports and pastimes, done by Number.

FINIS.

A COLLECTION OF such Tables as are contained in this Treatise.

A Large Table of Numeration.
A Table of Multiplication.
A Table of all the Gold and Siluer Coynes in this Realme, with the moste vsuall gold Coines throughout Christendome, with their seuerall waights of pence and graynes, and what they are worth in currant money English.
Certaine Tables or Notes of the Contents of Ale, Beere, Wine, Butter, Sope, Salmon, Eeles, &c. both what such vessels ought to containe by the Statute, and what those vessels emptie ought to weigh.
A Table of the quantitie of drie measures, as Peckes, Bushels, Quarters, Weyes, &c.
A Table of the proportion of measure, touching Lengths or Breadths: to wit, from the inch to the foot, and so to the yard, the Elle, with their partes, the pearch, the rod, the furlong: the myle, &c.
A Table made by Progression Arithmeticall, which containeth a double Table of Multiplication.
A Table or demonstration of a figure or measure for the perfect vnderstanding of Fractions

ons of Fractions.

A Table of the contents of the Statute for the Assise of the waight of bread. From one shilling the quarter to 20 shillings, faithfully corrected.

A necessarie Table of the Statute of measuring of grounde, vppon the breadth giuen, what lenght it ought to containe: faithfully corrected according to the equitie of the Statute, wherein the Author declareth how necessarie this worthie Arte of Arithmetike is vnto Gentlemen students of the Lawe, and such other as are desirous of infallible trueth.

Briefe Tables of the readie reducing of English, French, and Flemmish money, each into others common currant moneyes.

A briefe Table or collection of the common and vsuall moneyes of most places of Christendome for traffike, the manner of their paymentes or exchangyng from one Citie or Towne to an other: right necessarie for Merchants, and others Occupiers, Trauellers, &c.

Tables of the Waights, Measures, and Customes of moste places of Europe for traffike, whereby through the aide of the Rule of three, the ingenious may easily reduce our measure to the perfect valuation of other countries measure or waight, and likewise theirs to ours.

Lastly,

Lastly, a Table demonstrating the true solution of 3 diuerse things hidden of three seuerall persons in pastime.

Note that this ▲ Caracter representeth the crowne by exchange.

Item whereas I haue augmented diuerse necessarie questions and rules vnto the Author. The beginning of which Additions are marked with this ¶ note, And where they end, with this * note.

FINIS.

BEFORE THE INTRO-

duction of Arithmeticke, it were very
good to haue some vnderstanding
and knowledge of these figures
and Notes.

i.	1	one.	xx.	20	twentie.
ii.	2	two.	xl.	40	fourtie.
iii.	3	three.	l.	50	fiftie.
iiii.	4	foure.	lx.	60	sixtie.
v.	5	fiue.	lxx.	70	seuentie.
vi.	6	sixe.	xc.	90	ninetie.
vii.	7	seuen.	C.	100	a hundred.
viii.	8	eight.	CC.	200	two hundred.
ix.	9	nine.	Ↄ	500	fiue hundred.
x.	10	ten.	ƆC.	600	sixe hundred.
xi.	11	eleuen.	M.	1000	a thousand.
xii.	12	twelue.	M.D.	1500	a thous. fiue hund.
&c.	&c.	&c.	&c.	&c.	&c.

112 ℔ make a C. waight.
56 halfe a C. waight.
28 a quarter of a hundred waight.
16 ounces make a pound.
A pound Troy which is ỳ standard of England, wherby is onely wayed Gold, Siluer & Bread, is 12 ounces.

12 inches make a foote.
3 foote make a yarde.
The yarde is parted into 4 quarters.
And ech quarter into 4 nailes.
5 quarters of a yarde make an Elle. Which Elle is also parted in 4 quarters.
And each quarter into foure nayles.

A Dialogue betweene the Master *and the Scholer, teaching the Art* and vse of Arithmetike with penne.

The Scholer speaketh.

SYR, such is your authoritie in mine estimation, that I am content to consent to your saying, and to receiue it as truth, though I see none other reason that doth lead me thereunto: whereas else in mine owne conceit it appeareth but vaine, to bestow any time priuately in learning of that thing, that euery childe may and doth learne at all times and houres, when he doth any thing himselfe alone, and much more when he talketh or reasoneth with other.

Master. Loe, this is the fashion and chance of all them that seeke to defend their blind ignorance: that when they thinke they haue made strong reason for themselues, then haue they proued quite contrarie. For if numbring be so common (as you graunt it to be) that no man can do any thing alone, and much lesse talke or

bargaine with other, but he shall still haue to do with Number: this proueth not number to be contemptible and vile, but rather right excellent and of high reputation, sith it is the ground of all mens affayres, so that without it no tale can be tolde, no communication without it can be long continued, no bargayning without it can duely be ended, or no businesse that man hath iustly completed. These commodities (if there were none other) are sufficient to approoue the worthinesse of Number. But there are other innumerable farre passing all these, which declare Number to exceede all prayse. Wherefore in all great workes are Clerkes so much desired: Wherefore are Auditors so richly feede: What causeth Geometrians so highly to be enhaunced: Why are Astronomers so greatly aduaunced: Because that by Number such things they finde, which else should farre excell mans minde.

Scholer. Verily Syr if it be so, that these men by numbring their cunning do attayne, at whose great workes most men doo wonder, then I see wel I was much deceiued, and numbring is a more cunning thing than I tooke it to be.

Master. If Number were so vile a thing as you did esteeme it, then neede it not to be vsed so much in mens communication. Exclude Number, I answere to this question. How many yeeres old are you?

Scholer.

Scholer. Mum.

Master. How manie dayes in a weeke: how many weekes in a yeere: What landes hath your Father: how many men doth hee keepe: How long is it since you came from him to me:

Scholer. Mum.

Master. So that if number want, you answere all by Mummes: How many mile to London:

Scholer. A poake full of Plummes.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking, it maketh men dumbe, so that to most questions, they must answere Mum.

Scholer. This is the cause sir, that I iudged it so vile, because it is so common in talking euery while: For plentie is not deintie, as the common saying is.

Master. No, nor store is no sore: perceiue you this: The more common that a thing is, being needfully required, the better is the thing, and the more to be desired: But in numbring, as some of it is light and playne, so the most part is difficult, and not easie to attaine. The easier parte serueth all men in common, and the other parte requireth some learning. Wherefore as without numbring a man can do almost nothing, so with the helpe of it, you may attaine to all things.

Scholer. Yea sir: why: then it were best to

learne the Art of Numbring first of all other learning, & then a man neede learne no more, if all other come with it.

Master. May not so: but if it be first learned, then shall a man be able (I meane) to learne, perceiue, and attaine to other sciences, which without it, he should neuer get.

Scholer. I perceiue by your former words, that Astronomie and Geometrie depend much on the helpe of Numbring: but that other sciences, as Musicke, Phisicke, Lawe, Grammar, and such like, haue any helpe of Arithmetike, I perceiue not.

Master. I may perceiue your great Clerkliness by the ordering of your sciences: but I will let that passe now, because it toucheth not the matter, that I intend, and I will shew you how Arithmetike doth profite in all these, somewhat grossely, according to your small understanding, omitting other reasons more substantiall.

First (as you recken them) Musicke hath not onely great helpe of Arithmetike, but is made, and hath his perfectnesse of it: for all Musicke standeth by number and proportion.

And in Physicke, beside the calculation of Criticall dayes, with other thinges which I omit, how can any man iudge the pulse rightly, that is ignorant of the proportion of Numbers?

And as for the Lawe, it is playne, that the man

man that is ignorant of Arithmetike, is neither meet to be a Iudge, neither an Aduocate, nor yet a Proctour. For how can hee well vnderstand another mans cause appertayning to distribution of goods, or other debts, or of summes of money, if he be ignorãt of Arithmetike? This oftentimes causeth right to be hindred, when the Iudge either delighteth not to heare of a matter that he perceiueth not, or cannot iudge it for lacke of vnderstanding: This commeth by the ignorance of Arithmetike.

Now as for Grammar, mee thinketh you should not doubte in what it needeth number, sith you haue learned that Nounes of all sorts, Pronounes, Uerbes, and Participles, are distinct diuersly by numbers: besides the varietie of Nounes of number, and Aduerbes. And if you take awaie number from Grammar, then is all the quantitie of Syllables lost. And many other wayes doth number helpe Grammar. Whereby were all kindes of meeters founde and made? Was it not by Number?

But how needefull Arithmetike is to all partes of Philosophie, they may soone see, that doo reade eyther Aristotle, Plato, or any other Philosophers writing. For all their examples almoste, and their probations, depende of Arithmetike. It is the saying of Aristotle, that hee that is ignorant of Arithmetike is meete for no Science. And Plato his master wrote a like

sentence ouer his Schoolehouse doore. Let none enter in hither (quoth he) that is ignorant of Geometrie. Seeing hee would haue all his scholers expert in Geometrie, much rather hee would the same in Arithmetike, without which Geometrie cannot stand.

And how needfull Arithmetike is to Diuinitie, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much do write of it. And if I should go about to write all the commodities of Arithmetike in ciuill actes, as in gouernaunce of common weales in time of peace, and in due prouision and order of armies in time of warre: For numbring of the host, summing of their wages, prouisions of victuals, viewing of Artillerie, with other armour: Beside the cunningest poynt of all, for casting of ground, for encamping of men, with such other like. And how many wayes also Arithmetike is conducible for all priuate weales, of Lordes and all possessioners, of Merchants, and all other occupiers, and generally, for all estates of men, besides Auditors, Treasurers, Receiuers, Stewardes, Bailifes and such like, whose offices without Arithmetike are nothing. If I should (I say) particularly repeat al such commodities of this noble science of Arithmetike, it were ynough to make a very great booke.

Scholer. No, no sir, you shall not need: For I doubt not, but this that you haue said, were

ynough

inough to perſwade any man to thinke this Art to be right excellent and good, and ſo neceſſarie for man, that (as I thinke now) ſo much as a man lacketh of it, ſo much he lacketh of his ſenſe and wit.

Maſter. What, are you ſo farre chauged ſince, by hearing theſe fewe commodities in generall? By likelihood you would bee farre chaunged, if you knew all the commodities particular.

Scholer. I beſeech you Sir, reſerue thoſe commodities that reſte yet behinde vnto their place more conuenient. And if yee will be ſo good as to vtter at this time this excellent treaſure, ſo that I may be ſomewhat inriched thereby, and if euer I ſhal be able, I wil requite your paine.

Maſter. I am very glad of your requeſt, & will do it ſpeedily, ſith that to learne it you be ſo readie.

Scholer. And I to your authoritie my wit doe ſubdue, whatſoeuer you ſay, I take it for true.

Perſeuerance in ſtudie.

Maſter. That is too much, and meete for no man to be beleeued in all thinges, without ſhewing of reaſon. Though I might of my ſcholer ſome credence require, yet except I ſhew reaſon, I do it not deſire. But now ſith you are ſo earneſtly ſet this Arte to attaine, beſt it is to omit no time, leaſt ſome other paſſion coole this great heat, and then you leaue off be-

foꝛe you ſee the end.

Scholer. Though many there be ſo vnconſtant of minde, that flitter and turne with euerie winde, which often begin, and neuer come to the end, I am none of their ſoꝛte, as I truſt you partly know. Foꝛ by my good will, what I once begin, till I haue it fully ended, I would neuer blin.

Maſter. So haue I found you hitherto in deed, and I truſt you will increaſe rather than go backe. Foꝛ better it were neuer to aſſay, than to ſhꝛinke and flie in the miſle way. But I truſt you will not ſo do, thereforꝛe tel me briefly. What call you the ſcience that you deſire ſo greatly?

Scholer. Why ſir? you know.

Maſter. That maketh no matter. I would heare whether you knowe, and therefoꝛe I aſke you. Foꝛ great rebuke it were to haue ſtudied a Science, and yet cannot tell how it is named.

Scholer. Some call it Arſmetrike, and ſome Augrime.

Maſter. And what doo theſe names betoken?

Scholer. That, if it pleaſe you, of you would I learne.

Maſter. Both names are coꝛruptly written, Arſmetrike foꝛ Arithmetike, as the Greekes call it, and Augrim foꝛ Algoꝛiſme, as the Arabians ſound it, which both betoken

the

the Science of Numbring. For Arithmos in Greeke, is called Number: and of it commeth Arithmetike, the Art of Numbring. So that Arithmetike is a Science or Art teaching the maner and vse of Numbring. This Art may bee wrought diuersly, with Penne or with Counters. But I will first shewe you the working with the Penne, and then the other in order.

Scholer. This I will remember. But how many thinges are to be learned, to attaine this Arte fully?

Master. There are reckoned commonly seuen partes or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuision, Progression, and Extraction of rootes: to these some men adde Duplation, Triplation, and Mediation. But as for these last three, they are contained vnder the other seuen. For Duplation and Triplation, are contayned vnder Multiplication, as it shall appeare in their place. And Mediation is contained vnder Diuision, as I will declare in his place also.

Scholer. Yet then there remaine the first seuen kinds of Numbring.

Master. So there doth: Howbeit, if I shall speake exactly of partes of numbring, I must make but fiue of them: For Progression is a compound operation of Addition, Multiplication and Diuision. And so is the Extraction

tion

tion of rootes. But it is no harme to name them as kindes seuerall, seeing they appeare to haue some seuerall working. For it forceth not so much to contende for the number of them, as for the due knowledge and practising of them.

Scholer. Then you will that I shall name them as seuen kindes distinct. But nowe I desire you to instruct mee in the vse of each of them.

Master. So will I, but it must be done in order: for you may not learne the last as soone as the first, but you must learne them in that order, as I did rehearse them, if you will learne them speedily and well.

Scholer. Euen as you please. Then to begin, Numeration is the first in order: what shal I do with it?

Master. Firste you muste know what the thing is, and then after learne the vse of the same.

Numeration.

NUmeration is that Arithmeticall skill, whereby we may duely value, expresse and read any number or summe propounded: or else in apte figures & places, set downe any number knowne or named.

Scholer.

Scholer. Why: then me thinketh you put a difference betweene the value and the Figures.

Master. Yea so doe I: For the value is one thing, and the figures are an other thing: and that commeth partly by the diuersitie of figures, but chiefly of the places wherein they be set.

Scholer. Then I muste know here three thinges: the Value, the Figure, and the Place.

Master. Euen so: but yet adde Order to them as the fourth. And firste marke, that there are but ten figures, that are vsed in Arithmetike, and of those ten, one doth signifie nothing, which is made like an o, and is priuately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying figures, and be thus figured. A Cipher.

1 2 3 4 5 6 7 8 9
And this is their value.
i. ii. iii. iiii. v. vi. vii. viii. ix.

But here you muste marke, that euerie Figure hath two values: One alwayes certaine that it signifieth properly, which it hath of his forme: and the other vncertain, which he taketh of his place.

A place is called the seate or roume that a Figure standeth in. And looke how manie figures A place.

gures are written in one ſumme, ſo manie places hath that whole number. And the firſt place muſt be called that that is next to the right hande, and ſo reckoning by order towardes the left hande, ſo that that place is laſt, that is next to the left hand. As for example: If there ſtood before you ſixe men in a rowe, ſide by ſide, and you ſhould tell them as they ſtand in order, beginning with the man that were next to your right hande: then he that were next him ſhould be called the ſecond, and ſo forth to the furtheſt from your right hande, which is the ſixt and the laſt.

Scholer. Syr, I perceyue you wel: ſo might I recken letters or any other thing. As if I ſhould write eight letters after this order, a, b, c, d, e, f, g, h, then muſt I ſay, h is the firſt, g the ii, f the iii, e the iiii, d the v, c the vi, b the vii, and a the viii.

Maſter. That is well done. And after the ſame ſort vſe hereafter, that what I declare by one example, do you expreſſe by another, and ſo I ſhall perceyue whether you vnderſtand it or no. And ſo paſſe ouer nothing, till you perceiue it well, and be expert therein.

Scholer. Sir, I pray you how many of theſe places be there in all?

Maſter. There is no certaine number of them, but they are ſometimes more & ſometimes fewer, according to the ſumme that is expreſſed. For ſo many as the figures are, ſo many are the

the places: and the laſt place is ſo called, not becauſe it is laſt of al other, but it is the laſt of that preſent ſumme, and it may be the middle place in another ſumme.

Scholer. Wee ſeemeth I perceyue this very well, as touching the order of reckoning of the places: But as for the number of them, you ſay there is no certaintie. Nowe there reſteth to declare the value of the figures by the diuerſitie of places, which you called the Value vncertaine. Value vncertaine.

Maſter. But firſt let me heare whether you know perfectly the certaine value.

Scholer. Yes ſir, as you wrote them, ſo I marked them.

Maſter. How wrote you then fiue?

Scholer. By this figure 5.

Maſter. And how ſixe?

Scholer. Thus, 6.

Maſter. Write theſe three numbers ech by it ſelfe, as I ſpake them vii. iiii. iii.

Scholer. 7.4.3.

Maſter. How write you theſe foure other, ii, i, ix, viii?

Scholer. Thus (I trow,) 2,1,6,8.

Maſter. Nay, there you miſſe: Looke on mine example againe.

Scholer. Sir, truth it is, I was to blame, I tooke 6 for 9, but I will bee warer hereafter.

Maſter. Now then take heede, theſe certaine

taine values euery figure repzesenteth, when it is alone wzitten without other figures ioyned to him. And also when it is in the firste place, though manie other do follow: as foz example: This figure 9 is ix. standing now alone.

Scholer. How: is he alone and standeth in the middle of so many letters:

Master. The letters are none of his fellows. Foz if you were in France in the middle of a M. French men, if there were no English man with you, you would reckon your selfe to bee alone.

Scholer. So it is. Then 9 without moze figures of Arithmetike, betokeneth ix, whatsoeuer other letters be about it.

Master. Euen so, and so doth it, if it be in the first place ioyned with other, how many soeuer do follow, as in this example, 3679. you see 9 in the first place, and doth betoken nine, as if he were alone.

Scholer. I perceiue that. And doeth not 7 that standeth in the second place, betoken vii: and 6 in the third place, betoken vi: And so 3 in the fourth place, betoken three:

Master. Their places be as you haue said, but their values are not so. Foz as in the first place, euery figure betokeneth his owne value certaine onely, so in the second place euerie figure betokeneth his owne value certaine ten times: as in the example, 7. in the second

place

place is seuen times x, that is, lxx. And in the thirde place, euerie figure betokeneth his owne value a hundreth times, so that 6 in that place betokeneth vi. C. And in the fourth place, euerie figure betokeneth his owne value a M. times, as in the aforesaid number, 3 in the fourth place, standeth for 3 M. And in the fifth place, euery figure standeth for his owne value x. M. times. And in the vi. place a C. M. times: and in the vii. place a M. M. times: and in the viii. place x. M. M. so that euery place exceedeth the former x. times.

Scholer. As thus: if I make this number at all aduentures, 91359684, heere are eight places. In the first place is 4, and betokeneth but foure: in the second place is 8, and betokeneth x. times 8, that is 80, In the third place is 6, and betokeneth sixe hundred: In the fourth place 9 is nine thousande. And 5 in the fifth place is x. M. times 5, that is, fiftie M. So 3 in the first place, is a C. M. times 3, that is CCC. M. Then 1 in the seuenth place, a M. M. And 9 in the eight place ten thousand thousand times 9, that is xc. M. M. But now I cannot easily nor quickly read it in order.

Master. That shall you practise by this meanes. First put a pricke ouer the fourth figure, and so ouer the seuenth. And (if you haue so many) ouer the tenth, thirteenth, sixteenth, and so foorth, still leauing two figures betweene each two prickes. And those two

roomes

Ternaries. roomes betweene the prickes, are called Ternaries.

Then begin at the last pricke, and see how many figures are betweene him and the ende, which cannot passe three: reckoning himselfe for one: then pronounce them as if they were written alone from the reste, and adde at the end of their value so many times thousande as your number hath prickes.

After that come to the next three figures, and sound them as if they were apart from the reste, & adde to their value so many times thousands, as there are pricks betweene them and the first place of your whole number. And so doo by euery other three figures following, if you haue mo. As in example, 91359684. this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the seuenth place, and then no more, (for your places come not to tenne) as thus: 91359684.

Now go to the last pricke ouer 1, and take it and the figure 9 that followeth it, & value them alone.

Scholer. 91 that is xci.

Master. So is it: but then adde for the number of your prickes twice M.

Scholer. That is xci, thousand thousand.

Master. So is it. Then take the three other figures from one to the next pricke, and value them.

Scholer.

Scholer. 359. that is CCC. lix.

Master. Now adde for the one pricke, that is betweene them and the first place, Y.

Scholer. CCC. lix. thousand.

Master. Then come to the other three figures that remaine.

Scholer. 684, that is, vi. C. lxxxiiii.

Master. Now haue you valued all. And at the ende of the last number you shall adde nothing, because there remayneth no pricke nor number after it: yet proue in an other number, as thus 2308640891053̇40.

Scholer. 2308640891053̇40. I haue pricked them as you taught me: but I am in doubt, whether I haue done well or no, because of the Cyphers: for I remember, you told me that they do signifie nothing, and therfore I doubt whether I should recken them for a figure in setting of the prickes: & againe, I know not wherefore they serue.

Master. That will I tell you now. Indeed they are of no value themselues, but they serue to make vp number of places, and so make the figure following them to bee in a further place, and therefore to signifie the more value: as in this example 90, the Cypher is of no value, but yet he occupieth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is, xc. so that two Ciphers thrusteth the figure following them, into the third place, and so forth.

Scholer. Then I perceiue in the example aboue, I haue pricked well inough: for though that Cypher that is pricked signifie nothing, yet must he haue the pricke, because he came in the viii. place. Then will I prooue to number that summe. First there is 230. M. M. M. M. and then followeth 864. M. M. M. And what shall I now doe? There is a Cipher in the third place, & no figure after him, but they that I haue reckoned.

Master. Hee did serue for them that you haue alreadie reckoned, to make them in a place further than they should be, if he were away: and therefore now you shall let him goe. And so doe alwayes when he occupieth that place next before any pricke, which is the last of that Ternarie, and a Cipher in the last place doth nothing.

Scholer. Then shall I say but 89. M. M.

Master. So, but go forth.

Scholer. 105 thousand. Now are all my prickes spent, and yet remaine 340, so that I must value them CCC. xl. onely.

Master. Now can you recken after this sorte: and remember, that euery such roume so parted, is called a Ternarie or Trinitie: for you haue numbered or valued the summe moste truely, and by the aide of the prickes each Denomination is distinct most plainely.

Scholer. What call you Denominati-

ons?

ons?

Master. It is the laste value or name added to any summe. As when I say: CCxrii. poundes: poundes is the Denomination. And likewise in saying: 25 men, men is the Denomination, and so of other. But in this place (that I spake of before) the last number of euery Ternarie, is the Denomination of it. As of the first Ternarie, the Denomination is Unites, and of the seconde Ternarie, the Denomination is thousandes: and of the thirde Ternarie, thousande thousandes, or Millions: of the iiii. thousande thousande, thousandes, or thousande Millions: and so foorth.

Scholer. And what shall I call the value of the three figures that may bee pronounced before the Denominators? as in saying: 203 000 000, that is CC iii. millions. I perceyue by your wordes, that millions is the denomination: but what shal I call the CCiii. ioyned before the millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value or number. Numerator. Summe.

Scholer. Now is there any thing else to be learned in Numeration? or else haue I learned it fully?

Master. I might shew you here who were the first inuentors of this art, and the reason of

all these thinges that I haue taught you, but that I will reserue till ye haue learned ouer all the practise of this art, least I should trouble your wit with ouer many thinges at the first.

Three kindes of number.

But yet this must you marke, that there are three kindes of number: one called Digits, an other Articles, and the thirde mixt numbers.

Digit.

A Digit is any number vnder 10, as this:

1,2,3,4,5,6,7,8,9.

Article.

And 10 with all other that may be diuided into ten parts iust, and nothing remaine, are called articles: such are 10,20,30,40,50,&c. 100,200,&c. 1000,&c.

Mixt or cõpound.

And that number is called mixt, that containeth articles, or at the least one article, and a digit: as 12,16,19,21,38,107,1005. and so forth. And for the more ease of vnderstanding and remembrance, marke this: The digit number is neuer written with more than one figure, but the Article and the mixt number are euer written with more than one figure. And thus they differ, that the article hath euermore this Cipher 0, in the first place: and the mixt number hath euer there some digit.

Scholer. By these last words, I perceiue it much better than I did before, and now (I thinke) I will neuer misse to know those three asunder.

Master.

Maſter. If you remember now all that I haue ſaide, you haue learned ſufficiently this firſt kinde of Arithmetike, called Numeration. Howbeit, I will exhort you now, to remember both this that I haue ſaid, and all that I ſhall ſay, and to exerciſe your ſelfe in the practiſe of it: For rules without practiſe, are but a light knowledge: and practiſe it is, that maketh men perfite and prompte in all things.

And as you haue learned to gather and expreſſe the value of a ſumme propounded, and ſet downe before you: ſo muſt you practiſe to marke, note, or write downe, with apt figures, and in due places, any number, onely named or recited to you, or of your ſelfe imagined: as for a proofe. How note you, or write downe this ſumme, fiue thouſand, two hundred, fiftie and ſeuen.

Scholer. This troubleth me now, whether I ſhould begin at the firſt or at the laſt. For reaſon (me thinketh) ſhould cauſe me to begin at the firſt: and yet if I write it as you ſpeake it, I muſt begin at the laſt.

Maſter. When you know your places perfectly, you may begin where you liſt. But the more eaſe for your hand is to begin with the laſt, that is to ſay, as I did ſpeake them. Yet for the more ſuretie, a while you may begin with the firſt, repeating my wordes backward thus: Seuen, fiftie, two hundred, fiue thou-

sand: or else sounding them all by their digit or valuer: as thus: seuen, fiue, two, fiue: for that way is easiest. But then must you looke well, whether there be any Cypher in your summe, that he may be set in his place. As if your last valuer of your summe (as you call it) be aboue 9, then is there a Cipher in the first place. And if it be a hundred or aboue, then is there two Ciphers, one in the first place, and an other in the second, and so forth.

But because this thing is such that can not be set foorth without many wordes, I thinke best here now at the ende of Numeration to adde a Table easie and readie for the first exercise of it.

Loe this is the Table.

1. 2. 3. 4. 5. 6. 7. 8. 9. 0

333

The left side or hand.

The right side or hand.

The names of digits, values certaine, or valuers.

The denominatours of the place or value vncertaine.	Vnites.	Tennes.	Hundreds.	Thoulands.	C. of thoulands	M. of millions.	C. of millions.	D. of millions.	M. of millions.	C. of millions.	D. of millions.
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8	8
Seuen.	7	7	7	7	7	7	7	7	7	7	7
Sixe.	6	6	6	6	6	6	6	6	6	6	6
Fiue.	5	5	5	5	5	5	5	5	5	5	5
Foure.	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cipher.	0	0	0	0	0	0	0	0	0	0	0
The order of the places.	First.	Second.	Third.	Fourth.	Fift.	Sixt.	Seuenth.	Eight.	Ninth.	Tenth.	Eleuenth.

This Table (as you may see) hath eleuen places, and in each of them are set all the digits, whose certaine value is written on the right hand of the Table, and the value vncertaine on the left hand. So that by this Table you may learne both how to expresse any number that you list, (if that exceede not

eleuen places) that is to say, ℔C. thousand millions, and so may you by the helpe of it, value all summes proposed vnder the said number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fiftie and seuen. And if you will expresse it, take the first number (as I speake it) which is fiue M. whose valuer or certaine value is 5, and his vncertaine value or denomination is M. First you shall seeke at the right hand of the valuer 5. Then seeke along vnder the title of denomination toward the left hand, till you finde thousands, and vnder it right at the foote of the Table, is the number of the place, that is the fourth, wherin you must write your digit or valuer 5.

Afterward come to the second parte of the number, two hundred, whose valuer is 2, and his denomination C. Seeke two at the right hand of the Table, and go along vnder the denomination toward the lefte hand, till you come vnder C: then looke to the Table, and there you shall see the number of the place, that is to say, three, wherein you must set your digit 2.

Then do so by your other two numbers that remaine, and you shall finde 5 in the second place for your fiftie, and 7 in the first place for your seuen. And thus may you do with other numbers.

Scholer.

Scholer. Master, I thanke you heartily. I perceiue you seeke to instruct me most plainly and briefly, and not to hide your knowledge with subtile wordes as many do. For this rule is so plaine, that I can desire it no plainer. And though it seeme somewhat long, yet I perceiue it to be a sure way.

Master. So is it, and though it be long, yet it is neyther too long, neyther too plaine for young learners that lacke practise: for this table is in stead of a teacher, to them that lacke one. But now I trust I haue said inough of Numeration: which after you haue well practised, then may you learne forth.

Scholer. Yet I pray you in one thing to tel me your iudgement. Why do men recken the order of the places backward, from the right hand to the left?

Master. In that thing all men do agree, that the Chaldees, which first inuented this Arte, did set these figures as they set all their letters: for they write backward, as you terme it, and so do they reade. And that may appeare in all Hebrue, Chaldee, and Arabike bookes, for they be not onely written from the right hand to the left, and so must be read, but also the right end of the booke is the beginning of it: whereas the Greekes, Latines, & al Nations of Europe, do write and reade from the left hand toward the right: & all their bookes begin at the left side.

Scholer.

Scholer. That reason doth satisfie me.

Master. It neither satisfieth me, neyther liketh me well, because I see that the Chaldees and Hebrues doe not so vse their owne numbers, as at an other time I will declare. But this playne reason may best satisfie you presently: That seeing in pronouncing of nũbers we keepe the order of our owne reading, from the lefte hand to the right: and againe, we doe euer name the greater numbers before the smaller: it was reason, that the lesser places cõtaining the lesser numbers, should be set on the right hand, and the greater places containing the greater numbers, to proceede toward the left hand.

Scholer. This reason is to me so plaine, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make me fully ready and expert in it. And in the meane season, I desire to learne the other kinds of Arithmetike.

Master. That is well said: but what should you next learne, can you tell?

Scholer. I remember you said that Addition was next.

Master. Euen so, and what that is, must you first know.

Addi-

Addition.



Addition is the gathering togither & bringing of two numbers or more into one totall summe: as if I haue 160 bookes in the Latine tongue, & 136 in the Greeke tongue & would know how many they be in all, I must write these two nũbers one ouer an other, writing the greatest number highest, so that the first figure of the one, be vnder the first figure of the other. And the second vnder the second, & so forth in order.

When you haue so doone, drawe vnder them a right line, then will they stande thus. Now begin at the first places, toward the right hand alwayes, and put togither the two first figures of those two numbers, and looke what commeth of them, write vnder them, right vnder the line. As in saying, 6 and 0, is 6. Write 6 vnder 6: as thus.

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160
136
---
  6
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And then go to the second figures, and do likewise: as in saying 3 and 6 is 9: write 9 vnder 6 and 3, as here you see.

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160
136
---
 96
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And likewise do you with the figures that be in the third place, saying: 1 and 1 be 2: write 2 vnder thẽ, and then will your whole summe

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160
136
---
296
```

ay-

appeare thus.

So that now you see, that 160, and 136 do make in all, 296.

Scholer. What: this is verie easie to doo: me thinketh I can do it euen since.

There came through Cheapside two droues of cattell: in the first was 848 sheepe, and in the second was 186 other beasts.

Those two summes I must write as you taught me, thus.

	848
	186

Then if I put the two first figures togither saying: 6 and 8 they make 14. That must I write vnder 6 and 8, thus.

848	
186	
14	

Master. Not so, and here are you twice deceiued. First, in going about to adde togither two summes of sundrie thinges, which you ought not to do, except you seeke onely the number of them, and care not for the thinges. For the summe that should result of that addition, should be a summe neither of Sheepe, nor of other beastes, but a confused summe of both. Howbeit sometimes ye shall haue sums of diuerse Denominations to bee added, of which I will tell you anon: but first I will shew you, where you were deceiued in an other point, and that was in writing 14 (which came of 6 and 8) vnder 6 and 8: which is vnpossible. For, how can two figures of two places be written vnder one figure, and one place?

Scholer.

Scholer. Truth it is: but yet I did so vnderstand you.

Master. I said indeede, that you should write that vnder them, that did result of them both togither: which saying is alwayes true, if that summe do not exceed a digit. But if it be a mixt number, then muste you write the digit of it vnder your figures, as I haue said before: but and if it be an article, then write o vnder them, and in both sorts you shall keepe the article in your minde. And therefore when you haue added your second figures, which occupie the place of tennes, you shall put that 1 thereto, which you kepte in your minde: for though it were ten indeede, yet in that place it is but as one, because, that euery 1 of that place, is ten, for it is the place of tens. And in like maner: if you haue in the second place so great a number, that it amounteth aboue 9, then write the digit, and reserue the article in your mind, euer adding it to the next place following: and so of all other places, how many soeuer you haue. And if you haue a mixt number, when you haue added your last figures, then write the digit vnder the last figures, and the article in the nexte place beyond them: so shall your number resulting of Addition, haue one place more than the numbers which you should adde togither.

Scholer. Now do I perceiue you, and the reason of this is, (as I vnderstand) because that

that no one place can containe aboue 9, which is the greateſt figure that is, and then all tens or articles muſt be put to the next place following: for euery place (as I may ſee) exceedeth the other place next before him, by 10.

Now (if it pleaſe you) I will returne to my example of cattell. But I remember you ſaid, I might not adde ſummes of ſundrie thinges together, and that might I ſee by reaſon.

Maſter. Trueth it is, if you ſeeke the due ſumme of any thing, but if you onely ſeeke a bare ſumme, and haue no reſpect to the thing, then were it better to name the ſumme onely without any thing, as in ſaying 848, without naming ſheepe, or any thing elſe. And likewiſe 186, naming nothing.

Now let me ſee: how can you adde theſe two ſummes?

Scholer. I muſt firſt ſet them ſo, that the two firſt figures ſtande one ouer an other, and the other each one ouer his fellow of the ſame place: then ſhall I drawe a line vnder them both. And ſo likewiſe of other figures, ſetting alwaies the greateſt number higheſt, thus, as followeth.

Then muſt I adde 6 to 8, which maketh 14, ẏ is mixt nũber: therfore muſt I take the digit which is 4, and write it vnder 6 and 8 keeping ẏ article 1 in my minde thus.

Next that do I come to the ſe-

848
186
848
186
4

cond

cond figures, adding them togither, saying, 8 and 4, make 12, to the which I put the one reserued in my minde, and that maketh 13, of which number I write the digit 3 vnder 8, and 4, and keepe the article in my minde thus.

```
 848
 186
----
  34
```

Then come I to the thirde figures, saying: 1 and 8, make 9, and 1 in my minde maketh 10. Sir, shall I write the Cipher vnder 1 and 8?

Master. Yea.

Scholer. Then of 10 I write the Cipher vnder 1 and 8, and keepe the article in my minde.

Master. What needeth that, seeing there followeth no more figures?

Scholer. Sir, I had forgottē, but I wil remember better hereafter. Then seeing I am come to the last figures, I must write the cipher vnder them, and the article in a further place after the cipher, thus.

```
 848
 186
----
1034
```

Master. So now ye see, that of 848, & 186 added togither, there amounteth 1034.

Scholer. Now I thinke I am perfect in Addition.

Master. That will I proue by this example.

There are two armies of souldiers: in the one are 106800, and in the other 9400: How many are there in both armies, say you?

Scho-

Scholer. First I set them one ouer another, beginning with the first numbers of the right hand, thus.

But the neather number will not match the ouer number.

106800	
9400	
116200	

Master. That forceth not.

Scholer. Then do I adde 0 to 0, and there amounteth 0, that must I write vnder the first place, thus.

106800	
9400	
	0

Master. Well said.

Scholer. Then likewise in the second place I adde 0 to 0, and there ariseth 0, which I write vnder the second place, thus.

106800	
9400	
	00

Then I come to ỳ third place saying 4 & 8 make 12, of which I write the digit 2, and keep the article 1 in my mind, thus.

106800	
9400	
	200

Then I adde 9 to 6, which maketh 15, to that I adde the article 1 that was in my minde, and it is 16. I write 6 vnder 6 & 9, and keepe one in my minde, thus.

106800	
9400	
	6200

Master. Why do you not write both figures, seeing you are come to the last couple of numbers.

Scholer. ☞ Nay reason sheweth mee, that I must adde that article that is in my minde, vnto the next figure of the ouer summe, though there

there be no more in the neather summe.

Master. That is well considered: then do so.

Scholer. Then say I, 0 in the ouer summe, and 1 in my mind, maketh 1, that write I ouer 0: Then followeth there yet one more in the ouer summe, which hath none to be added to it, for there is none in the neather summe, nor yet in my minde, therfore I thinke I must write that euen as it is.

Master. Yea.

Scholer. Then doth my whole summe appeare thus.

	106800
	9400
	116200

Master. If you marke this, you haue learned perfectly the common addition of all summes which are of one denomination: so that ye obserue this also, that in Addition you must haue two numbers at the least, or else how can you say that you do adde: And euer let the greatest number be written highest: for that is the best way, though it be not necessarie.

And forget not this, that if you haue manie numbers to adde together, you shall haue oftentimes an article of a greater value than 10, sometimes 20, sometimes 30, sometimes more, yea peraduenture 100. Therefore as you did with the article 10, so do with them, reseruing them in your minde, and adding to the number next following, so many as their

valuer or value certaine is: that is to say, 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and so foorth of other like. So that if the Article bee 100, then must you set downe the 0, and keepe 10 in mind to be caried to the next row of figures or place, if any such happen to come, which for your better vnderstanding take this example for all.

I would adde these xiii. sums into one, which I set after this maner: then doe I begin and gather the summe of the first rowe of figures which commeth to 107, for I take 9 there x. times and that is 90, then 9 and 8 is 17, that is in all 107, of which summe I write the 7 vnder the first rowe of figures, and then for that 100 is x. tens, I keepe x. in mind: which ten I must adde vnto the nerte rowe of figures which are in the second place,

4889
4599
2299
3699
1399
4090
1099
3198
299
699
899
499
389

which second rowe of figures when they are added together with the x. that I had in my minde, make in all 125, of which summe, I write the digit 5 vnder the second rowe, & then for that 120 conteineth xii. tens. I keepe xii, in minde to be added to the third place or rowe of figures, which being added together make in all 60, the cipher 0, I set downe vnder the rowe of figures in the third place.

And

And the figure 6 I keepe in minde to be added to the rowe of figures in ỹ fourth place, which when they are added together, make 29: The figure or digit 9 I set downe vnder the fourth place, and because it is my last worke, I set downe the 2 also that I haue in minde to the 9 in the fifth place: so those summes do make in all 29057.

	48.8.9.
	45.9.9.
	2.2 9.9
	36.99.
	2399.
	4090
	1.099.
	3198.
	29.9.
	6'9.9.
	8.9.9.
	49.9.
	3.8.9
	29057

¶ But for your more ease in worke when you haue an addition of so many summes to be added together, you were best part that summe into two or three partes, and worke them seuerall, and so put their additions together, and this were the best thing you could do, when ouer many summes fall to be added. *

Scholer. This seemeth somewhat hard, by the reason of so many numbers together. Howbeit I thinke if I doe often proue euen with this same example either by working of it alone, or else by parting it, as you said euen now, that I shall be able to do so shortly with any other summe.

Master. So shall you. For it is often practise that maketh a man quicke and ripe in all things: But because as well in great summes as in small there may chaunce, to be some er-

rour, I will teach you how you shall proue whether you haue done well or no.

Scholer. That were a great helpe and ease.

The proofe of Addition.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not hauing regard to their places, but as though they were all vnites: and still as your number increaseth aboue 9, cast away 9. Then goe forth, euer casting away 9, as often as it amounteth thereto: and so doe till you haue gone ouer all the numbers that you intended first to adde, and whatsoeuer remaineth after such addition and casting away of 9, write it in some voyde place by the end of a line for the better remembrance: and thus is the first parte of your worke proued. Then secondly put together the Figures that result of the addition vnder the line, still casting away 9 also. And then that that remaineth, write at the other ende of that line: and if those two Figures be like, then haue you well done, but if they be vnlike, then haue you missed. As for example in this present summe: The first figure of the ouer line is 9, let him goe: then 8 and 8 is 16, take away 9, resteth 7, and adde that 7 to 4 that followeth, and it maketh 11, from which if you take 9, there resteth 2. Then come to the next rowe, whose first and second numbers are 9, therefore ouerpasse them both, and take the 5 to

the

the 2 which did remayne in the first rowe, that maketh 7, put thereto the 4 following, and that maketh 11, thence take 9, and there remayneth 2. Nexte to that go to the thirde line, whose two first numbers you may let passe because they are nines, then take the two figures of 2, which with the other two that remayned in the second rowe make 6. Then goe to the fourth rowe, whose two first numbers let goe, and take the 6 to the 6 that remayned, and that maketh 12, take away 9, and there resteth 3, which with the 3 that is nexte, maketh 6. And so goe through all the other numbers, and you shall finde that there remayneth 5, after you haue caste away 9 as often as you can finde it: therefore write 5 at the ende of a line in a voyde place, thus 5———

Then gather all the Figures to the totall summe which is vnder the lowest line, and caste away 9 as often as you can finde it, as thus: 7 and 5 make 12, take away 9, there resteth 3: to that if you adde the 2 that is last (for you may omit the 9) then doth it make 5, which 5 you must write at the other ende of the line that you made in the voyde place thus. 5———5

And then you see that those two figures be like, whereby you may know that you haue done well, and so may you proove in any other.

Scholer. If it please you, I will proove in

another ſumme.

Maſter. With a good will.

Scholer. Then will I take one of your former examples, which was this.

First in the higheſt line, 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that followeth, & that maketh 6. Then come I to the ſecond line, where I find firſt 4 which with 6 maketh 10, from that I take 9, and there reſteth 1, the next figure is 0, and therefore I let him alone, ſo finde I one remaining. which I ſet at the end of a line thus. 1 ——

	106800
	9400
	116200

Then I come to the totall ſumme, and there I finde that all the figures put together make 10, from which I take 9, and there reſteth 1 alſo, which I put at the other end of the line thus. 1 —— 1

And becauſe they be like, I know that I haue well added.

Maſter. So you know now both how to adde two ſummes or more together, and alſo how to proue whether you haue doone well or no: and now will I teach you how to adde ſummes of diuerſe denominations togither: which thing can neuer be but when the one denomination is ſuch that it containeth the other certaine times. And yet you ſhall adde them to the other, not after this ſort as you did them that were of one denomination, but after ſuch

Addition of numbers of diuers denominations.

such a sorte as I will now shew you, that is to say.

If you haue a summe of diuerse denominations, then looke that ye set euerie denominations by himselfe, with some note or figure of his denomination, as they be wont to be written. Then write your other summes so vnder that first, that euery one be set vnder the other of the same denominations, as for example: if your denominations be pounds, shillings, and pence, write poundes vnder pounds, shillings vnder shillings, and pence vnder pence, and not shillings vnder pence, nor pence vnder pounds.

Scholer. Now that you haue spoken it, me thinketh it needeth not to warne me of it, for it were against reason so to confound summes: but yet if you had not spoken of it, peraduenture I should haue bene deceiued in it.

Master. If you do say it is so plaine, I will speak no more of it, but with an example make the matter to appeare euidently.

Firste, one man oweth me 22 l, 6 s̄, 8 d̄. An other oweth me 45 l, 16 s̄, 6 d̄. And an other oweth me 4 l, 3 s̄ I would know what this is altogether. Therefore must I first set downe my greater summe, and then the other, euery one vnder his denomination agreeing to the greatest summe. as here you see with a

l	s̄	d̄
22	6	8
45	16	6
4	3	

line vnder them.

Then must I begin at smallest numbers, (which must alwayes bee set next the right hande) and adde them together, and if the summe of them will make 1, or 2, or 3, of the next denomination, then must I keepe it in my minde till I come to that place, and vnder that first place must I note the residue, if there remaine any of the same Denomination, but if there remaine none, then needs I to write vnder it nothing. And this is all that you muste marke in this Addition: for all other thinges are like to the other maner of Addition before mentioned. Therefore the chiefest point of this Addition is, to know the values of common Coynes and rated summes. As how many shillings be in a pound: how many pence in a shilling, of which and of other like things I will instruct you hereafter in teaching of Reduction: But now I may not disturbe your wit from the thing that we are about.

Therefore let vs returne to that former example which I proposed of three debters, which summes when I had set orderly, they stood thus with a line vnder them.

l	s	d
22	6	8
45	16	6
4	3	

Then to adde them vnto one summe, I must beginne at the right hande, where the smallest denomination is, and adde them together first, saying: 6 and 8 make 14. Now seeing these

14 are pence, which containe one shilling and 2 pence. The 2 pence I sette downe vnder the line of pence. And the one shilling I keepe in my minde to carie to the next rowe being the place of shillings.

l	s	d
22	6	8
45	16	6
4	3	
		2

Then doe I adde the shillinges together, saying 1 in minde and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the second place which standeth for 10, make 26, which is 1 pound, 6 shillinges. The 6 shillinges I set downe vnder the place of shillings, as appeareth in the example. And the one pound I keepe to carie to the pounds.

l	s	d
22	6	8
45	16	6
4	3	
	6	2

Then come I to the poundes, adding them altogether, saying 1 that I keepe, and 4 make 5, and 5 make 10, and 2 make 12. The figure or Digit 2 I set downe right vnder that place or rowe of poundes where I gather them. And the Article 1 I keepe to carie to the next place, saying 1 in minde, and 4 is 5, and 2 is 7, which 7 I set downe directly vnder that rowe also. And then appeareth my whole summe thus.

l	s	d
22	6	8
45	16	6
4	3	
72	6	2

And thus must you do with any such like summes whatsoeuer, whether they be money, weight

weight or measure, which if you practise your selfe well therein by setting downe of diuerse summes, you shall be well acquainted with the feate of Addition.

But now can you tell how to proue this Addition or such other like of diuers denominations, and to trie whether you haue well done or no?

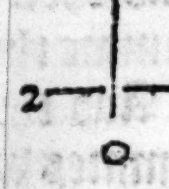
Scholer. I would I could.

Master. That shall you do by this meanes.

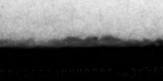
Proofe of addition of diuers denominations.

You must make a crosse which shall haue so many lines as you haue sundrie denominations in your Addition: As if you haue but two Denominations, then you may make it thus, that the ouer parte and the neather part may serue for one denomination. And if you haue three denominations, as pounds, shillings and pence, then muste you make three lines thus. The vpright line may serue for poundes, and the highest thwart line for shillings, and the lowest for pence: as for example by our summe which we last wrought.





£	s	d
22	6	8
45	16	6
4	3	0
72	6	2



For

For the proofe of the which, because it containeth three denominations, I must make a crosse of 3 lines, as in the example before. The I reckon first at the right hand the pence: 6 & 8 make 14, from which I take 12 for the next denomination, that is to say, a shilling, & there resteth 2, which I must write at one end of the neather thwart line:

After that I gather the summe of the shillings, 3, 16, 6, which maketh 25, to them I put 1 that I tooke of the pence, and that maketh 26: from those I take 20, the quantitie of the nert greater denomination, that is to say, a pound, and there resteth 6, which I write at the end of the highest thwart line.

Thirdly, I adde togither the pounds, 4, 5, and 2, which make 11, to them I adde the 1 that came of the shillings, and they make 12, from whence I cast 9, and there resteth 3. That three I ioyne to the 4 in the next place, and they make 7, then the 2 more make 9, which I cast away, and so there is left 0, which 0 I set at the vpper end of the crosse also. (And so should there be set also any number that were left vnder 9.) And thus is the first part of my worke prooued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I finde but 2, and cannot take 9 from him, therefore I set him at the other end of the neather thwart line: Then I

come

come to the shillings, where I finde onely 6, which because it is lesse than 9, I set it at the other ende of the line of shillings, that is, the ouermost thwart line.

Last of all, of the 72 l, I take eight times 9, which is 72, and there remaineth 0, which I write vnder the vpright line, either else I may reckon them simplie without any respect of their valuation or place: in saying 2 and 7 make 9, which I cast away, and so resteth 0:

Then I consider euerie number, comparing it to the number that is against it, and because I finde them to be euery one like his match, I know that I haue well done.

Scholer. This crosse I perceiue doth serue for those three denominations, poundes, shillings, pence. But what if I had l, s, d, ob. and q.

Master. These lines as I haue said, doe serue for three denominations, such as they be: as here they do serue for pounds, shillings, and pence: but if yee haue no pounds in your sum, then may they serue for shillings, pence, and halfe penies: yea for d, ob, & q.: or in weight for C. q. and l, or in measures for Elles, quarters, and nailes, if you haue no greater denomination, so that you remember that the vpright line serueth for the greatest denomination, and the highest thwart line for the next, and the lowest for the least.

And so if you haue foure Denominations

you

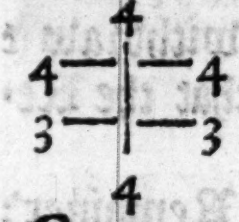
you must make your crosse with so many lines. And if that your summe be of moze denominations, make so many lines in your crosse. And thus will I make an ende of Addition, sauing that here foz the better vnderstanding of this Rule, I haue set you downe certayne examples both of money, weight, and measures, with their wozkes and proofes.



Examples of Addition.

l.	s.	d.
23	10	4
45	6	8
37	2	9
15	13	6
121	13	3

l.	s.	d.
130	17	10
28	6	8
13	13	4
120	0	0
292	17	10



The proofes.



C	q.	l.
24	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards	q.	nailes
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3



The proofes.



Subtraction.

Scholer.



When haue I learned the two firſt kinds of Arithmetike: now, as I remember, doth follow Subtraction, whoſe name me thinketh doth ſound contrary to Addition.

Maſter. So is it in deede: for as Addition encreaſeth one groſſe ſumme by bringing many into one, ſo contrary waies, Subtraction diminiſheth a groſſe ſumme by withdrawing of other from it, ſo that Subtraction or Rebating is nothing elſe, but an Art to withdraw & abate one ſumme from another, that the Remainer may appeare.

Scholer. What do you call the Remainer?

Maſter. That you may perceiue by the name,

Scholer. So me thinketh: but yet it is good to aſke the truth of all ſuch things, leaſt in truſting to mine owne coniecture, I be deceiued.

Maſter. So it is the ſureſt way. And as I ſee cauſe, I will ſtill declare thinges vnto you ſo plainely, that you ſhall not need to doubt. Howbeit, if I doe ouerpaſſe it ſometimes (as

the

the maner of men is to forget the small knowledge of them to whom they speake) then doe you put me in remembrance your selfe, & that way is surest.

And as for this word that you last asked me, take you this description: The Remainer is a summe left after due Subtractiõ made, which declareth the excesse or difference of the other two numbers: as if I would abate or subtract 14 out of 18, there should remaine 4, which is called the Remainer, and is the difference betweene those two numbers 14 and 18. Remainer.

Scholer. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Master. That shall you do by this meanes. First you must consider, that if you should go about to rebate, you must haue two sundrie summes proposed, the first which is your grosse summe or summe totall: (& it must be set highest) & then the rebatement or summe to be withdrawen, which must be set vnder the first (whether it be in one parcel or in many) and that in such sort, that the first figures be one iust ouer an other, & so the second and third, and all other following, as you did in Additiõ: then shal you draw vnder them a line, & so are your summes duely set to begin your working.

Then begin you at the right hand (as you did in Addition) and withdrawe the neather number out of the higher, and if there remaine

maine any thing, write that right vnder them beneath the line: and if there remaine nothing (by reason that the two figures were equall) then write vnder them a Cipher of nought. And so do you with all the other figures, euermore abating the lower out of the higher, and write vnder them the Remainer still, till you come to the ende. And so will there appeare vnder the line what remaineth of your grosse summe, after you haue deducted the other summe from it, as in this example.

I receyued of your father 48 s̃, of which I haue layde out for you 36 s̃: now would I know what doth remayne, and therefore I set my numbers thus in order: First I write the greatest summe, and vnder him the lesser, so that the figures at the right side be euen one vnder another, and so the other, thus.

	s̃
	48
	36

Then do I rebate 6 out of 8, and there resteth two, which I write vnder them right beneath the line, thus.

	48
	36
	12

Then I goe to the second figures, and do rebate 3 out of 4, where there remaineth 1, which I write vnder them right, and then the whole summe and operation appeareth thus.

	48
	36
	12

Whereby it appeareth, that if I withdraw

36

36 out of 48, there remaineth 12.

Scholer. Now will I proove in a greater summe: And I will subtract 2367924 out of 3468946. Those sums I set in order thus.

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3468946
2367924
1101022
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Then doo I begin at the right side, and deduct 4 out of 6, and there resteth 2, which I write vnder them. Then goe I to the second figures, and withdrawe 2 out of 4, and there remayne two, which I set vnder them also: then I take 9 out of 9, and there resteth 0, which I write vnder them: for you say, that if the figures be equall, so that nothing remayne, I must write this Cypher 0 vnder them.

Master. It was well remembred, now go forth.

Scholer. Then I come to the fourth place and drawe 7 out of 8, and there remaineth 1, which I write vnder them also. Then in the fifth place I take 6 from 6, and there resteth nought, for it I write vnder them a cipher, 0: Then in the 6 place 3 rebated from 4, there remaineth 1, which I write vnder them: and likewise in the vii. and last place, 2 taken from 3, there is lefte 1, which I write vnder them: so haue I doone my whole worke, and my summes appeare thus: Whereby

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3468946
2367924
1101022
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I see, that if I rebate 2367924, out of 3468946, there remaineth 1101022.

Master. This is well doone. And that you may be sure to perceiue fully the Art of Subtraction, let me see how can you subtract 52984732 out of 8250003456.

Scholer. First I set downe the greatest summe, and after that I write vnder it the lesser number, beginning at the right side: and then my figures will stand thus.

8250003456
52984732
8197018724

Then take I 2 from 6, and the rest is 4, which I write vnder them: then doo I withdraw 3 from 5, and there remaine 2, which I write vnder them. Then take I 7 out of 4, but that I cannot, what shall I now doo?

Note. Master. Marke well what I shall tell you now, howe you shall doo in this case and in all other like. If any figure of the neather summe bee greater than the figure of the summe that is ouer him, so that it cannot be taken out of the figure ouer him, then must you put 10 to the ouer figure, and then consider how much it is, and out of that whole summe withdrawe the neather figure, and write the rest vnder them. Can you remember this?

Scholer. Yes, that I trust I shall. Now then in mine example where I should haue taken 7 out of 4, and could not, I put 10 to

that 4, which maketh 14, from it I take away 7, and there resteth 7 also, which I write vnder them.

Master. So haue you doone well, but now muste you marke an other thing also: that whensoeuer you doo so put 10 to any figure of the ouer number, you must adde one still to the figure or place that followeth next in the neather line, as in this example there followeth 4, to which you muste put 1, and make him 5, and then go on as I haue taught you. Note.

Scholer. Then shall I say: 4 and 1 (which I must put to him for the 10 that I added to 4 before) make 5, which I should take out of 3, but that cannot be, therefore must I put to it also 10, and then it will be 13, from which I take 5, and there resteth 8 to bee written vnder them: and because of that 10 added to the 3, I must adde 1 to 8 that followeth in the neather line, & that maketh 9, which I should take out of 0, and cannot, therefore I put thereto 10, and that maketh 10: from 10 I take 9, and there remayneth 1, which I write vnder them.

Then doo I adde one likewise to the nert figure beneath, which is 9, and that maketh 10, that 10 should I take out of the figure aboue, but I cannot, for it is 0, therefore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to bee written vnder them.

Then come I to the next figure, which is 2, and to him do I adde 1, which maketh 3, that 3 I cannot take out of nought, therefore of that nought I make 10, and thence doo I take 3, so remaineth there 7 to be written vnder them. Likewise doe I put 1 to 5 that followeth, and then is it 6, that would I take out of 5, and cannot, therefore I adde 10, to that 5 and make it 15, from which I rebate 6, and there remaineth 9, which I write vnder them. Now haue I spent all the neather figures, and what shall I doo more?

Master. You should haue added 1 to the next figure following (if there had beene any) because you added 10 to the last figure before of the ouer line: but seeing there is no figure following, you must adde that 1 to the place following, & then deduct that 1 from the number aboue.

Scholer. Then shall I say because I borrowed 10 to the ouer 5, I must put 1 in the next place beneath, that is vnder 2: then must I subtract that 1 from 2, and there resteth 1, to be written vnder that, in the ninth place. Now I haue no more to subtract, for there is not any figure remaining beneath, neither yet any vnite to be added, because I borrowed not 10 to the figure last before, and yet is there 8 remaining in the ouer line, which (I thinke by reason) should bee set at the ende of the Figures in the lowest rowe which is vnder

the

the line, for because there was nothing taken from it.

Master. That is well considered, and reason teacheth so in deede.

Scholer. But sir, I beseech you, shal I alwaies when any number so remaineth alone (as this 8 did) write him vnder ỹ line straight against his owne place?

Master. Yea, what elle? Whether they be one or many: and this well remembred, you haue sufficiently learned Subtraction. Howbeit, because of certaine thinges that might deceiue you, if you did not take good heede to your working, I will propose to you an other example of many numbers to be subtracted, as thus.

I receiued of a friend of mine to keepe 2869 crownes, of which at one time I delinered him againe 500, at an other time 368, and at an other time 440, and at another time 80, and another time 64: now would I know how many doo rest behinde. Therefore first I set downe my grosse summe, and a line vnder it: and vnderneath it I set all the parcels, thus: and vnder them a double line.

2869	*Crownes receiued.*
500	*Delinered.*
368	
440	
80	
64	

Then first I begin at the first place, and gather togither the summe

of all those lines (saue the ouermost) in their first figures, and so I doo with all the figures of the second place, and so fooyth, as I did in Addition, saue that I leaue out the highest rowe of numbers (as the line warneth mee)

Note. and that summe so gathered betweene the double line, is the summe deliuered in all, which summe I do afterwardes subtract out of the highest rowe of numbers, and the remayner do I set vnder the neathermost line: as foy example.

I set the summes as before: then do I gather the first figures of all the parcels deliuered togither, where I finde but 4 and 8, that make 12 (foy thyee Ciphers increaseth no summe in Addition, as you learned before) of the 12, therefoye doo I wyte the digit 2 betweem the double line, and keepe the article 1 in my minde, till I come to the second place, where I finde 6,8,4,6, that make 24, to them I put the article in my minde, and it is 25, of which I wyte 5 vnder the second place, and keepe the digit 2 in my minde foy the thirde place, where I finde 4, 3, 5, that make 12, to the whiche I adde the 2 in my minde, and it maketh 14, thereof I wyte the 4 vnder the third

2869	*Crownes receiued.*
500	*Deliuered.*
368	
440	
80	
64	
1452	*Deliuered in all.*
1417	*Reſt behind.*

place:

place: and because there remaineth no more figures to be added, I write the digit 1 in the fourth place, as you see in the example, and so it appeareth I haue deliuered in all 1452 crownes.

Then come I to the subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceed to subtract it from the ouermost summe, saying: 2 from 9, remaine 7, to bee written vnder them beneath the lowest line. Then in the seconde place I take 5 from 6, and there resteth 1, to be written vnder them. Then in the third place, 4 from 8, resteth 4. Last of all in the fourth place, 1 from 2 remaineth 1. And thus I see that after those 5 sums are subtracted from 2869, the Remainer is 1417.

Master. This is a very sure and easie way for a learner: therefore I counsell you to practise it well.

Scholer. This I perceiue: but is there no shorter way and more speedier?

Master. Yes, when you are a while exercised in it: for you may, as fast as you can gather the numbers together, withdraw them out of the highest summe. But if in quantitie those numbers added together, exceede the highest summe or vpper number then shall you (as before hath bin taught you) imagine to borow 10, 20, or 30, or mo, as neede shall re-

An abridgemēt of the former manner of subtraction.

quire, and put them to the vpper number, to helpe to further the abatement, reseruing or restoring the Articles that you borowed to the next place againe, and so still go forwarde till you haue ended your worke, as for example: In the laste summe proposed, I gather first in the first place 4 and 8, that maketh 12: which 12 I should deduct or take out of 9 in the vpper number aboue the line. But I cannot, therefore I adde to 9 an Article of 10, and that maketh the vpper number 19: from whence I take 12, then there resteth 7. Then for the Article 10, I adde 1 to the nexte place of money deliuered: saying, 1 that I bring, and 6 make 7, and 8 make 15, and 4 make 19, and 6 make 25, which 25 I should take out of 6 in the vpper number, but I cannot. Therefore I adde 2 tennes or 20 vnto 6 in the vpper number, and that maketh 26, then 25 out of 26 resteth 1, then the 2 tennes which I borrowed, or haue in mind, I adde to the next row, or summe deliuered, saying: 2 that I bring, and 4 make 6, and 3 make 9, and 5 make 14, then 14 out of 8 I cannot take, but 14 out of 18, resteth 4. Now because there are no more places to be added, the 1 that I borowed or haue in my mind, I rebate from 2 in the vpper line, and there remaineth 1 which I set downe in the remainer line, and so my summe appeareth, as before, to be 1452 crownes. Loe thus haue you now a shorter way.

Scho-

Scholer. I like both wayes well, and I perceiue both well, yet as in the one the worke king seemeth somewhat long, so in the other it leaneth verie much (nee seemeth) to remembraunce, and therefore may cause errour quickly, except a man haue a quicke and an exercised remembrance. But yet for the sharping of my wit, by your patience, if you will giue me leaue, I will trie what I can doo in such a like summe, to worke it the shortest way: whereupon I would subtract out of 40308964, these three parcels

	40308964
20003428	20003428
10002342	10002342
10101461	10101461

Therfore I set them first in order due: & then I gather the parcels of the first place, which are 8, 2, 1, that is 11, which I should take or deduct out of 4, which is ouer him, but I can not, therefore I adde an article or one ten vnto 4, which maketh 14, then 11 out of 14, there resteth 3, to be written vnder the first place betweene the two lines.

Then come I to the second place, saying: 1 that I borowed or haue in minde, and 6 make 7, and 4 make 11, and 2 make 13, which I cannot take from 6, therefore I adde 10 to 6, which maketh 16, then 13 from 16, resteth 3, which I write vnder the secõd place betweene

the

the two lines.

Then come I to the third place, saying, one that I borowed or haue in minde, and 4 make 5, and 3 is 8, and 4 make 12, which I should take out of 9 that is ouer them, but I cannot, therefore I adde 10 to 9, which make 19, then 12 out of 19, resteth 7.

Then come I to the fourth place, saying, one in minde and 1 is 2, and 2 make 4 and 3 make 7, which because it is a digit, I take from 8, and there remaineth 1.

After that I come to the fifth place where are onely three Ciphers, which make nothing, then should I take that, that is to say nothing, from the figure ouer them, which is also a Cipher, therefore I must say thus: if I take nought from nought, there remaineth nought: so must I write a cipher vnder them. Then in the first place I finde but 1, which I take out of 3 ouer him, and the Remainer is 2, that must be written betweene the two lines in the first place. So I go to the seuenth, where I finde onely ciphers, and in the grosse summe ouer them a cipher also, therefore must I write their Remainer (which is nothing) with a cipher also. Then in the eight and last place, I gather 1, 1, 2, that make 4, which if I take out of that 4 that is ouer them, there will nothing remaine. And that must be noted with a cipher betweene the two lines, as I haue often said, and so haue I ended my worke, and

the

the figures ſtand thus.

But Syr, I remember you taught me that Cyphers ſhould not come in the laſt place, for becauſe they ſerue onely to increaſe the value of other figures which follow thē, and ſerue not thoſe figures that goe before them: and now in my example I haue ſet two Ciphers in the two laſt places.

40308964
20003428
10002342
10101461
00201733

Maſter. I commend you for your remembrance. And trueth it is, you ſhould not haue ſet them here, but onely becauſe that I would make you plainely to perceiue the Art of Subtraction. Therefore ſeeing that you do now perceiue it, whēſoeuer you ſhould write downe a Cipher, looke whether any other figures be yet behind. And if not, then let go the Cipher alſo, for it needeth not to write him in any latter places, where no other figure doth follow, except it be (as I did now ſuffer you) to teach the vſe of Subtraction the plainer.

Therefore your figures muſt ſtand thus when the worke is ended.

40308964	*Charge.*
20003428	*Diſcharge.*
10002342	
10101461	
201733	

Scholer. Syr I do think with that that you taught me before and by theſe two ſummes that you taught

taught me last also, that now I could subtract any summe.

Master. So may you, if you haue marked what I haue taught you. But because this thing (as all other) must be learned surely by often practise, I will propound here two examples to you, wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly, for in them is contained all the obseruances of whole number. And because you shall perceiue somewhat both how to do it, and also whether it be well done when you haue proued to do it, therefore haue I written vnder them both the Remainers.

30606 *Lent.*		30606 *Debt.*	
10354	*Repaide.*	10354	*Paide.*
10249		10249	
163		163	
20766 *Paid in al.*		9840 *Rest.*	
9840 *Rest to pay.*			

Scholer. Sir, I thanke you. But I thinke I might the better doo it, if you did shew me the working of it.

Master. Yea, but you must prooue your selfe to do some thinges without my aide, or else you shall not be able to do any more than you were taught: And that were rather to learne

learne by rote (as they call it) than by reason. And againe there is nothing in this example or any other of whole number, but I haue taught you the rules of them already, sauing I haue not yet shewed you how you shall proue this worke.

Scholer. Sir I pray you then shewe it me also.

Master. For the performance whereof if you marke well what I said in Addition, you may easily perceiue what is to bee doone for the proofe of Subtraction, & that is onely performed by the aide of Addition, thus. Draw vnder the lowest number (which is your Remainer) a line, and then adde the summe paid in all (which before was made by the particulars) and the remainer together. And if these twaine being added together in one summe, do make the contents of the vpper number aboue the line, then is the Subtraction well wrought, or else not.

For the triall whereof I will propone one of our examples which are done, which was of 2869 crownes receiued to keepe of a friend.

Crownes 2869 *Received.*

Of which at sundrie tymes I deliuered these particular sums

500
368
440
80
64

which

which particulars in one Addition make, 1452. *deliuered in all.*
so resteth to my frīēd 1417. *Rest to pay.*
2869 *Proofe.*

The proofe of Subtraction.

Now put or adde this remainer, and the summe deliuered together, saying: 7 and 2 make 9, and 1 and 5 make 6, and 4 and 4 is 8, and 1 and 1 is 2, which in one summe amounteth to 2869, and is equall with the sum 2869 aboue the line, whereby I know the subtraction is well wrought, and this is the order and proofe hereof: and of all such like workes of subtraction. *

Scholer. Then I trust by practise to attaine the vse of it. And this is all that I shall learne of Subtraction.

Master. Yea, sauing that (as you haue seene in Addition) there are numbers of diuers denominations, in which the working is not much vnlike, yet without some instructions be giuen of it, it might seeme to a learner more difficult, than in deede it is. Therefore I will briefly shewe you the vse of it onely by one example or two.

A certaine man owed to me 14 ℔, 12 s̄, 8 d̄, of which he paide me at one time 4 ℔, 6 s̄, 8 d̄, at an other time 3 ℔, and at an other 2 ℔, 3 s̄, 4 d̄, and last of all, 6 s̄, 8 d̄.

Now would I know what remaineth vnpaide yet, therefore I set my summes thus, e-

uerie

uerie one in their due place: As pounds vnder pounds, shillings vnder shillinges, pence vnder pence.

℔	s̄	d̄
14	12	8
4	6	8
3	0	0
2	3	4
	6	8

Scholer. Sir, I pray you why doo you write 2 ℔, for the common speach vseth rather to say 40 s̄.

Master. Wee must here vse the Denomination that is greatest in any summe, so that we may not write according as we vse to speake, saying: 16 d̄, 18 d̄, or likewise, 7 grotes, 8 grotes, 24 s̄, 40 s̄, 48 s̄, and such other, but wee must write euery denomination that is in any summe by it selfe, namely shillings and pounds. So must we write for the last summes now named, 1 s̄, 4 d̄: 1 s̄, 6 d̄: 2 s̄, 4 d̄: 2 s̄, 8 d̄: 1 ℔, 4 s̄: 2 ℔, 8 s̄: and so forth of other like.

Note how the pen differeth frō common order of counters.

Scholer. So that we may not write in Arithmetike pence, when the summe amounteth to shillings, nor shillings when the summe maketh pounds. Now if it please you, end your example.

Master. When my summes are so set as I shewed, then according to the rules of Addition I gather all the particular summes which he paide me into one totall summe: directly to be set vnder them betweene the two lines, not medling with the 14 ℔, 12 s̄, 8 d̄, as the line warneth me, therefore must I first begin with

the

the ſmalleſt denomination, ſaying, 8, 4, 8, is 20 d. which maketh one ſhilling and 8 pence: the 8 d I ſet downe vnder the place of pence, and the one ſhilling I keepe in minde to carie to the nexte Denomination of ſhillings. Then come I to the ſhillinges, and ſay, one that I bring or haue in minde, and 6 is 7, and 3 is 10, and 6 make 16, which becauſe it conteineth not one poũd, I ſet directly vnder the place of ſhillings. Then come I to the pounds whoſe parcels are 2, 3, 4, that is in all 9, that 9 doo I ſet downe directly vnder the pounds: And ſo the totall or whole Addition of all the particulars paide, amounteth to 9 l. 16 s. 8 d.

l	s	d	
14	12	8	
4	6	8	
3			
2	3	4	
	6	8	
9	16	8	
4	16	0	*Reſt.*

Now for the worke of Subtraction I muſt rebate that totall ſumme of Addition out of the higher number, that is to ſay, from the 14 l. 12 s. 8 d.

Therfore to performe the worke, I ſay 8 d out of 8 d, remaineth or reſteth nothing, therefore in the place of reſt or remaine, right vnder that denomination I ſet downe a 0. Then comming to the ſhillings, where I finde 16, which ſhould be taken out of 12, but I cannot, therefore I imagine to borrow one of the nexte de-

denomination, that is of the 9 l, and put that one pound so borowed vnto 12 s, that maketh 32 s.

Now 16 s out of 32 s resteth 16 s, which 16 s I set downe directlie vnder the place of rest.

Lastly comming to the pounds, saying, one pound in mind that I borowed, and 9 make 10, then 10 out of 14, there resteth 4.

So doth my whole rest or remaine appeare to be 4 l, 16 s, 0 d.

This I account the easiest way for a yong beginner to practise, though it be something long.

Scholer. Is there then any shorter way for this worke also?

Master. Yes, as in this last example I will also shewe you, for you may adde together the particular summes as they are set in order: beginning with the pence, saying, 8, 4, 8, make 20 d, which 20 d you should take out of the 8 d aboue the line, but you cannot: therefore shall you borow one of the next denomination, that is to say, one of the shillings, and put it to the 8 d that maketh 20 d, now 20 d out of 20 d, rest 0, which Cipher I set downe directly vnder them.

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

Then one shilling that I borowed or had

in minde, and 6 make 7, and 3 make 10, and 6 make 16, then 16 out of 12 I cannot take, therfore of the nert denomination I do borowe one pound, and put it to 12 s̄, which maketh 32 s̄, then 16 s̄ out 32 s̄, resteth 16 s̄.

Lastly I come to the pounds, saying: one pound in mind or that I borowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there resteth 4.

So doth my remaine or rest appeare as before to be 4 l̄, 16 s̄, 0 d̄.

Scholer. This doe I perceiue very well, and if there be none other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckened to be the nexte order.

Master. We haue doone in deede with the Art of Subtractiō, as touching the working. But yet before wee go to Multiplication, I will instruct you how to examine your worke whether it be well doone or not. For the performance whereof, if you marke what I said right nowe in the last maner of proofe, you may easilie perceiue what is to be doone for this proofe, which is onely made (as before was taught you) by the aide of Addition, thus.

The proofe of Subtraction, in diuers denominations.

Drawe vnder the lowest number (which is your Remainer) a line, and then adde the summe paide in all, and the Remainer together. And if these twaine added together in one summe make the contentes of the vpper num-

number aboue the line, which in this example is 14 ℔, 12 s̄, 8 d̄. Then is the Subtraction well wrought or else not.

As for example, in our first summe which stood thus.

℔	s̄	d̄	
14	12	8	
4	6	8	
3	0	0	
2	3	4	
	6	8	
9	16	8	*Paid in al.*
4	16	0	*Rest.*
14	12	8	

Where in the title of pence I finde 8 and 0: the 8 I set downe directly vnder him in the place of pence.

Then in the place of shillinges I finde 16 and 16, which make 32 shillings, wherein is conteined one pound and 12 s̄, the 12 s̄ I set downe directly vnder them in their due place of shillings, and one pound I keepe.

Then comming to the pounds, saying: one that I keepe, and 4 is 5, and 9 is 14, which 14 in due order I set downe directly vnder them, as this figure sheweth. Which is also 14 ℔, 12 s̄, 8 d̄, agreeing with the vpper number aboue. So I finde the worke is good, & the Subtraction well wrought.

Now for the proofe of the latter Subtraction which we reckoned for the shorter worke. Drawe vnder the Remainer also a line, then adde that remainer and all the other numbers that ye did subtract before together, and write that that amounteth vnder the lowest line.

And if the summe that commeth thereof be equall to the highest number aboue, then is the Subtraction well wrought, or else not.

As for example also in the last summes which stood thus.

£	s	d
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
9	16	8
4	16	0
14	12	8

First in the title of pence I adde 8, 4, 8, that maketh 20 d, which containeth one shilling and 8 pence.

The 8, I set downe vnder the lowest line in the rowe or title of pence, and the one shilling I keepe to carie to the next denomination or place of shillings.

Then returning to the shillings, saying: one in minde, or that I keepe, and 16 make 17, and 6 make 23, and 3 make 26, and 6 make 32 shillings, which amounteth to one pound, 12 s, the 12 s I set downe vnder the title of shillings, £ 1 pound I keepe or haue in minde to carie to the nexte denomination or place of pounds. Then come I to the pounds, saying, one that I bring, and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then do I write 14 vnder the pounds, and so haue I ended the Addition, and I see that the lowest line is like vnto the vppermost line in number, wherefore I know that I haue well doone.

And thus now haue I taught you the Arte of Subtraction, & the meanes to proue whether

ther it be well wrought, or not.

Many other worke and proofes might bee shewed of Subtraction, but one of these, whether of them you please, are most aptest & best allowed of any other worke or proofe, whether it be of l', s, d, or any other grosse summe whatsoeuer.

Scholer. Sir, I thanke you most heartily, for now I vnderstand well both the feate of Subtraction and his proofe.

Master. Therefore here will I make an end of Subtraction, and will instruct you in Multiplication.

Multiplication.



Multiplication is such an operation, that two sūs product ỹ third: which third sum so manie times shal containe the first, as there are vnites in the second. And it serueth in stead of many Additions. As for exāple. When I would know how many are 30 times 48: if I should adde 48 thirty times, it would be a long worke. Therefore was this worke of Multiplicatiō deuised, which shall do ỹ at once

that Addition should do at many times.

Scholer. I perceiue the commoditie of it partly, but I shall not see the full profit of it, till I know the whole vse of it. Therefore sir I beseech you teach me the working of it.

Multiplication of Digits.

Master. So I iudge it best, but because that great summes can not be multiplied, but by the multiplication of digits, therefore I thinke it best to shew you first the way of multiplying them: As when I say, 8 times 8, or 8 times 9, &c. And as for the small digits vnder 5, it were but follie to teach any rule, seeing they are so easie, that euery child can do it. But for the multiplication of the greater digits, thus shall you do.

First set your digits one ouer the other right, then from the vppermost downeward, and from the neathermost vpward, drawe straight lines, so that they make a crosse, commonly called Saint Andrewes crosse, as you see here. Then looke how many each of them lacketh of 10, and write that against each of them at the ende of the lines, and that is called the difference: as if I would know howe many are 7 times 8, I must write those digits thus.

The difference.

Digit difference.

8

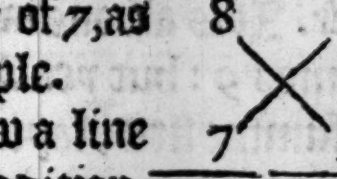
7

Then doe I looke howe much 8 doth differ from 10, and I finde it to be 2, that 2

doe I wrıte at the right hand of 8, at the ende of the line thus.



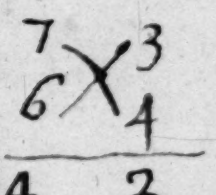
After that I take the difference of 7 likewise from 10, that is 3, and I wrıte that at the right side of 7, as you see in this example.



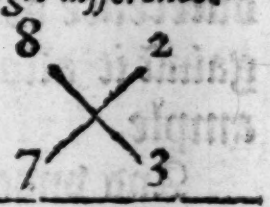
Then doe I drıaw a line vnder them, as in Addition, thus.

If y differences multiply above 20 ffor seldom if w it above & carry one to y digits & 26

Last of all, I multiplie the two differences, saying, 2 times 3 make 6, that must I euer set vnder the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other digit (not from his owne) as the lines of the crosse warne me, and that that is left must I wrıte vnder the digits. As in this exaumple. If I take 2 from 7, or 3 from 8, there remaineth 5: that 5 must I wrıte vnder the digits: & then there appeareth the multiplication of 7 times 8, to be 56. And so likewise of any other digits, if they be aboue 5, for if they be vnder 5, then will their differences be greater than themselues, so that they can not be taken out of them. And againe, such little summes euerie childe can multiplie, as to say: 2 times 3, or 4 times 5,



Digit difference.



and such like.

Scholer. Trueth it is. And seeing me seemeth that I vnderstand the multiplying of the greater digits, I will proove by an example how I can doe it. I would know how many are 9 times 6.

Master. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying: 6 times 9, and so of all other summes.

Scholer. Then would I knowe, howe many are 6 times 9: therefore I set the digit thus, and make the crosse thus.

9
×
6

Then do I set their differences from 10 at the right side: the differēce of 9 which is 1 against it, & the difference of 6 which is 4 against it also, as in this example.

9 1
×
6 4

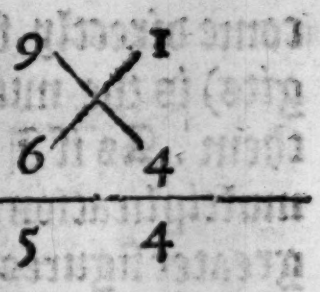
And vnder them I drawe a line. Then do I multiplie the digits together, saying: one time 4 maketh 4, that 4 doe I write vnder the differences thus.

9 1
×
6 4
———
4

Then take I one of the differences from the other Digit, as one from 6, or else 4 from 9, and eache wayes there resteth 5, which I doe write vnder the

digits

digits thus. And so appeareth the multiplication of 6 times 9, to be 54. Thus I seethe feate of this maner of multiplication of digits.



Master. Now might you goe straight to the multiplication of greater numbers, saue that both for your ease and suretie in working, I will draw you here a Table, whereby shall appeare the multiplication of all digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last digit in the greater figures, and from it go right forth toward the right hand, till you come vnder the number of your second digit, which is in the highest rowe: and then the number that is in the meeting of the rowes of little squares (which

come

come directly from both your propounded digits) is the multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then go right fourth toward the right hand, till you come vnder 9 of the highest rowe, in which place where you so come vnder the other digit (as here for example you come vnder 9) is alwayes contayned the ofcome or product, which you seeke: and that place we terme to be in the common angle, in respect of the two numbers so taken on the outsides, as here in that common angle, where the rowes of litle squares (directly proceeding frō 7 & 9) do meet, you haue 63, which 63 is the sum of this multiplication of 9 by 7.

Scholer. This is verie good and readie. And so may I finde the multiplication of any digits. But now how shall I doe in greater summes?

Master. When you would multiplie anie sum by another, you shall marke that it is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number vnder it, for that is the place of the multiplier, or Multiplicator, that is to say, the number by which multiplication is made: and is in English alwaies put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times,

is

is that which must be multiplied.

Therefore when I would multiplie one number by another, I must write the greatest highest, and the lesser vnder it, as in Addition. And vnder them must I draw a line. As for example: If I would multiplie 264 by 29, I must set them thus.

264
29

¶ Of which numbers thus set downe to be multiplied, may be formed a question as thus: There are 29 men, and each man hath 264 lambes. The question is how many lambes they haue in all. *

To the performance whereof I must multiplie euerie figure of the higher rowe by euerie figure of the neather row: and that that amounteth, I must set vnder the line, as thus. First I do multiplie 4 by 9, saying: 9 times 4 (or 4 times 9, which is all one) and that maketh 36, as the Table before of digits doth declare, of that 36 I must write the 6 that is the digit, vnder the 9, and the Article 3 I keepe in minde to carie to the next place.

264
29
6

Then come I to the second figure of the higher row, which is 6, and say: 9 times 6 make 54, and with the 3 in my minde make 57, the 7 I set downe vnder the 2, and the 5 I keepe in minde.

264
29
76

After

After that I come to the next figure which is 2, and doe multiplie it by 9, and that maketh 18, and with 5 that I haue in minde maketh 23, wherefore because it is the laste worke of that multiplier. I set it downe in order as you see.

264
29
2376

And so haue I ended the first figure of the multiplier. Wherefore I giue it now a fine dash with my pen.

Then begin I with the next figure, and multiplie it into all the higher figures, as thus.

264
29
2376
8

First 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the digit or first figure of Multiplication that amounteth of the first figure of the higher number, must be set vnder the multiplier of it, and the other in their order, toward the left hand.

Scholer. I vnderstand you thus: that the digit of the summe amounting of the Multiplication of the first figure of the higher rowe, by the first figure of the lower rowe or multiplier, must be set vnder the first place: and that that amounteth of the same first figure by the second multiplier, must be set vnder the second place, and so of the other, if there be more multipliers.

Master. So meane I in deed: and if there amount but a digit, then must it be set vnder the

the multiplier.

And now to goe foorth: I multiplie by the same 2, the second figure of the higher rowe, which is 6, saying: 2 times 6, make 12: whereof I write the digit 2 vnder the third place, and the article 1 I keepe in mind.

Then do I multiplie the last figure of the higher summe by that same 2, saying: 2 times 2 is 4, and with the one that I haue in minde maketh 5, which 5 I write vnder the fourth place. And so haue I ended the whole multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched anie Digit by which I multiplie: And the summes stande thus.

Then must I drawe a line vnder all those sums that amount of the multiplication, and must adde all them into one summe, as in this Example you may see.

264	
29	
2376	
28	
264	
29	
3376	
528	
264	
29	
3376	
528	
8656	

Where in the first place I finde but 6, and therefore write I it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and so foorth, as you learned in Addition. And so appeareth the whole summe to be 8656, which amoun-

amounteth of the Multiplication of 264 by 29, and that is the iuſt number of Lambes the 29 men had.

Scholer. If there be no more to be obſerued in it, then can I doe it, I ſuppoſe, as by this example I ſhall proue.

¶ There is a peece of ground which conteineth 1365 yardes in length: And 236 yards in breadth, I would know how many yardes ſquare there is in all this peece of grounde, which numbers I ſet downe with the greater aboue, & the leſſer vnder, as you ſee.

Then doe I multiplie 5 by 6, ſaying: 6 times 5 make 30: of which I write the Cipher in the firſt place and the Article 3 I keepe in minde to carie to the nexte place.

Then doe I by the ſame 6 multiplie the ſecond figure of the higher ſumme, which is 6, ſaying: 6 times 6 make 36, and 3 in my mind make 39, of which I write the 9 vnder the ſecond place, and the article 3 I keepe in minde.

Then do I multiply the third figure which is 3, by the ſame 6, and that maketh 18: and with 3 in my mind make 21. Then 1 I ſet downe, & 2 I keepe in minde.

1365	
236	
0	
90	
190	

Then

Then come I to the last figure of the higher summe, and multiplie it by 6, saying: 6 times 1 make 6, and 2 in my minde make 8, that 8 doe I wzite vnder the fourth place. And so haue I ended the first multiplier, and dash him slightly with my pen.

Then begin I with the second multiplier, and say first 3 times 5, that maketh 15, of which I set the 5 vnder the second place, because that the multiplier is there, and the article 1 keepe in mind.

Then come I to the second figure, that is 6, and multiplie it by 3, which maketh 18, and with one in minde maketh 19, the 9 I set downe vnder the third place, and 1 I keepe in mind.

Then come I to the third figure, which is 3, and multiplie it by 3, saying: 3 times 3 make 9, and with one in my mind make 10, the Cipher I set vnder the fourth place, and the article 1 I keepe in minde.

And then comming to the last figure 1, I multiplie it by 3, and it maketh 3, and with the one in mind it maketh 4: which 4 I set in the fift place, and then haue I

```
1365
 236
8190

1365
 236
8190
   5

1365
 236
8190
 95

1365
 236
8190
 095

1365
 236
8190
4095
```

ended

ended two of the multipliers, and the summes stand as you may see in the latter ende of the page going before. And then I giue this dash.

Then come I to the thirde multiplier, and multiplie it into euery figure of the higher sum, and first I say: 2 times 5 make 10, of which I set the cipher vnder the multiplier in the thirde place, and the Article 1 I keepe in minde.

1365
236
8193
4095
0

And so multiplying the second figure 6, by that same 2, there amounteth 12, and with 1 in my minde make 13, wherof I write the digit 3 vnder the fourth place, and the article 1 I keepe in mind.

1365
236
8190
30

Then do I multiplie the last 2 by the third figure of ỳ higher summe, which is 3, and that maketh 6, and with the one in mind make 7, which 7 I set downe vnder the fifth place, as appeareth by the example.

1365
236
8190
4095
730

Then come I to the last place, and multiplie that 1 by 2, and there amounteth 2, which I set in the first place, and then doeth the summe stand thus.

1365
236
8190
4095
2730

And so haue I ended the whole

whole multiplication.

But now(as you taught me) to know what this whole sum is, I must adde al those parcels together, and then vnder the line will appeare as you may see, the grosse or totall summe, that is, 322140. Whereby I know there is so many yardes square in that peece of ground. *

	1365
	236
	8190
	4095
	2730
	322140

Master. This is well doone, and like a workeman.

Scholer. Then me thinketh I could call it well doone, when I know whether I had well doone or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by Diuision, and therfore I will reserue that proofe by Diuision till you haue learned the Arte of Diuision. And anon I will shew you how it is commonly proued.

¶ But first for your further instruction in this exercise of Multiplication, I will with one example more trie your cunning, and so make an ende. And the question is this. I would know how many dayes it is since the Natiuitie of our Lord and Sauiour Iesus Christ vnto this present yeare, 1590. Which to performe, you must multiplie by the daies in one whole yeare, which are 365.

Scholer. Now for that you haue giuen me

so much light into the question, you shall see I will handsomely finish the work, for according to your former instructiõs I set them downe with a line vnder them, thus.

Then say I 5 times 0 is 0, the 0 I set down vnder the first place, as here appeareth: Then say I 5 times 9 make 45, the digit 5, I set downe in the second place vnder 9, and the article 4, I keepe in minde to be added to the next multiplication. Then saying 5 times 5, make 25, and 4 in mind makes 29, the 9 I set downe in the third place, and 2 I keepe in minde. Then comming to the last figure I say once 5 is 5, or 5 times 1 is 5, and 2 in minde make 7, that seuen do I set downe vnder the fourth place. And thus haue I ended my first multiplier, and therefore I giue it a dash with my pen.

1590	
365	
7950	

Then come I to the second multiplier, which is 6, and do likewise multiplie it into the vpper number, saying, 6 times 0, is 0: the 0 I set downe in the second place right vnder his multiplier. Thē say I 6 times 9 make 54, the 4 I set downe vnder the third place, and 5 I keepe in minde. Then say I 6 times 5, make 30, and 5 I keepe in minde make 35, the digit 5 I set downe in the fourth place, and 3 I

1590	
365	
7950	
9540	

keepe

keepe in minde. Then say I 6 times 1 is 6: or once 6, is 6: and 3 in minde make 9, which I set downe next: and so haue I ended two multipliers: wherefore I dash the 6 with my pen.

Then begin I to multiplie the third multiplier into the ouer number, saying, 3 times 0, is 0: the 0 I set downe in the third place right vnder his multiplier. Then say I 3 times 9 make 27, the Digit 7 I set downe in order next, and 2 I keepe in mind, then I say 3 times 5, is 15 and 2 in mind is 17: the 7 I set downe and one I keepe. Lastly I say once 3 is 3, and 1 I keepe is 4, which I set downe orderly next: And so haue I ended the Multiplication, and my figures stand thus.

	1590
	365
	7950
	9540
	4770

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe, it will be 580350, which is the grosse or totall summe of that Multiplication, and declareth the number of daies since our Lord and Sauiour his Incarnation, vnto the end of 1590 yeares, besides 397 daies and 12 houres, for leape yeares.

Scholer. This is maruelous me thinketh, that such great matters may so easily be atchieued by this Art, which heretofore I euer thought had beene impossible, as infinite sortes

of people are of that minde.

Master. Truth it is that knowledge hath no greater enemie than ignorance, for this is one of the least of ten thousand thinges that may be doone by this Art, as hereafter you shall be able to iustifie.

Scholer. This maner of multiplication I perceiue, if there be no more in it.

Master. Yes there are other formes and helpes for ease and shorter labour of the worke of Multiplication, but I will remit them till you haue a litle tasted Diuision, where also the like helpe into Diuision may be vsed, and so therefore vnder one example for both, will I shew you both ease in Multiplication and also in Diuision.

But sith the other formes and workings do nothing differ from these workes in effect, but onely in setting of the numbers, I will ouerpasse them till a more meeter place and time. And now will I instruct you in Diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholer. Yes sir I thanke you, but I doe not perceiue how to examine my worke, to trie whether I haue well doone or no, therefore as you promised me ere while, I pray you first shew me how I shall proue it.

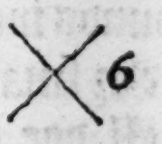
Master. That is commonly vsed by the proofe of 9, as you learned before in Addition, sauing that it differeth from that forme in di-

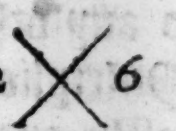
uers

uers respects: As for example.

First you must make a crosse after this manner. 

Then must you examine your sum that should be multiplied, and looke what remaineth after casting away of 9, that set you at the one side of the crosse: then examine the multiplier, and whatsoeuer remaineth in it, after casting away 9 so often as you can, write that at the otherside of the crosse: then must you multiplie those two numbers together, and looke what amounteth thereof, if it be vnder 9, write it at the higher parte of the crosse: but if it be aboue 9, then take thence 9 as often as you can, and write the rest at the head of the crosse. As first for example, wee will proue the exãple you put forth of the peece of ground that contained 1365 yards in length and 236 yards in breadth.

Therefore first I cast away all the nines from the summe to be multiplied, saying: 5 and 6 make 11, cast away 9, rest 2, then 3 and 2 make 5, and 1 is 6, that 6 I write at the one side of a crosse thus. 

Then do I examine the multiplier, which is 236, wherein when the 9 is cast out, there remaineth 2, that 2 therefore I set at the other side of the crosse. 

Then doe I multiplie 6 by 2, and it ma-

keth 12, from which 12 I withdrawe 9, then resteth 3, that 3 doe I set at the head of the crosse. Then doe I examine the grosse summe amounting of the Multiplication, which is 322140 where I finde 9 once, and 3 remaining, that 3 I set at the foote of the crosse, & then I see it to agree with the other 3 at the top of the crosse, & so know I that I haue doone well: for if they two did differ, then were my worke vayne, and the multiplication false.



This is the cõmon proofe, but the most certaine proofe is by Diuision, of which I will anon instruct you.

Scholer. Sir, what is the chiefe vse of multiplication?

Master. The vse of it is greater than you can yet vnderstand: howbeit, these playne commodities it hath, that if you would resolue any great and whole value into many small and lesse portions: as if you would change pounds into shillings, peuce or any other greater or smaller parcels, by Multiplication, yee shall doe it speedily and easily. Also if you should neede to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more speedily, readily, easily and surely, than by often and sundrie Additions. Take you these commodities grossely shewed for an answere at this time,

and

and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholer.



Well Sir, then in Diuision I pray you to instruct me. But me thinketh by the name of it, that it should bee all one with Multiplicatiō: for I call that Diuision, when any thing is parted into diuerse and many partes.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrary to Multiplication, and doth not parte one thing or fewe thinges into manie, but contrarie wayes it bringeth many parcels into fewe, but yet so, that these fewe taken together, are equall in valure to the other many: for by Diuision pence are turned into shillings, and shillings into pounds: as for example of 120 shillings, it maketh 6 pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the denominators, you shall see that

they are such, that one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they doe farre differ, and the latter summe is the lesser, and so is it alwaies in Diuision: howbeit, yet in the working, the summe is parted by another, & thereof doth it take the name.

Scholer. I thinke I shall better vnderstand the reason of the name, when I know the vse of the worke, therefore now would I gladly learne that.

Diuision what it is.

Master. Diuision is a distributing of a greater summe by the vnites of a lesser. Or Diuision is an Arithmeticall producing of a third number, in respect of two propounded numbers: which third number shall so often containe an Unite, as the greater of the two propounded numbers doth containe the lesser. So that, euen as Multiplication did seeme to serue in stead of many Additions, so Diuision may seeme to be in place of many Subtractions: Because the third number briefly expresseth, how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more plainely appeare. Therefore (as you may perceiue) vnto Diuision are required three numbers: the first, which should be deuided, and that must (generally) be the greater: and the second, by which the other must bee diuided, and that is (generally) the lesser, and is called

the

the Diuisor. And the third which answeareth to the question. How many times: and therefore is called the Quotient.

The first must be first written, and the second so set vnder it that the last figure of the lower number be right vnder the last of the higher, contrariwise to the workes of the other kindes of Arithmeticke: for in them the two first figures were set euer meete one vnder the other, but in Diuision the last figures must be set meete, except it chance so, that the last figure of the Diuisor be greater than the last of the higher number, for then you shall set the last of the Diuisor, vnder the last saue one of the higher number, as for example. A general rule for placing the figures. An exception.

a. know how manie monthes there be in a yeare then

If you should diuide 365 (which are the summe of the dayes of a yere) by 28, which are the daies of a common moneth, then should you set them thus.

365
28

b. know how manie weeks there be in a yeare then

But if you would diuide those 365 dayes, by 7, which is the number of ~~weekes~~ daies in one ~~yeare~~ weeke, then should you set them thus.

365
7

Likewise if I would diuide the same 365 by 4, which is the summe of the quarters of a yere, then must I set them thus.

365
4

Scholer. Sir, this doe I vnderstand, but how now should I doe to diuide the one by the other?

Master.

☞ Master. You must begin with the last figure next the left hande, and see how many times the last figure of the Diuisor may be taken out of the last figure of the ouer number, and that shall you note within a crooked line toward your right hand. As for example.

I would diuide 365 by 28, then set I those two summes thus.

365 (
28

And I looke how many times I may finde 2 (which is the last figure of the Diuisor) in 3, (which is the last of the number to be diuided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

Quotient numbers.

1
365 (1
28

Then must I goe to the next figure of the Diuisor, and take it likewise so many times out of the figures that be ouer it, & looke what doth remaine, that I must write ouer them, and cancell them, as in this example.

Therefore now doe I take once 8 out of 16, and there remaineth 8, which I must set ouer the 6, and cancell or crosse out the 16, and the 8 of the Diuisor: And then will the figures

gures stand thus.

And so haue I once wrought. 18

Scholer. So I perceiue that 365 (1
you take the neather figure not 28
onely out of the other that is
right ouer him, but out of that with the other also that remaineth before, and are written toward the left hand.

Master. So must you do: for you must so take the Diuisor out of the ouer number, that there remaine not ouer it so great a summe as it selfe is, for then were your worke in vayne.

But yet againe here must you marke, that when you seeke how many times the laste figure of the diuisor may be found in the number ouer him, that you looke also whether you may as often finde all the figures following in those that are aboue them, (considering all the remainders, if there be any) if not, take your Quotient lesse by one, and then proue againe, and so still, till you finde a meete Quotient: And by that meet Quotient must you alwaies multiplie your diuisor, and the product set vnder your diuisor, so that his first figure stande vnder the first figure of your diuisor, and the second vnder the second, and so fooyth: and then subtract that product from the number to be diuided, that standeth directly ouer it, as you haue seene me doe. Note.

When you haue thus wrought once, then must

must you begin againe, and write your diuisour a new, nearer toward the right hand by one place, as in this example, you shall set 2 vnder 8, and 8 vnder 5, thus:

```
 1 8
3 6 5 (1
2 8 8
  2
```

Then (as before) seeke how many times you may take your deuisour out of the number ouer him now.

Scholer. That may I do here 4 times.

Master. Truth it is that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholer. No, neither yet once.

Marke how to consider this kind of remainer.

Master. Therefore take 2 out of 8, once lesse.

Scholer. That is 3 times.

Master. Well, then 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following, make 25, in which summe I find 8 three times also, and therefore I take 3 as a true quotient, and write it within the crooked line of the Quotient, before the one thus.

```
 1 8
3 6 5 (13
2 8 8
  2
```

Then say I: 3 times 2 make 6, then 6 out of 8 resteth 2, therfore I cancel the 2 and the 8, & write ouer it the 2 that doth remaine, thus.

```
 1 8
3 6 5 (13
2 8 8
  2
```

Then

Then doe I take 8 as many times out of 2, saying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1: so then I cancell 25 and 8, and ouer the 5 I set 1, thus.

2
1 8 1

Or you might (after you found 3 to be a fit Quotient) straight way haue multiplied the whole diuisor 28, by that 3, at once: which giueth 84, which beyng set vnder 28, and duely subtracted from 85, of the number diuided, giueth 1, the remainer of the whole diuisiō: as before you had. Worke which way you list: here you see also the forme.

3 6 5 (13
2 8 8
2
8 1
3 6 5 (13
2 8 8
2
8

And now haue I doone with diuiding, for I can finde my diuisor 28 no more in the ouer summe.

Scholer. No except you would part the 1 that remaineth into 28 partes.

Master. That is well said, and so must we doe in such cases, when there remaineth any thing: but I will let that passe now, and will make you perfect in diuision of whole numbers, and will hereafter teach you particularly of broken Numbers, called Fractions.

Now if you doe perceiue the order of Diuision, then do you diuide this summe 136280 by 452.

Scholer. First I set downe the number

that

that should be diuided, then doe I set the diuisour vnder it, so that the last figure of it bee right vnder the last figure of the ouer number. Then will it bee thus.

1 3 6 2 8 0
4 5 2

Master. Can you take the last of your diuisour (which is 4) out of 1, which is the last of the ouer number?

Scholer. I had forgotten, because the last of the Diuisor cannot be taken out of the last of the ouer number, in so much as it is the greater, therfore must I set the diuisor one place more forward, toward the right hand thus.

1 3 6 2 8 0
4 5 2

And then must I looke how often I may finde the last figure of the Diuisour (that is 4) in 13, which thing I may doe 3 times, therefore doe I say: 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then doe I make at the right hand of my summes a crooked line, and write before it my Quotient 3: and I cancell 13 and 4, and ouer the 3 I set the 1 that remaineth, & then the figures stand thus.

1
1 3 6 2 8 0 (3
4 5 2

Then doe I multiplie the same Quotient into euerie figure of the diuisor, and withdraw the summe that amounteth out of the numbers

bers ouer them, as first I say: 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth, thus.

```
  1 1
1 3 6 2 8 0  (3
    4 5 2
```

Thus doe I say likewise, 3 times 2 makes 6, which I take out of 12, and there resteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I write the 6 that remaineth, thus.

```
  1 1 6
1 3 6 2 8 0  (3
    4 5 2
```

Then shoulde I set forward the diuisor, into the next place toward the right hand, thus.

```
  1 1 6
1 3 6 2 8 0  (3
    4 5 2 2
      4 5
```

Master. But you may see, that ouer the 4 is no figure, therefore must I set the Diuisor yet forwarder by an other place.

And marke, whensoeuer it chaunceth so, that you should set forward the diuisor, & that it cannot stande there, because there is no number ouer the last place, or if there be any, it is lesser than the last figure of the Diuisor, then must you remooue the diuisor yet once againe: and because that his first place of remouing serued not to subtract him so much as once, therefore you shall write in the quotient a cipher 0. And if you should by chance neede to doe so oftentimes, for euery time write a

cipher

cipher in the Quotient. The reason of this, will I shew hereafter.

Scholer. Then must I set my summes thus.

And because I remoued ỹ diuisor, so that I ouerskipped one place, I must write a Cipher in the quotient: & then must I seek a new quotient, as in this example I must say. How many times 4 is there in 6 (and sith it can be but once, therefore doe I write 1 in the quotient: and then say I: 1 time 4, taken out of 6, remaineth 2, I cancell the 6 and the 4, and write 2 ouer thē, thus.

Then say I againe, once 5 out of 28, remaineth 23, I let the 2 stand as it did, and ouer that 8 I set 3, cancelling the 8 and the 5 vnder it, thus.

```
116                    116                    116
136280 (              136280 (301            136280 (301
45252                 45252                  45252
                      4                      4
                      2                      2
                      116                    116
                      136280 (301            136283
                      45252                  45252
                      4                      4
```

Master. You might as well haue said, once 5 out of 8, and so remaineth 3, but now go forth.

Scho-

Scholer. Then once 2 out of 0, can not be, what shall I now doe?

Master. Borow of the next number that is behinde, (for there is 230) and doe as you learned in Subtraction in a like case.

Scholer. Then must I borow 1 of the 3 comming behinde next, and make that 0 to be 10: & then take I 2 out of 10, and there resteth 8. And because I borowed one of the 3, I must cancell the 3, and write 2 ouer it: thē doth the figure stand thus.

22
1136280 (301
452
4

Master. Now haue you doone, and yet remaineth 228, and your quotient sheweth you, that if you diuide 136280 by 452, you shall find your diuisor in your greater number 301, that is CCC times and once, and 228 remaining.

And in the other example, where I diuided 365 by 28, the quotient was 13, and 1 remained: whereby I know that in a yeare (which containeth 365 dayes) there are 13 monethes reckoning 28 dayes (or 4 weekes) iust to a moneth, and 1 day more.

Scholer. Why then do we call a yeare but 12 monethes?

Master. Of that at a more conuenient time will I fully instruct you: but now it is not conuenient to entangle your minde with o-

ther things, than doe directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short maner of diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shewe you certaine other proper points touching it.

Scholer. Then I pray you, yet tell me, how I shall examine and trie my worke, whether I haue doone well or no, that though no man be by to tell mee, yet I may perceiue it my selfe.

Master. Some men (yea and commonly) doe trie it by the rule of 9, as in all the other kindes, saue that their order is. First they cast away 9, as often as they can, out of the diuisor, and that remaineth, they set at one side of a crosse: As in our first example, the diuisor was 28, from which you may take 9 three times and 1 remaineth, which they set by a crosse, thus.



Then doe they likewise examine the quotient (which in our example is 13) and from thence they caste away 9 as often as they can, and the remainer they set at the other side of the crosse, and then multiplie they together those two remainers: and to it that amounteth, they adde the remainer of the Diuision, if there were any, from that whole sum they withdrawe 9 as often as they can,

and

and the reſt they ſet at the head of the croſſe: as in our example the quotient is 13, from which take 9, and there remaineth onely 4, & therefore muſt you ſet 4 at the other ſide of the croſſe, thus.



Then multiplie 4 by 1, and it yeeldeth but 4, thereto adde the remainer of the diuiſion (which was 1) and it will be 5, which ſumme doth not amount to 9, & therefore muſt be ſet wholly at the head of the croſſe, as you ſee here.



And this nũber on the head of the croſſe, is the firſt proofe, to which if you finde another like in the number that was diuided, then haue you done well.

Therefore now ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foote of the croſſe: and if it be equall to that in the head of the croſſe, then haue you well done, or elſe not.

As in our exãple the whole ſum was 365, which maketh 14, from that take 9, & there reſteth 5, which ſet at the foot of the croſſe, thus.



And you ſhall ſee that they agree: therefore haue you well done.

Scholer. Now will I likewiſe examine

our second example, where the deuisour was 452, which maketh 11: from thence I take 9, and the 2 that remaineth I set at the right side of a crosse, thus.



Then examine I the quotient, which was 301, where I finde but onely 4, that doe I set at the other side of the crosse thus.



Then do I multiplie 4 by 2, and it maketh 8, to that doe I adde the remainer of the diuision (which was 228, and maketh 12) & they two make 20, wherein I finde twise 9, and 2 remaining, that 2 must I set at the head of the crosse, thus.



Then doe I examine the whole number to be diuided, which was 136280, where I find twise 9, and 2 remaining, which I set at the foote of the crosse, thus.



And because that it doeth agree with the figure at the head of the crosse, I know that the diuision was well wrought.

Maister. This is the common proofe, howbeit, the more certaine working is by the contrary kinde, as to proue Diuision by multiplication, thus.

Againe of the proofe of Diuision.

Multiplie the quotient by the diuisor, and

if

if the summe that amounteth be equall to the summe that should be diuided, then haue you well diuided, else not.

Howbeit, this must you marke, that if there remained any thing after the Diuision, that must you adde to the summe that amounteth of the Multiplication: as in our first example the quotient was 13, and the Diuisour was 28: Now multiplie the one by the other, and the summe will be 364: to that if you adde the one that remained after the Diuision, then will it be 365, which was the summe that should be diuided, and therefore I know that I haue well doone.

Scholer. Now will I proue the same in the second example whose diuisour was 452, and the quotient 301: these doe I multiplie together, and there amounteth 136052, to which if I adde the 228 that remayned, then will it be 136280, which was the whole summe to be diuided, and therefore I perceiue that I haue well doone.

Master. This is the surest way, to examine Diuision by Multiplication: and contrariwise the surest proofe of Multiplication, is by Diuision.

And therfore according to my promise, now will I shew how you may proue Multiplication by Diuision.

A proofe of Multiplication by Diuisiō.

When you haue ended Multiplication, and would know whether you haue well doone

or not, set the grosse summe that amounteth of the multiplication ouermost, and diuide it by the multiplier: and if the quotient be the same number that should be multiplied, then haue you well wrought, else not: as in that example where we multiplied 264 by 29, the grosse summe was 7656.

Now if you will know whether that multiplication be true, you shall diuide that 7656 by the multiplier 29, and you shall perceiue that the quotient will be 264, and that is a token that you haue well wrought.

Scholer. By your patience I will proue that: and first I set downe the grosse summe and the multiplier, not after the rule of multiplication, but after the rule of diuision, for now that number is become the diuisor, that was before the multiplier, I shall set thē therfore thus.

7656	
29	

Then shall I seeke how many times 2 in 7, that may be 3 times, and 1 remaineth: but then may not 9 be found so often in 16, therefore must I take a lesser quotient, that is to say, 2: then say I twise 2 maketh 4, which I take out of 7, and there remaineth 3, then doe I cancell 7 and 2, and ouer 7 I write 3, and in the quotient I set 2, so the figures stād thus.

3	
7656	(2
29	

Then say I fourth, 2 times

9 make

9 make 18, which I abate out of 36, and there resteth 18, then cancel I 3, and ouer him set 1, and likewise I cancell 6 and 9, and ouer them I set 8, so that thus stand the figures.

1
3 8
7656 (2
29

Then doe I set forward the diuisor by one place, and seeke a new Quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times, but then can I not finde 9 so many times in 5, therefore I take a lesser quotient, as to say, 8: but yet is that too great: for if I take 8 times 2 out of 18, there remaineth but 2, and I can not finde 8 times 9 in 25, therefore yet I take a lesser quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seeke a lesser quotient, as to say, 6: then say I, 6 times 2 make 12, that I take out of 18, and there remaineth 6, so I cancell the 18 and the 2, & write 6 ouer 8, thus.

1
16
38.
7656 (26
299
2

1
16
381
7656 (26
299
2

Then say I forth: 6 times 9 maketh 54, that take I out of 65, and there remaineth 11, and the figures stand thus.

Then must I set forth the diuisor againe, and seeke a new quotient, which will be 4: for

though I may find 2 in 11 fiue times, & 1 remaine, yet I can not finde 9 so often in 16, therefore I set the figures thus.

And the 4 in the quotient I multiplie into the figures of the diuisor, saying: 4 times 2 maketh 8, which I take out of 11: and there resteth 3, therefore I cancell the 11 and the 2, and set 3 ouer the first place of 11, thus.

And then doe I say forth, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing, so that the quotient of this Diuision, where 7656 is diuided by 29, is 264, which doth declare that if 264 be multiplied by 29, the sum will be 7656. And thus I perceiue now how both Multiplication is prooued by Diuision, & Diuision also by Multiplication.

```
1
16
381
7656 (264
2999
22

1
163
381
7656 (264
2499
22
```

Master. Now haue I ended the fiue common kindes of Arithmetike: for as touching Mediation, Duplation, Triplation, and such other, they are no seuerall kindes of Arithmetike, but are contained vnder the other: for Mediation is contained vnder Diuision, and is nothing else but dividing by 2: and so are Duplation & Triplation contained vnder Multiplication: for Duplation is nothing else but multiplying by 2, & Triplation is multiplying

by

by 3, of which I will onely propose an example, for the rules you haue heard already.

If you would mediate or diuide into 2, this sum, 4531010, you shal set 2 for the diuisor, worke as you learned before, as thus.

An exãple of Mediation.

```
4531010 ( 2
2
```

Then I finde 2 in 4 two times, therefore my quotient must be 2: so I cancell 4 and 2, and remoue the diuisor forward, thus: as the worke requireth, & as before in diuision hath beene declared.

```
4531010 (2
 22
```

Which mediation or diuision by 2, beeing finished, you shall haue for your Quotient 2265505, which is the halfe of 4531010, as you may trie by duplation: for double that quotient, or multiplie it by 2, and the same number will amount.

Duplation.

I wil no lõger tarie about these, seeing they are but members of the other kinds. But here now according to my promise I wil teach you certaine easie formes both of Multiplication & of Diuision, and first of Multiplication.

Easie formes of Multiplication.

If you would therefore multiplie any sum by 10, you shall need to do no more but adde a cipher before his first place, as for example: 36 multiplied by 10, make 360.

Likewise if you would multiplie any sum by 100, put two ciphers at his beginning.

So if you would multiply any sum by a thousand, adde three ciphers to the beginning of it.

Scholer.

Scholer. This do I well perceiue, and also the reason of it.

Master. I will omit all reasons til our next meeting, when I shall tell you the reasons of all other partes of Arithmetike also: and as to our matter now, looke (as I haue told you) that you both remember it, & also often practise it.

And now haue you learned how to multiplie easily by 10, 100, 1000: and of like maner may you do with any other of that sort.

But now, if you will multiplie by 20, 30, 40, and so forth: or by 200, 300, and such like, where there is one cipher in the first place, or many orderly in the first places, you shall take away those ciphers, and multiplie the summe onely by the other figure or figures, (if they be many) and then at the beginning of the summe that amounteth, you shall set so many ciphers as you tooke away. #

Example of 2873, which I would multiplie by 300. First I omit the 2 ciphers frō the multiplier, & I multiplie the summe by onely 3 that is left, & it amounteth to 8619: before which I put the two ciphers that I before omitted or tooke away, and then is it 861900. And that is the summe that amounteth, when 2873 is multiplied by 300.

Scholer. And if there were two or more figures beside the ciphers, I must onely take away the ciphers, and multiplie by the other figures

Vnlesse in progresse of diuision by multiplication or reason in diuision some figures must be taken away after by reason of a cipher or ciphers in the first places of the diuisor or else in stead of the cipher or ciphers put to the beginning of the figure that was taken away, as diuide 6428 by 20: you must take away 7 and diuide the rest by 2 & in the beginning of the quotient by 2 you must put the 7 to beginning of the same amounting

figures, as I learned before: as if I would multiplie 93648 by 25000, I should take away the three ciphers, & multiplie the same by 25, and then at the beginning of that totall summe, should I adde the three ciphers againe.

Master. Euen so: but and if it chance the number that should be multiplied, or both the summes as well the number that should be multiplied as the multiplier, to haue ciphers in the first places, euermore omit the Ciphers, and worke by the rest. But remember to restore as many ciphers to the amounting summe, as you bated before, as in this example: 30200 shall be multiplied by 206: I shall onely take away the two ciphers from the greater number, and then multiplie 302 by 206, and afterward adde the two ciphers againe. But if I would multiplie the same 30200 by 2060, I shall not onely take away the two Ciphers from the number that should be multiplied, but also I may take away the one Cipher from the multiplier, and then must I adde 3 ciphers to the summe that amounteth: but take heed that you take away no Cipher that commeth after any signifying figure, as in this last example, you may not take away that in the fourth place of the higher number, neither that in the third place of the multiplier: howbeit, yet this you may doe: If one cipher or more come in the middest

of

of your summes, you may multiplie by the other figures, and ouerskippe them, but so, that you giue euerie figure his due place, as thus: I will multiplie 3026 by 2004, therefore I set them thus.

3026
2004
12104

And thus doe I multiplie them: First 4 times 6 make 24: I set the 4 vnder the first place, and keepe the 2 still in my minde: Then say I againe: 4 times 2 maketh 8, and the 2 that is in my minde maketh 10, I set downe the cipher 0, and keepe the article 1 in my minde. Then 3 times 4 is 12, ~~and the 1 in my minde maketh 13~~, I set downe the figure 3, and keeping the 1 still in my minde hauing no more places of the vpper number to multiplie it withall: I put it downe next 2 in the fift place.

But now when I come to the next place being a cipher 0: I let it go, because it multiplieth nothing: and likewise the second cipher.

But then when I doe come to the 2, and multiplie it into the 6 of the ouer number, you must take heede (according as I taught you in Multiplication) that the first number amounting of the multiplication, be set right vnder the multiplier, and the other orderly toward the left hand, according as you may see in this example,

3026
2004
12104
6052
6064104

which

then because you haue nothing more to multiplie but the cipher we maie be omitted, set downe the one in your mynd.

which being finished with the Addition thereof gathered togither, will stand as this example sheweth.

Which is in deede wrought so much sooner and shorter by ouerskipping of the two ciphers: which otherwise if the same example were wrought at length, it would haue had two workings more as by the same example here also set downe doth appeare.

			3026		
			2004		
		12104			
		0000			
	0000				
6052					
6064		104			

Scholer. Sir, I thanke you: for I see great ease in this way of Multiplication, and if you can shewe me such like in Diuision, you shall greatly further me.

Master. Yes, I will teach you some easie wayes in Diuision also, and first this: If you would diuide any summe by 10, you shall onely with your pen make a square line, betweene the first figure of your summe and the second, and then haue you done: for the whole number that followeth the line, standeth for the quotient, and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, will stand thus.

364 | 8

Where 364 is the quotient, and betokeneth that so many times are 10 in 3648: and the 8 after the line, is the remainer, which cannot be diuided into 10, but by breaking it into

into fractions, wherwith I wil not medle yet.

And so likewise if you would diuide any summe by 100 with your pen, you shall cut away the two first figures, and if ye would diuide by 1000, you must cut away the 3 first figures, and so of any other diuisor, whose last figure is 1, and the other be ciphers, looke how many ciphers the diuisor hath, and so many figures at the beginning shall you cut away with the squire line, and they stand alwayes for the remainer because they are lesse than the diuisor, and cannot be diuided by it, and the other figures that be behind the line, stand for the Quotient.

But now if your diuisor haue any other figure in his last place than 1, and in all his other places haue ciphers, looke how many ciphers there be, cut away so manie of the first figures of the number that should be diuided, and diuide the rest that followeth the line, by that figure that is in the last place, as if it were the whole diuisor.

Example of 64284, which I would diuide by 300, here must I cut away the two first figures, (for so many ciphers my diuisor hath) and must diuide the rest by 3 which is the figure in the last place of the diuisor. First therefore I part away the two first figures, and the sum standeth thus:

642 | 84

Then doe I diuide 642 by 3, and the quotient

tient is 2 1 4, for in 6 I finde twise 3, and in 4 once, and 1 remaining, which 1 with the 2 next before, doth make 12: wherein I finde 3 foure times: & this is a ready way to turne shillings into pounds: for sith one pound doth containe 20 shillings, I must diuide the whole number of shillings by 20 therefore easily to do it, I see that my diuisor hath one cipher, and therefore I cut away one figure from the beginning of the whole summe of shillings, and then I doe mediate or diuide by 2 the other figures or summe that followeth.

Scholer. I will put an example.

If you would diuide 6 4 2 8 7 shillings by 20, that is to say: If I would turne so many shillings into pounds, I must cut awaie the first figure, that is 7, and diuide the rest, that is 6 4 2 8 by 2, so shall the quotient be 3 2 1 4 whereby I knowe that 6 4 2 8 7 shillinges make 3 2 1 4 pounds, and 7 shillinges remayning.

Master. Now proue by Multiplication whether you haue well doone or no.

Scholer. The quotient is 3 2 1 4, which I doe multiplie by the diuisor 2, and it doth amount to 6 4 2 8.

Master. Hereby may you perceiue not onely that you haue well doone, but also how by Diuision you may turne shillings easilie into pounds: And contrariwise, by Multiplication you may turne pounds into shillings.

But

But here shall you see amongst diuers mẽ, diuers formes of such diuisiõ, but if you marke what I haue told you, you shall perceiue easilie all their wayes: for some men do not cut away so many of the first figures of the summe that they would diuide, as there are Ciphers in the first places of their diuisor, but they set all their Ciphers orderly vnder the first places of the number that they would diuide, and then with the other figure (or figures if they be many) they diuide the rest of their summe. Example. If they would diuide 7 2 5 9 3 1, by 3 4 0 0, they set their summes thus.

An other maner of the abridgement.

7 2 5 9 3 1	
3 4 0 0	

And then doe they diuide orderly till they come to the Ciphers: for there they stay and end their worke, as in this example: They seeke how often 3 may be found in 7, which is 2 times, and one remaining, therefore they set 2 in the quotient, and cancel 3 and 7, and ouer 7 they set the 1 that remained, thus.

1	
7 2 5 9 3 1	(2
3 4 0 0	

Then do they go forth saying: two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus.

1 4	
7 2 5 9 3 1	(2
3 4 0 0	

Then renew they the diuisor forward, and seeke how often 3 may be found in 4, which is

is but once, & 1 remaineth, then set they 1 in the quotient, & cancell 3 and 4, and ouer them they set that 1, thus.

Then take they once 4 out of 15, and there resteth 11: Or elle more easilie: Take once 4 out of 5, and there resteth 1, so they cancell the 4 and 5, and set 1 ouer them, thus.

Then set they forth the diuisor againe, and seeke how many times 3 are in 11, which they find 3 times, and 2 remayning: so they set 3 in the quotient, & cancell 11 and 3, and ouer them set 2, thus.

Then do they multiplie 4 by 3, which maketh 12, that withdraw they out of 29, & there resteth 17, of which the 7 must be set ouer the 9, and the 1 ouer the 2, thus.

And now are the two ciphers next ensuing, so that the deuisor can no more be set forward, and therefore is the deuision ended, and the

remainer is 1731.

Now the quotient, which is ~~213~~ doth declare, that if you diuide 725931, by 3400, you shall finde it therein ~~213~~ times, and there remaineth ~~1731~~, so shall you finde it, if you worke as I taught you, by cutting away the two first figures, because of the two ciphers.

1731 ~~2131~~

Note.

But this must you marke (as you may perceyue by this last example) that if there be left any other remainer in the summe that was behinde the squire line, that the remainer must be set to the latter end of the first remainer, which was cut away with the squire line: as if you would diuide 725931 by 3400, after the forme that I taught you, then would your summes appeare thus.

						1
					1	7
				1	4	7
7	2	5	9	3	1	(213
		3	4	4	4	
			3	3		

In seuering such a some y^t figures 31 are not to be added but to be put to ye beginning of y^t some wch comes of mult.pl. they are not to be accomted to be of y^t wch is dimded y^r of 2 17 are to be adde to the other figure but not put before as 31 are.

So that 17, which remaineth after ye line, must be set to the 31 (that was cut away with ye line) in higher places, as you see here: where that 17 with the 31, do make 1731.

¶ And here will I make an end of Diuision, sauing that I doe request you to exercise your selfe well herein by many summes, till you haue attained some expertnesse therein. For the reasons and conclusions thereof are so many and so auaileable for all sortes of men whatsoeuer, that if I should speake of the infinite

finite vses thereof, I should rather lacke woꝛds then matter. And therefoꝛe recommending it to your iudgement hereafter, vpon your further trauell into the Art, I will here end this treatise, repꝛesenting vnto you one example oꝛ simple question of Diuisiõ and Multiplication in stead of many, which is this.

There are 4 bꝛasse peeces: The first of them at a shot spendeth 9 pound of poulder, the second spendeth 5 pound, the third 4 pound, and the fourth 2 pound. They are all appointed against the batterie of a hold, and there is allowed by the master Gunner 700 pound of poulder to be spent by these 4 peeces in this assault. The question is two fold: The first how many shot each peece shall iustly make about with this 700 pound of poulder. And lastly how many pound of poulder ought iustly to be allowed to each peece foꝛ his true pꝛopoꝛtion.

Scholer. Why sir you make me smile, to beare me in hand, that these two demaundes may be simply resolued by Multiplication and Diuision.

Master. Truely that they may, and that may you by and by woꝛke your selfe with a litle labour: First adde together their quantities of poulder, that is 9 pound, 5 pound, 4 pound, and 2 pound, all which make 20: Diuide the 700 pound of poulder by that 20, and your quotient giueth 35, as here appeareth,

which sheweth for most certaintie that they shall make iust 35 shottes about.

1
700
200 (35
2

Scholer. Syr all this haue I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To trie the truth of the same, multiplie the first peece, that spends 9 pound by 35, and you shall see his allowance, which is 315 pound of poulder. Multiplie also the second peece that spends 5 pound by 35, and you shall find 175 pound his allowance: then 4 by 35, & you shall find 140 pound his allowance. Lastly multiplie 2 by 35, and you shall find 70 pound his allowance. All which 4 particular summes you shal adde together by Addition, as here appeareth, and it maketh iust 700 pound, and so is the question truelie absolued.

315
175
140
70

700

Scholer. Truely Sir these excellent conclusions do wonderfully more and more make me in loue with the Art.

Master. It is an Art that the further you trauell, the more you thirst to go on forward. Such a fountaine that the more you draw, the more it springs. And to speake absolutely in one worde (excepting the studie of Diuinitie, which is the saluatiõ of our soules) there is no studie in the world comparable to this for delight

light in wonderfull & godly exercise: For the skil hereof is well knowen, immediatly to haue flowed from the wisedome of God into the hart of man, whom he hath created the chiefe image and instrument of his praise & glory.

Scholer. The desire of knowledge doth greatly encourage me to be studious herein: and therefore I pray you cease not to instruct me further into the vse hereof. *

Master. With a good will, And now therfore for the further vse of these two latter, that is Multiplication and Diuision, I will briefly shew you the feate of Reduction.

Reduction.

REduction is, by which all summes of grosse denominatiõ may be turned into sums of more subtile denomination. And contrariwise, all sums of subtile Denomination, may be brought to summes of grosser denomination.

Scholer. What call you grosse denomination, and subtile denomination?

Master. That I call a grosse denominatiõ Grosse denominatiõ.

which doth containe vnder it many other subtiller or smaller: As a pound in respect to shillings, is a grosse denomination: for it is greater than shillinges, and containeth many of them. And shillings in comparison to pounds, are a subtile denomination, for because they are lesser than pounds, and many of them are contained in one of the other: as so, likewise of other thinges, whatsoeuer thing is compared to other, if it be a greater and containeth many of them, it is a grosser denomination: but if it be lesser, so that many of them are in the other, then are they called subtile denominations: whereby you may perceiue, that one denomination may be called a grosse denominatiõ, and also a subtile (that is to say, a great and a small) in diuers comparisons. For shillings compared to pounds are a subtile or small denomination: but compared to pence, they are a grosse or great denomination.

Subtile denominations.

Scholer. Now I vnderstand the name, I pray you teach me the vse.

Master. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any summe of a grosse denomination into a summe of a smaller or subtiller denomination, you must consider how many of that subtiller denomination do make one of the grosser denomination, and by that number or numerator do you multiplie the other summe: as if you would reduce 20 pounds

To reduce grosse denominatiõs to subtile.

into

into shillinges, you must consider that in a pound are included 20 shillings, therfore multiplie the one 20 by the other 20, and there will amount 400, whereby you may know, that in 20 pound are contained 400 shillings. Likewise if you would reduce 30 shillings into pence, considering that in 1 shilling are 12 pence, you must multiplie 30 by 12, and it will be 360: whereby you finde, that in 30 shillings, are contained 360 pence. And thus may you reduce any grosse denomination into a more subtiller, by Multiplication, if you know how many of the lesser doe make the greater: of which thing I will anon giue you a briefe Table for the most accustomed kinds of money, weights, measures and time, and such like, whereby you may know how often each subtile denomination is contained in the Grosser, when you shall neede it for the foresaid kinde of Reduction. And also the same shall serue you, if you would reduce any summe of a subtiler denomination, into a summe of a grosser denomination: For in such Reduction you must consider (as in the other forme) how many of the smaller do make the greater, and by that number must you diuide the other summ, and the quotient will declare how many of the greater denomination: are comprehended in that summe: as for example: If you would know how many shillings, are contained in 3240 pence, consider that 12 peece doe make

To reduce subtile denominations to grosse.

1 shilling: you must diuide that 3240 by 12, and your quotient will be 270, whereby you know that so many shillinges are in 3240 pence. But and you would know further, how many pounds are in those 270 shillings, seeing that euerie pound containeth 20 shillings, diuide that 270 by 20, and it will be 13, and 10 remayning, whereby you may know that in 3240 pence, or 270 shillinges, are 13 pounds and 10 shillings. For euermore the remainer must be named by the name or denomination of the summe that was diuided, which in this place were shillings. And thus may you doe with any other kindes of denominations.

¶ Wherefore to the intent you may haue a certaine light or knowledge in most common coines, weights & measures (which is the most chiefe and principallest thing in traffike to be knowē) I haue in ech Reductiōs as they come in order, set down certaine instructiōs incident therunto. And first I haue hereunto added this Table, wherin is comprehended not onely our currant and common coines, but also the most part of the vsuall coines of Christendome, with their iust waights and value currant in this realme of Englād, intēding at the latter end of my Addition to this booke, to write of the ordinarie money vsed in diuers places, & their common values currant for traffike, with the maner of their exchanges from place to place, &c.

A Table of the names and valuation of the moſt vſuall Gold Coines throughout Chriſtendome with their ſeuerall weight of Pence and Graines: & what they are worth of currant money Engliſh.

The names and titles of the Gold.	The waight. Péce.	Graines.	The Value. Shil.	Pence.
Royall	4	23	15	0
Halfe Royall	2	11	7	6
Old Noble	4	9	13	4
Halfe old Noble	2	4	6	8
Angel	3	8	10	0
Halfe Angel	1	16	5	0
Salute	2	5	6	4
2. parts of Salute	1	11	4	2
George Noble	3	0	9	0
Half George Nob.	1	12	4	6
Firſt crowne K.H.	2	9	6	4
Baſe crowne K.H.	2	0	5	0
Great Soueraigne	10	0	30	0
Souera. K.H. beſt.	3	14	10	8
Soueraigne K.H.	4	0	10	0
Edward Souer.	3	14	10	0
Elizab. Soueraigne	3	14	10	0

The names and titles of the Gold.	The waight. Péce.	Graines.	The Value. Shil.	Pence.
Elizab. crowne	1	19	5	0
Vnicorne of Scot.	2	10	6	0
Scotiſh crowne	2	5	6	0
French Noble	4	16	13	4
All the ſortes of French crownes	2	5	6	0
Old French crown	2	5	6	0
Flaunders Rider	2	6	6	6
Gelders Rider	2	2	3	6
Phillips Royall	3	10	10	0
Phillips crowne	2	5	5	0
Collen Gilden	2	2	4	8
New Andr. Gild.	2	3	5	0
Flaunders Noble	4	10	12	0
Half Flaun. noble	2	6	5	0
Flem Angel beſt	3	6	9	0
Flād. royal or Key.	3	10	10	0
Carolus Gilden	1	21	3	6
Flaunders royal	2	6	5	0
Saxon Gilden	2	2	4	8
Flaunders crowne	2	5	6	0
Phillips Gilden	2	3	4	2
Halfe Phil. Gilden	1	1	2	1

The names and titles of the Gold.	The weight. Péce.	Grains.	The value. Shil.	Pence.
Golden Lion	2	16	7	8
3. parts of gold Li.	0	21	2	5
2.3. parts of gol. Li.	1	19	4	11
Dauids Gilden	2	2	4	0
Horne Gilden	1	12	4	11
Old Andre, Gild.	2	3	4	10
Cruſado long cros	2	6	6	0
Cruſado ſhort cros	2	6	6	2
Myl rayes	4	20	13	4
Halfe Mill rayes	2	10	6	8
Portigue. 1. ounce	2	16	68	0
Golden Caſtilio	2	23	8	10
Ducket of Aragō	2	6	6	6
Hungarie Ducket	2	7	6	4
Double Piſtolat	4	8	11	8
Single Piſtolat	2	4	5	10
Ducket of Floren.	2	5	6	4
Double Ducket	4	11	13	0
Single Ducket	2	6	6	6
Dou. duc. of Rome	4	13	12	8

Of Siluer coines currant in this Realme.

The Edward Crowne of 5 s.
The Edward halfe Crowne of 2 s, 6 d.
The Edward Shilling, half Shilling, and the Three-pence.
Phillip and Maries Shilling, & halfe shilling.
The Marie Groat, and Marie Two pence.
Queene Elizabeths Shilling, 6 d, 4 d, 3 d, 2 d, 1 d, 3 ob, 3 q, and ob.

Note. It is to be vnderstood (gentle Reader) that whereas the waight is called by the name of a penny, it is not meant a penny of siluer money, but a penny of Goldsmithes waights, which is 24 Barly cornes drie. And 20 of these pence make an Ounce: and 12 of these ounces make a Pound Troy: So that if a man haue not the waight wherewith to weigh any peece that may come to his hand, he may do it with the Barly graines or cornes being drie, and taken out of the middle of the eare. *

Here would I now expresse the values of sundrie other coines of diuers Countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the Statutes of this Realme. An other cause is, by reason they are so vncertaine, that they be neuer long at one rate. And againe they are so different in so many places, that it were mat-

matter inough for a great booke, to speake sufficiently of them all. Howbeit, yet because you shall not be altogether ignorant of them, I wil shew you the values of some that are most in vse, and first of Fraunce.

The most common money are Deniers, Soulx and Franks. 12 Deniers make 1 s̄, 20 Soulx make one Frank, so that as you see, these three kindes are like in the rate to pence, shillings, and pounds with vs, but that this is the difference, that their Denier is but the ninth parte of our pennie, and so their Soulx (commonly called Sowses) goe 9 to our shilling, and 9 of their Frankes to an English pound of money: So that 3 of their Frankes make a Noble. And by those 3 may you practise how to reduce French money into English money, according as I haue set foorth here following. French coines.

2160 deniers make 240 d̄, or 20 s̄.

3240 deniers make 360 d̄, or 30 s̄.

8352 deniers make 929 d̄, or 2 l̄, 17 s̄, 4 d̄.

2160 soulx make 240 shillings. And so of other in like rate. As for the rest of their coines I omit them till hereafter that you haue some vnderstanding in broken numbers.

But now, as for the Coines of Flaunders they be so changeable, that you must know them from time to time, else you cannot reduce them into our money certainelie. But yet that you may haue an example of their money Flanders coines.

money to exercise you withall, you shall take those that be most common: as Stiuers both single & double, Groates Flemmish, Carolus, and Gildens. A Flemish Groate is a little aboue three Farthings English. A single Stiuer is 1 d, ob. q̃, half farthing. The double stiuer is 3 d, farthing. The double stiuer Carolus is 4 d, ob. halfe farthing. Then there is also the Carolus Gilden which is worth 20 stiuers. And the Flemish Noble is worth 3 Carolus Gildens, and 12 Stiuers.

So that if you would conuert Flemish money, or any other kinde of money whatsoeuer it be, iustly into sterling, you must reduce it first into the smallest part of English money that is in that coyne, as for example. If I would reduce 368 double stiuers into English money, considering that a double stiuer containeth 3 d farthing, you shall first looke how many farthings be in the double stiuer, & you shall finde them 13, therefore multiplie the summe of the stiuers by 13, and then haue you their value in farthings, which is 4784. Now if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) and then will the quotient declare the summe of the shillings.

Likewise, if you would reduce any sum of single stiuers into English money, you must multiplie the summe first by 13, and then haue

you

you reduced them into a certaine summe, that is to wit, halfe farthings: which summe if you diuide by 8, then will amount the summe of pence: or if you diuide it by 96, the summe of shillings will appeare.

But marke this in all Diuision, when yee do reduce to bring one denomination into another, if there be any remainer after the Diuision, that must be named by the denomination of the grosse summe that was diuided: as for example. I would bring 254 farthings into pence, therefore I do diuide that 254 by 4, for so many farthings make 1 penie, and the quotient is 63, which is the summe of the pēce, and then remaineth yet 2, which are farthings still, as one may proue by diuiding. And this must be marked in all Diuision, namely when it is doone for Reduction. Note wel.

¶ Touching Danske money, they haue their Souls, whereof 20 is a Liuer: which is 2 s̄ sterling. They haue also their Grash, whereof 30 make a Gilderne, which is 4 s̄ sterling. They haue also Dollors, and their common or old Dollor is 35 Grash. New Dollors they haue, which be diuerse, some valued at 24 Grash, some at 26, and some at 30. And thus much I thought good to adde to the authour touching Danske money.

Concerning Spanish money, whereof the most common, are Cornados, Marueides, Ryals and Duckets: 6 Cornados make a Marueide,

Marueide, 36 Marueides maketh 1 Ryal, and 11 Ryals maketh one Ducket, so the Ducket containeth 374 Marueides, which is about 5 s. 10 d sterling. Therefore if you would conuert 124 l 5 s sterling into Duckets, consider that pence is the least value or Denomination named in this question, therefore reduce 124 l 5 s into pence, and it maketh 29820 pence: which if you diuide by the pence that a Ducket is worth, which is 70, you shall haue for your quotient 426 Duckets, your desire. *

Weights.

Thus much haue I said of money, now will I shew you in like sort the distinction of weightes, after the Statutes of England, where the least portion of weight is commonly a Graine, meaning a Graine of Corne or wheate, drie and gathered out of the midle of the eare. Of these Graines in times passed, 32 wayed iust 1 penie of Troy, and then was but 40 pence in an Dunce. But now are there 48 pence in an Dunce, so that there are ~~not~~ fully 24 Graines in one peny. But now of Dunces after Troy rate (which is the standard of England) 12 do make 1 pound.

A graine.

A penie of Troy.

An ounce.

Haberdepoise weigts.

But cõmonly there is vsed another weight called Haberdepoise, in which 16 Dunces make a pound. Therefore when you would reduce Dunces into pounds, you must consider whether your weights be Troy weight or Haberdepoise: and if it be Troy weight, you must

The weight of a penie white Loafe in Troy weight.

Pounds	Ounces	Penie wayght.
27	2	8
18	1	12
13	7	4
10	10	11 ½
9	0	16
7	9	5 7/9
6	9	12
5	0	10 ½
4	6	5 5/9
4	4	11
3	5	6 1/10
3	4	4
3	3	1 7/9
2	4	2 6/9
2	3	2 2/9
2	2	1 7/9
2	1	1 3/9
2	0	3/9
1	2	16 8/11
1	1	15 1/11
1	0	14

The weight of a penie white Loafe in Troy weight.

Pounds	Ounces	Penie wayght.
13	7	4
9	0	16
6	9	12
5	5	8 2/3
4	6	5 1/3
4	0	1
3	4	16
3	3	12 4/11
3	2	8 8/11
3	1	4 4/11
3	0	1
2	5	13 5/11
2	4	10 10/11
2	3	7 9/11
2	2	4 8/11
2	1	1 7/11
2	0	11 2/11
1	2	8
1	1	4 4/11
1	0	1

The price of a Quarter of Wheate.

£	s	d
1		
1	1	
1	1	1
1	1	2
1	1	3
1	1	4
1	1	5
1	1	6
1	1	7
1	1	8
1	1	9
1	1	10
1	1	11
1	2	0
1	2	1
1	2	2
1	2	3
1	2	4
1	2	5
1	2	6
1	2	7
1	2	8
1	2	9
1	2	10
1	2	11
1	3	0

* The weight of a farthing white loafe in sterling money by assise.

£	s	d	q
4	3	0	
3	4	6	
2	5	10	2 2/3
2	5	7	1
2	4	6	0
2	3	5	1
2	2	4	2
2	1	3	3
2	0	2	0
1	19	1	1
1	18	0	2
1	16	11	3
1	15	10	0
1	14	9	1
1	13	8	2
1	12	7	3
1	11	6	0
1	10	5	1
1	9	4	2
1	8	3	3
1	7	2	0
1	6	1	1
1	5	0	2
1	4	0	3
1	3	0	0
1	2	0	1
1	1	0	2
1	0	0	3
0	19	0	0
0	18	0	1
0	17	0	2
0	16	0	3
0	15	0	0
0	14	0	1
0	13	0	2
0	12	0	3
0	11	0	0
0	10	0	1
0	9	0	2
0	8	0	3
0	7	0	0
0	6	0	1
0	5	0	2
0	4	0	3
0	3	0	0
0	2	0	1
0	1	0	2
0	0	0	3

The weight of a farthing white loafe in Troy waight by assise.

Pounds	Ounces	Penie wayght.
2	3	4
1	6	2 2/3
1	4	1 1/3
1	2	0
1	0	16
0	15	14 2/3
0	14	13 1/3
0	13	12
0	12	10 2/3
0	11	9 1/3
0	10	8
0	9	6 2/3
0	8	5 1/3
0	7	4
0	6	2 2/3
0	5	1 1/3
0	4	0
0	3	14 2/3
0	2	13 1/3
0	1	12
0	0	10 2/3

The weight of a halfe penie white loafe in Troy waight by assise.

Pounds	Ounces	Penie wayght.
1	3	2
1	2	1 1/3
1	1	0
0	15	14 2/3
0	14	13 1/3
0	13	12
0	12	10 2/3
0	11	9 1/3
0	10	8
0	9	6 2/3
0	8	5 1/3
0	7	4
0	6	2 2/3
0	5	1 1/3
0	4	0
0	3	14 2/3
0	2	13 1/3
0	1	12
0	0	10 2/3

The weight of a penie white Loafe in Troy waight by assise.

Pounds	Ounces	Penie wayght.
1	3	4
1	2	2 2/3
1	1	1 1/3
1	0	0
0	15	14 2/3
0	14	13 1/3
0	13	12
0	12	10 2/3
0	11	9 1/3
0	10	8
0	9	6 2/3
0	8	5 1/3
0	7	4
0	6	2 2/3
0	5	1 1/3
0	4	0
0	3	14 2/3
0	2	13 1/3
0	1	12
0	0	10 2/3

...ited to be 26 stone.

¶ Now in Cheese, though it bee solde by the hundreth, and by the stone in some places, yet the verie weights of it are Cloues and

Cheese weights.

The waight of a halfe penie white loafe in Troy waight by Cheese

Pounds	Ounces	Penie weights
2	3	4
1	6	$2\frac{2}{3}$
1	1	12
0	10	17
0	9	$1\frac{1}{3}$
0	7	$15\frac{3}{7}$
	6	16
	6	$0\frac{8}{9}$
	5	$8\frac{4}{5}$
	4	$18\frac{10}{11}$
	4	$10\frac{2}{5}$
	4	$2\frac{7}{13}$
	4	$1\frac{3}{5}$
	3	$12\frac{8}{11}$
	3	8
	2	4
	2	$0\frac{4}{9}$
	2	$17\frac{5}{11}$
	2	$14\frac{2}{5}$
	2	$11\frac{1}{2}$
	2	$9\frac{1}{2}$
	2	$7\frac{7}{23}$
	2	$5\frac{2}{3}$
	2	$3\frac{1}{21}$
	2	$1\frac{1}{11}$
	2	$0\frac{8}{27}$

The waight of a farthing white loafe in Troy waight by affize.

Pounds	Ounces	Penie weights
2	3	4
1	6	$2\frac{2}{3}$
1	1	12
0	10	17
0	9	$1\frac{1}{3}$
0	7	$15\frac{3}{7}$
	6	16
	6	$0\frac{8}{9}$
	5	$8\frac{4}{5}$
	4	$18\frac{10}{11}$
	4	$10\frac{2}{5}$
	4	$2\frac{7}{13}$
	4	$1\frac{3}{5}$
	3	$12\frac{8}{11}$
	3	8
	2	4
	2	$0\frac{4}{9}$
	2	$17\frac{5}{11}$
	2	$14\frac{2}{5}$
	2	$11\frac{1}{2}$
	2	$9\frac{1}{2}$
	2	$7\frac{7}{23}$
	2	$5\frac{2}{3}$
	2	$3\frac{1}{21}$
	2	$1\frac{1}{11}$
	2	$0\frac{8}{27}$

The waight of a penie white loafe in Troy waight

£	s	d	Penie weights	grains
				12
				1 $\frac{7}{9}$
				17 $\frac{3}{5}$
				17 $\frac{2}{11}$
				1 $\frac{1}{3}$
				7 $\frac{1}{13}$
				3 $\frac{1}{5}$
				5 $\frac{1}{15}$
				16
				8
				0 $\frac{8}{9}$
				4 $\frac{10}{19}$
				8 $\frac{4}{5}$
				3 $\frac{11}{21}$
				19
				14 $\frac{14}{23}$
				11 $\frac{1}{3}$
				7 $\frac{1}{25}$
				3 $\frac{2}{13}$
				0. $\frac{16}{27}$

The waight of a farthing white loafe in Troy waight by affize.

Pounds	Ounces	Penie weights
2	3	4
1	6	$2\frac{2}{3}$
1	1	12
0	10	17
0	9	$1\frac{1}{3}$
0	7	$15\frac{3}{7}$
	6	16
	6	$0\frac{8}{9}$
	5	$8\frac{4}{5}$
	4	$18\frac{10}{11}$
	4	$10\frac{2}{5}$
	4	$2\frac{7}{13}$
	4	$1\frac{3}{5}$
	3	$12\frac{8}{11}$
	3	8
	2	4
	2	$0\frac{4}{9}$
	2	$17\frac{5}{11}$
	2	$14\frac{2}{5}$
	2	$11\frac{1}{2}$
	2	$9\frac{1}{2}$
	2	$7\frac{7}{23}$
	2	$5\frac{2}{3}$
	2	$3\frac{1}{21}$
	2	$1\frac{1}{11}$
	2	$0\frac{8}{27}$

The waight of a farthing white loafe in Troy waight by Cheese

Pounds	Ounces	Penie weights
2	3	4
1	6	$2\frac{2}{3}$
1	1	12
0	10	17
0	9	$1\frac{1}{3}$
0	7	$15\frac{3}{7}$
	6	16
	6	$0\frac{8}{9}$
	5	$8\frac{4}{5}$
	4	$18\frac{10}{11}$
	4	$10\frac{2}{5}$
	4	$2\frac{7}{13}$
	4	$1\frac{3}{5}$
	3	$12\frac{8}{11}$
	3	8
	2	4
	2	$0\frac{4}{9}$
	2	$17\frac{5}{11}$
	2	$14\frac{2}{5}$
	2	$11\frac{1}{2}$
	2	$9\frac{1}{2}$
	2	$7\frac{7}{23}$
	2	$5\frac{2}{3}$
	2	$3\frac{1}{21}$
	2	$1\frac{1}{11}$
	2	$0\frac{8}{27}$

The waight of a farthing white loafe in ſterling money by affize.

£	s	d	Penie weights	grains
				12
				1 $\frac{7}{9}$
				17 $\frac{3}{5}$
				17 $\frac{2}{11}$
				1 $\frac{1}{3}$
				7 $\frac{1}{13}$
				3 $\frac{1}{5}$
				5 $\frac{1}{15}$
				16
				8
				0 $\frac{8}{9}$
				4 $\frac{10}{19}$
				8 $\frac{4}{5}$
				3 $\frac{11}{21}$
				19
				14 $\frac{14}{23}$
				11 $\frac{1}{3}$
				7 $\frac{1}{25}$
				3 $\frac{2}{13}$
				0. $\frac{16}{27}$

The waight of a penie white loafe in Troy waight

£	s	d	Penie weights	grains
				12
				1 $\frac{7}{9}$
				17 $\frac{3}{5}$
				17 $\frac{2}{11}$
				1 $\frac{1}{3}$
				7 $\frac{1}{13}$
				3 $\frac{1}{5}$
				5 $\frac{1}{15}$
				16
				8
				0 $\frac{8}{9}$
				4 $\frac{10}{19}$
				8 $\frac{4}{5}$
				3 $\frac{11}{21}$
				19
				14 $\frac{14}{23}$
				11 $\frac{1}{3}$
				7 $\frac{1}{25}$
				3 $\frac{2}{13}$
				0. $\frac{16}{27}$

The waight of a penie white loafe in Troy waight

£	s	d	Penie weights	grains
				12
				1 $\frac{7}{9}$
				17 $\frac{3}{5}$
				17 $\frac{2}{11}$
				1 $\frac{1}{3}$
				7 $\frac{1}{13}$
				3 $\frac{1}{5}$
				5 $\frac{1}{15}$
				16
				8
				0 $\frac{8}{9}$
				4 $\frac{10}{19}$
				8 $\frac{4}{5}$
				3 $\frac{11}{21}$
				19
				14 $\frac{14}{23}$
				11 $\frac{1}{3}$
				7 $\frac{1}{25}$
				3 $\frac{2}{13}$
				0. $\frac{16}{27}$

The waight of a farthing white loafe in ſterling money by affize.

£	s	d	Penie weights	grains
				12
				1 $\frac{7}{9}$
				17 $\frac{3}{5}$
				17 $\frac{2}{11}$
				1 $\frac{1}{3}$
				7 $\frac{1}{13}$
				3 $\frac{1}{5}$
				5 $\frac{1}{15}$
				16
				8
				0 $\frac{8}{9}$
				4 $\frac{10}{19}$
				8 $\frac{4}{5}$
				3 $\frac{11}{21}$
				19
				14 $\frac{14}{23}$
				11 $\frac{1}{3}$
				7 $\frac{1}{25}$
				3 $\frac{2}{13}$
				0. $\frac{16}{27}$

The price of a Quarter of Wheate.

£	s	d
	3	0
	4	6
	6	0
	7	6
	9	0
	10	6
	12	0
	13	6
	15	0
	16	6
	18	0
	19	6
1	0	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

The waight of the penie white Loafe.

Pounds	Ounces	Penie weights
27	2	8
18	1	12
13	7	16
10	10	$12\frac{4}{5}$
9	0	4
7	9	$6\frac{2}{7}$
6	9	8
6	0	$2\frac{2}{3}$
5	5	$6\frac{2}{5}$
4	11 •	$16\frac{8}{11}$
4	6	12
4	2	$11\frac{1}{13}$
4	0	$4\frac{4}{5}$
4	5	$18\frac{3}{5}$
3	3	4
3	2	12
3	0	$1\frac{3}{7}$
2	10	$11\frac{11}{19}$
2	8	$3\frac{2}{5}$
2	7	$15\frac{3}{7}$
2	5	$8\frac{4}{11}$
2	4	$1\frac{21}{23}$
2	3	17
2	2	$10\frac{14}{25}$
2	1	$5\frac{7}{13}$
2	0	$0\frac{8}{9}$

The waight of the penie white loafe.

Pounds	Ounces	Penie waight
13	7	4
9	0	16
6	9	12
5	5	$5\frac{2}{3}$
4	6	8
3	10	$12\frac{4}{7}$
3	4	16
3	0	$5\frac{1}{5}$
2	8	$12\frac{4}{5}$
2	5	$13\frac{5}{11}$
2	3	4
2	1	$2\frac{2}{13}$
2	0	$9\frac{3}{5}$
1	8	$19\frac{1}{3}$
1	8	8
1	7	4
1	6	$2\frac{2}{3}$
1	5	$3\frac{11}{19}$
1	4	$6\frac{2}{5}$
1	3	$10\frac{6}{7}$
1	2	$16\frac{8}{11}$
1	2	$3\frac{12}{23}$
1	1	14
1	1	$1\frac{1}{25}$
1	0	$11\frac{1}{13}$
1	0	$1\frac{7}{9}$

The waight of the ſtarling white Loafe.

Pounds	Ounces	Penie weight
6	9	12
4	6	8
3	4	16
2	8	$12\frac{4}{5}$
2	3	4
1	11	$6\frac{2}{7}$
1	8	8
1	6	$2\frac{2}{3}$
1	4	$6\frac{2}{5}$
1	2	$16\frac{8}{11}$
1	1	12
1	0	$11\frac{1}{13}$
0	10	$4\frac{4}{5}$
0	10	$18\frac{3}{5}$
0	9	4
0	8	12
0	8	$1\frac{3}{7}$
0	7	$11\frac{11}{19}$
0	7	$3\frac{2}{5}$
0	6	$15\frac{3}{7}$
0	6	$8\frac{4}{11}$
0	5	$1\frac{21}{23}$
0	5	17
0	4	$10\frac{14}{25}$
0	4	$5\frac{7}{13}$
0	3	$0\frac{8}{9}$

The price of a Quarter of Wheate.

£	s	d
	3	0
	4	6
	6	0
	7	6
	9	0
	10	6
	12	0
	13	6
	15	0
	16	6
	18	0
	19	6
1	0	0
1	2	6
1	4	0
1	5	6
1	7	0
1	8	6
1	10	0
1	11	6
1	13	0
1	14	6
1	16	0
1	17	6
1	19	0
2	0	6

The waight of the ſtarling white Loafe.

Pounds	Ounces	Penie weights
6	9	12
4	6	8
3	4	16
2	8	$12\frac{4}{5}$
2	3	4
1	11	$6\frac{2}{7}$
1	8	8
1	6	$2\frac{2}{3}$
1	4	$6\frac{2}{5}$
1	2	$16\frac{8}{11}$
1	1	12
1	0	$11\frac{1}{13}$
0	10	$4\frac{4}{5}$
0	10	$18\frac{3}{5}$
0	9	4
0	9	12
0	8	$1\frac{3}{7}$
0	8	$11\frac{11}{19}$
0	7	$3\frac{2}{5}$
0	7	$15\frac{3}{7}$
0	7	$8\frac{4}{11}$
0	6	$1\frac{21}{23}$
0	6	17
0	6	$10\frac{14}{25}$
0	6	$5\frac{7}{13}$
0	6	$0\frac{8}{9}$

vnderstanding of this T[illegible] wherein according to our time Master Recorde alloweth 60 pence to the ounce, and 3 li 60 shillings to the pound. And thereupon to the pound Troy doth [illegible]ame or produce this his Table, beginning at 3 shillings the quarter till he comes to 4 shillings 6 pence the quarter, And this his not downe any one example to continue the worke, hath beene hard for many to conceiue or comprehend, and therefore the onely chefe cause why I haue written this rstanding of him therein. The first thing therefore that is sought for in this Table, as in the other afore said, is a Maxime grounded vpon the Statute, which is Wheate is sold for 2 shillings, then the farthing white Loafe shall way 68 shillings, whereby a shilling is meant $\frac{1}{20}$ of a pound, and by a penie $\frac{1}{20}$ of an ounce. Now to find what waight the farthing white loafe shall wey at 3 shillings the quarter, till you come to 40 shillings 6 pence the quarter, is thus to be wrought. Comming ing by the Backer Rule, say: If 2 shillings the quarter giue or allow the farthing white loafe to way 68 shillings, what waight ought the farthing white loafe to er? Worke and you shall find 45 shillings, 4 pence, as before in the correction of the first Table is noted. Then for the second worke say by the Rule of 3 direct, If [illegible] ounce, wha giueth 45 shillings 4 pence? Multiplie and diuide, and you shall finde 544 ounces, which 544 ounces being multplied by 3, for 3 pound or 60 shillings yeeldeth 1632 ounces, which diuided by 20, produceth 81 ounces, and $\frac{12}{20}$, or rather $\frac{3}{5}$ of an ounce equall vnto 12 penie waight, which is halfe an ounce, and 2 penie waight, and so maketh in all 6 pound, 9 $\frac{1}{2}$ ounces, and 2 penie waight. Now the next way to continue this Table, to know the waight of the halfe penie white loafe is thus. Multiplie 1632 ounces by 2, and it ringeth forth 3264 ounces, and diuided by 20, it yeeldeth 163 ounces, and $\frac{4}{5}$, which is equall to 13 pound, 7 ounces, and 4 penie waight, as Master Recorde his Table noteth. Thirdly for the waight of the penie white loafe multiplie 1632 ounces by 4, and diuide by 20, and after by 12, as afore. And you shall finde 27 pounde $\frac{1}{4}$ ounce, and 3 penie waight, &c. This method or else by doubling the farthing white loafe for the waight of the halfe penie white loafe, and so doubling the halfe penie white loafe for the waight of the penie white loafe is the order to continue this Table to the end thereof.

Thus much for the vnderstanding of this Table here placed by Master Recorde a man endued with rare knowledge in Arithmeticall and Geometricall proportions touching the statute of Coynage, and the standard thereof, as appeareth in his Epistle of this booke dedicated to King Edwarde the Sixt. Insinuating vnto his Highnesse that the standard of the Coyne is much altered from the 14 yeare of King Edwarde the third, (when this statute and assise was confirmed) to the standard in this our time. For it appeareth that in King Edward the thirds time, when this assise of bread and drinke was established, that a sterling pennie round without clipping, did then waye 32 Cornes of wheate, drie and taken out of the midle of the Eare, and 20 of these pence made an ounce, and 12 ounces made a pound Troy. And so from the waight of a pennie to 20 shillinges sterling, which then wayed 12 ounces, tooke bread his waight and proportion: and now finding 60 pence in an ounce. That onely cause I perceiue for zeale to a common wealth moued him to set downe this table in this priuate booke, meaning not thereby to alter any word of the statute, being so good an ordinance and of so long continuance, but as a kind of exposition by the way, that thereby the statute may be better vnderstood, and so consequenely better put in execution: which assise of his is three times greater than the statute now alloweth. Therefore also to gratifie such as are desirous of knowledge (according to these prices of a quarter of wheate) I haue added to this Author three other newe Tables, and reduced their prices into their iust proportion of sterling money, and also reduced the money into knowen waight Troy according to the statute. And thereafter according to proportion in my three other Tables, haue I noted the iust waight, that a farthing, halfe penie, and penie white loafe ought to way by the Statute. *

£	s	d
	3	0
	4	6
	6	0
	7	6
	9	0
	10	6

by assise.

£	s	weth gar / 'e qr /
2	5	4
1	10	2 2/3
1	2	8
0	18	1 3/5
0	15	1 1/3
0	12	1 3/

ht by assise.

Pounds	Ounces	Penie waig
2	3	4
1	6	2 2/3
1	1	12
0	10	17
0	9	1 1/

...e 1 ounce, what

1 6 3 2 ounces, which diuided

9 ½ ounces, and 2 penie waight. Now the next way to co

it yeeldeth 1 6 3 ounces, and 1/5, which is equall to 1 3 poun

by 20, and after by 12, as afore. And you shall finde 27 p

halfe penie white loafe for the waight of the penie white lo

Thus much for the vnderstanding of this Table here p

thereof, as appeareth in his Epistle of this booke dedicate

this statute and assise was confirmed) to the standard in th

ping, did then waye 32 Cornes of wheate, drie and taken

which then wayed 12 ounces, tooke head his waight and

booke, meaning not thereby to alter any word of the statut

ly better put in execution: which assise of his is three time

ded to this Author three other newe Tables, and reduced th

proportion in my three other Tables, haue I noted the iust

must diuide your Dunces by 12. to bring them to pounds, but if it be Haberdepoise, you must diuide them by 16. Now again, there be greater weights which are called an hundred, halfe a hundred, and a quarterne, and also a halfe quarterne, &c. A hundred weight.

Scholer. Why: so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they haue not common weights made to ther rate, as the other haue. And againe, these that I did name are not iust in number as they seeme by their name, for an hundred is not iust 100, but is 112 pound. And so the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And these bee the common weights vsed in most things that are solde by weight.

Howbeit there are in some things other names, as in wooll, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne, but a Stone, and the 7 pound halfe a Stone. Other names because they differ in many places, and agree in few, I let them passe. Wooll weights. Todde. Stone.

But a Sacke of Wooll by the statutes, is limited to be 26 stone. Sacke.

¶ Now in Cheese, though it bee solde by the hundreth, and by the stone in some places, yet the verie weights of it are Cloues Cheese weights.

and Weyes. So that a Cloue containeth 8 pound: and a Wey 32 Cloues, which is 256 pound, that is 12 score and 16 pound: And so much weyeth the Weigh of Suffolke cheese: And the like weight is or should be the barrell of Suffolke butter.*

The Wey of Essex Cheese containeth 16 score, and 16 pound: and so much is also the barrell of Essex Butter.

Measures for liquor.

Now of weights are made other measures, both for graine and liquor. For a pound in waight maketh a Pint in measure, so that 8 pound (or 8 pintes) doe make a gallon: halfe a Gallon is named a Pottle: & halfe a Pottle is called a Quart, which contayneth two Pintes. Now aboue a Gallon the next measure is a Firkin: then a Tertian, a Kilderkin, or halfe Barrell, and a Barrell. And by those measures are sold commonly Ale, Beere, Wine, and Oyle, Butter, and Sope: Salmon, Herrings, and Eeles.

A pint.
Gallon.
Pottle.
Quart.

Firkin.
Tertian.
Kilderkin.
Barrell.

Ale measures.

But as these be vnlike things, so the measures of their vessels doe differ: for the measures of Ale are as followeth.

Of Ale { the Firkin, ỳ Kilderkin, the Barrell } containeth { 8, 16, 32 } gallons.

Of beere { the Firkin, ỳ Kilderkin, the Barrell } cõteineth { 9, 18, 36 } gallõs

Sope

Sope measures.

Sope measures, both Firken, Kilderkin, and Barrel, should be equall to Ale measures. Moreouer, the statutes doe limite the waight of euerie of those three vessels being emptie.

A Barrell	to	26	pounds.
Halfe Barrell	weigh	13	
A Firken	emptie	6½	

Herriag.

Herrings also to be sold by ỳ same measures that Ale and Sope be sold by.

Herrings also are sold by the tale, 120, to the hundred, ten thousand to the last.

Salmon & Eles.

Salmon and Eeles haue a greater measure.

Salmon & Eeles	the Butte	holdeth	84	Gallõs
	the Barrel		42	
	halfe Bar.		21	
	the Firkin		10½	

Howbeit, some statutes did limite Eele vessels equall with Herring vessels.

Wine measures.

Now as for wine vessels they are seldome smaller than Hogs heads which are of 63 gallons: euery Hogges head is two Barrels: yet there are many other wine vessels, but of them all, see this Table, and marke the measures one to another.

Of wine and oile.	the Rondlet	holdeth	18½	Gallons.
	the Barrell		31½	
	ỳ Hogshead		63	
	the Tertian		84	
	the Pipe		126	
	the Tonne		252	

Tertians. But you shall marke that there bee other kindes of Tertians: for there bee Tertians (that is to say) Thirdles of Pipes, of Hogges heads, and of Barrels, as well of other things as of wine.

Butte. Also of Malueseys, and Secke, &c. the halfe Tonne is not called a Pipe, but rather a But.

And thus much haue I thought meete to tell you at this time.

Scholer. And is that alwayes true?

Master. I haue tolde you how it should be, but how it is, I may not say: how they doe differ dayly from their iust measure, that Gagiers can tell you better than I. But I will let this passe now, and speake briefly of the other measures.

Drie measures. And as of weights there did spring the liquid measures, (whereof I spake last) so of the same springeth drie measures: as Peckes, Bushels, Quarters, and such like, whereby are measured corne and like graines: also salt, lime, coales, and other like. And this is the order and quantitie of them.

A pecke. A Pecke is the measure of two Gallons.

A

A bushell containeth foure peckes. A Bushel.
A quarter holdeth eight bushels. Quarter.
A wey conteyneth sixe quarters. Wey.

These are the common names and measures, but in diuerse places there bee diuerse sortes.

The bushel in many places is 2 bnshels: but then is the bushell there called a Strike. Strikes.
And in some places halfe a quarter is called a Cornoke. But these diuersities are too many to tell you briefly them all. And againe, sith they are against the law and statutes, I count them vnmeete to be vsed.

But now remaineth yet another kinde of Measure.
measures, whereby men meate length and to meate
breadth, and thickneſſe, and those are an inch, length,
a foote, and such other: whose names and breadth,
quantities this table sheweth. and thick-
neſſe.

3 graines of barly in length make an inch. An inch.
12 Inches make a foote. Foote.
3 foote make a yarde. Yard.
3 foote and 9 inches make an Elle. Elle.
5 yardes and a halfe, make a perch. Perch.
5 yards a half 1 perche in breadth and 40 in length, doe wch is yards 220
make a rodde of land, which some call a roode, some a yarde land, and some a Farthendele.
2 farthendels make halfe an acre of ground.
4 Farthendels make an Acre. Acre.

¶ More 40 perches in length do make a fur- yards [redacted] 220
long: 8 furlongs make a mile, which contey- yards [redacted] 1760
neth 320 perches. yards 1760

So that an English mile grounded vpon the statute, is in length 1760 yards, 5280 foote, and 63360 inches.

Somewhat greater than the Italian mile of 1000 paces, and 5 foote to a pace.*

Here might I tell you many thinges else touching measure, and also howe to reduce strange measures to our measures, but because it cannot be well done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe till another time, that I haue taught you the knowledge of broken numbers,

The parts of time.

Scholer. But yet Sir of the parts of time I pray you tell me somewhat.

A day.
An houre.
Weeke.
Moneth.
Yeare.

Master. You knowe that a naturall day hath 24 houres, and euerie houre hath 60 minutes. It needeth not to tell you, that 7 dayes make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeare, lacking 1 day, and certaine houres, and minutes: But of that I shall instruct you hereafter.

Here will I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmetike, yet you see it is no lesse needefull to be knowne, or easier to bee done, than any of the other.

Scholer. Marie Sir, it seemeth vnto me much harder than any other sort, for it requireth the knowledge of so manie thinges:

but

but now ſir when you ſee time, I am readie to learne forth: for as much of reduction as you haue taught me, I remember, but and if I doe at any time forget, I ſhall haue recourſe to the tables you ſet forth for me.

Maſter. So doe you, for it will not be remembred without exercise. But in as much as you vnderſtand ſo much as we haue intreated of, I will nowe inſtruct you in Progreſsion.

Progreſsion.

ALthough vntill this day the moſt part of wryters haue defined Progreſsion as a compendious kinde of Addition, yet truely it is not ſo: for Progreſsion (as the verie nature of the worde dooth informe anie man) is a going forwarde and proceeding in numbers, and that regularly and orderly, whoſe place is aptly choſen to bee verie neere, or rather next after the expoſition of the foure principal parts of Arithmetike, for in it after a moſt eaſie maner, are all the foure former parts exerciſed & practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath bin the cauſe, why it hath ſo ſpecially bene named a kinde of Addition, and defined to be a quicke

and brieke Addition of diuers summes proceeding by some certaine and reasonable order.

You shall also vnderstand, that there are infinite kinde of Progressions, but for you (as yet) two are sufficient to be exercised in: of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall progression.

Arithmeticall Progression is a rehearsing or placing downe of many numbers, number after number, in such sort that betwene euerie two next numbers rehearsed or placed downe, the difference, diuersitie, or excesse bee equall and alike.

Scholer. Sir, I thanke you for that you haue both opened vnto mee what Progression is, truely, and also why it is here placed. But I pray you with an example make plaine your definition.

Master. Examples cannot want, seeing all reasonable creatures naturally vse the order of one kind of Arithmetical progression (which therefore is also named Naturall) whensoeuer they distinctly doe count or number any multitude by one, saying: 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, and euery one surmounting and exceeding his fellow next before by a like quantitie (which here is 1) declareth the same to be Arithmeticall Progression. And for the more plainnesse, I set it downe in this maner.

The

The common exceſſe 1 1 1 1 1



The Progreſsion 1 2 3 4 5 6

Scholer. This is moſt euident. And I thinke that I am able to tell you now of any Progreſſion Arithmeticall propounded, what is that common exceſſe or difference whereby it proceedeth if this order be kept in it.

Maſter. What ſay you of 3, 6, 9, 12, 15?

Scholer. They exceede each other by 3. And that may I ſet downe in ſuch euident order, as you did your example of naturall Progreſſion, in this wiſe.

The common exceſſe 3 3 3 3



The Progreſsion 3 6 9 12 15

Maſter. And do you not alſo now perceiue, that the whole Table of Multiplication may be made by the order of Progreſſion Arithmeticall: either if you will begin at the firſt number of any of them on the left hand, and ſo proceede right ouerthwart: or at any of the firſt numbers of the vpper row, and go directly downeward.

Scholer. I pray you let me conſider the thing a little, and I will anſwere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very well: for the common exceſſe or difference betweene any two next, is continually as much as the firſt number of euery row, either from the left hand ouerthwart taken, or from any of the vppermoſt ouerthwart rowes downward.

Maſter. Now then if of any ſuch Progreſſion you would ſpeedily know the totall ſum, much quicklier than by common rules of Addition: firſt tell how many numbers there are (which numbers here we call places or parcels) and if they bee odde, write their ſum downe by it ſelfe, as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7 as you may ſee: therefore ſet downe 7 in a place alone: then adde together the firſt number and the laſt, as in this example: adde 2 to 14 & that maketh

To know the totall ſumme of an Arithmeticall progreſſion

maketh 16, take half of it, & multiply by the 7, which you noted for the number of the ſame places, and the ſumme that amounteth, is the ſumme of all thoſe figures added togither, as in this example: 8 multiplied by 7 makes 56: and that is the ſumme of all the figures.

Scholer. That will I worke by another example. I would know how much this ſum is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tel the places, and they are 9, that I note. Then I put the firſt number 5, and the laſt 29, togither, and they make 34. I take the halfe of it, that is 17, and multiplie by 9, and it maketh 153. That you ſay is the ſumme of all the numbers.

Maſter. So ſhall you finde it if you trie it.

Scholer. How ſhall I trie it?

Maſter. By your common Addition: for if you adde all the parcels togither, you ſhall ſee the ſame ſumme amount, if you did worke well. And that maner of Addition trieth all kindes of ſumming any Progreſſion.

Scholer. Then can I ſumme a Progreſſion, if the numbers of the parcels be odde. But what if they be euen? as in this example, 1, 2, 3, 4, 5, 6, 7, 8.

Maſter. When the number of the parcels is euen, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the halfe of the number of the places doe you multiplie it: as in your example, the

parcels

parcels are 8, that I note: then adding the firſt ſume to the laſt, there amounteth 9, that doe I multiplie by the halfe of parcels, that is by 4, and it maketh 36, which is the ſumme of the parcels.

☞ But if you will take one rule for theſe both, doe thus. Multiplie the halfe of the one by the other whole, and the ſumme will amount all one. For ſometime it chaunceth that the number of the parcels bee odde, ſo that their halfe cannot be taken: and ſometime it chaunceth the Addition of the firſt number and the laſt, to bring forth an odde number, ſo that the halfe of it cannot bee taken: but they will neuer be both odde.

A generall rule.

Scholer. Then I perceiue this, if there be no more longing to it.

Maſter. This is inough for Progreſſion Arithmeticall: howbeit there is an other maner of Progreſſion called Geometricall, when the numbers increaſe by a like proportion, that is, if the ſecond number conteyneth the firſt, 2, 3, or 4, times, and ſo forth: then the third containeth the ſecond ſo many times alſo: and ſo the fourth the third, and the fifth the fourth, where fore I ſet theſe three examples.

Progreſſion Geometricall.

3,	6,	12,	24,	48,
1,	3,	9,	27,	81,
2,	10,	50,	250,	

Here in the firſt example you ſee, that euerie number containeth the other (that goeth

next before him) 2 times: and in the second example 3 times, in the third example 5 times. Now if you would know how to finde easily the summe of any such numbers, doe thus: Consider by what number they bee multiplied, whether by 2,3,4,5, or any other, and by the same number doe you multiplie the last summe in the Progression.

Scholer. I pray you worke it by this example, 2,8,32,128,512,2048, which I haue framed by proceeding from 2, and continually multiplying by 4.

Master. Then must I multiplie the last summe (which is 2048) by 4 also, and it will be, 8192. Now must I bate from this summe the first number of this progressiō, which here is 2, then resteth 8190, which summe I must diuide by 1 lesse then was the number that I multiplied by. Seeing then I multiplied by 4, I must diuide by 3, so diuiding 8190 by 3, the Quotient will bee 2730, which is the summe of all the Progression. And nowe to prooue whether you can doe the same, I giue you these numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholer. I cannot well tell by what number this Progression doth encrease.

Master. In any such doubt, do thus: Diuide the second number by the first, and the quotient will shew you the number that engendereth

gendreth the Progression.

Scholer. Then is that number in this example, 5, for so many times is three in 15.

Master. So is it. Now worke as I taught you.

Scholer. The last number is 46875, which I multiplie by 5, and it yeeldeth 234375, frō which I abate the first number of the Progression, that is 3, and there resteth 234372, which I diuide by 4, for that is one lesse than 5, and the quotient is 58593, which is the whole summe of the Progression.

Master. If you remember well this, you haue learned the Art of Progression, both Arithmeticall, and also Geometricall, which you may proue either by Subtracting of each number alone from the summe, and so will there nothing remaine: or else by adding togither of all the parcels, for so will the same summe amount.

¶ And nowe for the vse and better vnderstanding of both these rules, I will propone vnto you certaine pleasaunt and necessarie questions Arithmeticall and Geometricall: And to the performance of ther workings, such necessarie rules and documents, as are requisite for the vnderstanding of them, or any such like.

A certaine Mercer sold 20 yards of veluet to be paid in 12 weekes, by Arithmeticall proportion: that is to wit, to receyue the first

weeke

weeke 6 ſhillings, the ſecond weeke 12 ſhillings, the third weeke 18 ſhillings. And ſo fooꝛth increaſing the number of weekes by 6 ſhillings, till the twelfth and laſt weeke were expired. The queſtion is how many poundes he had foꝛ 20 yards of veluet.

To the perfoꝛmance of this queſtion and ſuch other the like, I ſet foꝛth the 12 paiments in ſuch ſoꝛt, as foꝛ example here appeareth in the margent.

6
12
18
24
30
36
42
48
54
60
66
72

Then touching the adding togither of theſe ſummes without the ayde of Addition, accoꝛding to the rules I taught you in Pꝛogreſſion Arithmeticall, I note the number of the places, which are 12, then adding the laſt number of the Pꝛogreſſion which is 72, and the firſt number togither make 78, and multiplying 78 by halfe the number of the places, which is 6, amounteth to 468 ſhillings, and in pounds maketh 23 ℔, 8 ſ. And ſo much hath the Mercer foꝛ his 20 yards of veluet, which is nie about 23 ſ, 5 d, a yard.

Scholer. I vnderſtand this woꝛke verie well: But is there anie pꝛooſe foꝛ the iuſtifiing hereof, as you haue of other woꝛkes.

Maſter. The woꝛke of it ſelfe (being ſo perfitely wꝛought) that in your pꝛoceeding and going foꝛward from number to number, each number exceeding his fellow by an equall oꝛ like quantitie, is all that is demaunded foꝛ iuſtifying of the ſame: yet notwithſtanding be-

cauſe

cauſe your requeſt is reaſonable, I will propone an example for the proofe hereof.

A certaine man is bounde to pay for 20 yardes of Uelluet, the ſumme of 23 ℔, 8 s̄. And it is to be paid weekly in 12 weekes or terms by Arithmeticall Progreſsion. The queſtion is therefore to know with what number the ſame progreſsion is to be begun, and continued in ſuch equall proportion Arithmeticall, that in 12 weekes the ſame may iuſtly bee accompliſhed.

The proofe of the laſt queſtion.

For the ſolution whereof, and of all ſuch other like, reduce 23 ℔, 8 s̄, all into ſhillings, which maketh 468 ſhillings.

A generall rule.

Then adde 1 vnto 12 the number of the termes, it maketh 13, which 13 you ſhall multiplie by halfe the number of the termes which is 6, it maketh 78: then diuide 468 by 78, and you ſhall find 6 in the quotient, which is the true number that ſhall begin and continue the ſayd Progreſsion. That is to ſay, the firſt weeke 6 s̄, the ſecond 12 s̄, the third week 6 s̄ more, which is 18 s̄, and ſo euery weeke as they riſe, 6 s̄ more than the weeke before, as is manifeſt in the queſtion aforeſaid.

but if y terme be odd put nothinge ꝑ to but multiply y m by y better halfe as 7. by 4. y of as if y tearme were 13 adde not 1. but multiplie y m by y better halfe viz 7.

A Farme is to bee ſolde to bee paid by the weekes in a yeare, the firſt weeke to pay 4 s̄, the ſecond weeke 8 s̄, the third weeke 12 s̄, and ſo forth encreaſing each number by 4 till the number of 52, which are the number of weekes in a yeare be expired. The queſtion is

what

what the price of the farme commeth to.

Scholer. I doubt not but by that you haue alreadie taught me, to end this queſtion verie well, wherefore I ſet foorth the Progreſſion with his exceſſe 52 times.

Maſter. Nay ſtay a while. And here for your further eaſe, (to abridge you of great labour that appeareth to fall out in this queſtion, and ſo may do in many other the like, if a queſtion were propoſed of 100 or 200 places or moe, and that this queſtion nor any other the like can be ended, vnleſſe you know abſolutely what the laſt number of the Progreſſion at the 52 place is or ought to bee) I will giue you a generall rule how to know the laſt number of any Progreſſion Arithmeticall, as well as if you had ordinarily proceeded by continual Addition, till you had come to the laſt worke, which is this.

Multiplie the exceſſe by a number leſſe by one then the number of the places, and thereto put the firſt number of the Progreſſion, and you ſhall haue your deſire. A geaerall rule.

Scholer. This rule is well worth the noting: for if I vnderſtand you aright, I conſider that my exceſſe is 4, which I multiplie by 51, which is one leſſe then the number of the places, and it maketh 204: whereunto I adde the firſt number of the Progreſſion which is 4, and then it is 208: which you ſay is or ſhould be the laſt number of the Progreſſion.

Maſter. This is a moſt approoued truth if there were neuer ſo many places.

Scholer. This rule is ſo eaſie, that I were much to blame, if I do not remember it. For by the benefite hereof I haue ſuch an eaſe and light into this excellent arte, that my firſt entrance doth ſeeme to paſſe a great many mens further ſtudie and longer continuance.

Maſter. Many moe conſiderations coulde I propound you in theſe Arithmetical progreſſions: but theſe are ſufficient for a taſt, to giue you occaſion to thinke that rules of knowledge and arts are infinite capable of enlargement.

Scholer. Happie were I, if I did but well vnderſtand that which is alreadie inuented and written. But theſe things in my ſimple fantaſie offer themſelues to be greatly beneficiall into the aide of Progreſſion. Therefore nowe I will go forward with your queſtion.

Now conſidering that the 52 & laſt place is 208, I adde thereunto the firſt number of the Progreſſion which is 4, it maketh 212, which I multiplie by halfe the number of the places, which is 26, & it amounteth to 5512 ſhillings. And ſo much is the totall ſumme or Addition of this Progreſſion: which maketh 275 l, 12 s, as appeareth here by my tables.

Maſter. I like well your labour, and commend you for your diligence. I will here propone one example moze, and therewithall for

this

this time will ende progreſſion Arithmeticall.

A certaine man bought 20 Elles of Holland to bee paide in 17 weekes or termes by progreſſion Arithmeticall. And the firſt weeke to pay 1 s̄,8 d̄, the ſecond weeke 3 s̄, 4 d̄, the third weeke 5 s̄,the fourth weeke 6 s̄,8 d̄, and ſo forth each weeke ſucceeding 20 pence more than the weeke before. The queſtion is what the ſumme of his 20 elles commeth to.

Scholer. Becauſe here is mention made both of ſhillings and pence, I feare there is ſome harder matter contained herein, than in the other before,therefore I pray you worke it your ſelfe,and I will diligently marke your labour.

Maſter. There is no more to bee done in this, than in the other before, but becauſe your requeſt is ſo reaſonable, be attentiue vnto me.

Firſt by our generall rule I ſeek to finde out the laſt number in the 17 place, what this Progreſſion ought to be. Therefore herein my tables multiplying the exceſſe 20 by 16, which is 1 leſſe than the number of the termes or places, and it commeth to bee 320, and thereunto adding the firſt number of the progreſſion which is 20 pence, all is 340 d̄ or 28 s̄,4 d̄: for ſo much ought the laſt number of the paiments to be.

Then finally to know what the whole 17 places amount vnto, I adde the firſt number

of the Progression and the last togither, both which make 360: Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360 which is 180: and multiplying 180 by 17, it commeth to 3060 pence, which maketh as you see by diuision 12 ℔, 15 s̄. And so much is the buier to pay for his 20 elles of Holland. Which 3060 pence if you diuide by 20, the number of elles that was bought, you shall finde 12 s̄, 9 d̄, and so much paide he for an Elle one with onother.

The proofe.

A certaine man doth owe 12 ℔ 15 s̄, to bee paide in 17 weekes or termes by Arithmeticall Progression. The question is to knowe with what number hee shall begin and continue the Progression in such equall proportion, as the same may be truely payde and satisfied in 17 weekes.

The answere.

First I reduce 12 ℔, 15 s̄, all into pence: which as you see here in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiplie by halfe the number of the weeks or termes, which is 8 ½, which 8 ½ multiplied

tiplied by 18, cannot well be done vnleſſe you were acquanted with Fractions or broken numbers, therfore you ſhall let that paſſe, and multiplie 17 by the halfe of 18: which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$: and the multiplication of 9 into 17 maketh as you ſee 153, with which number you ſhall diuide the 3060 pence before ſayde: and the quotient bringeth forth 20 pence, which is the firſt number or payment to begin the progreſſion withall: and ſo each weeke ſucceeding to riſe 20 d more than the weeke before, and thereby in 17 weekes ſhall the 12 l. 15 s. be paide: as before was ſufficiently declared. Thus much for progreſſion Arithmeticall.

Scholer. Certainely ſir I know not how to render you condigne thankes for theſe benefites ſhewed me, which me thinketh are ſo eaſie, delightfull and pleaſant, that I count my ſelfe happie to be in your companie.

Maſter. I am glad you delight ſo well herein, which is an arte of woonderfull dexteritie to all ſorts of men of what degree or profeſſion ſo euer they be. And now will I propone a queſtion or two of progreſſion Geometricall.

Progreſsion Geometricall.

A Mercer had 12 yardes of ſatten which he valueth at 16 ſhillinges the yard, and ſelleth the ſame 12 yards to another man to be paide as followeth. That is to wit for the firſt yard to haue one ſhilling, for the ſecond yard 2 s:

for the third yard 4 s̄, for the fourth yard 8 s̄, and so forth doubling each number following till the twelfth and last yarde. The question is who hath made the better bargaine of the buier or seller.

First you may set downe 12 the number of the yards, as you see here in this example. And against each number the number of shillings due to be paide, as the order of Duplation or the multiplication by 2 teacheth.

s̄	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12

Then resorting to the adding vp or summing of this progressiō, where I consider that the increase of this sum proceeded by the multiplication of 2. And therfore after I haue drawne a line vnder the 12 worke, and multiplie the last summe by 2 also, and it yeeldeth 4096: from whence I abate the first number of the Progression which is one, and then resteth 4095: which I should diuide by 1 lesse than I did multiplie by, but seeing it is one, I neede not to diuide it: for 1 (as I haue sayde before) doth neither multiply nor diuide, therefore I take that summe 4095 for the whole summe of the shillings, which by Reduction

One doth ney deuide nor multiply

amoun-

amounteth to 204 ℔, 15 s̃, and ſo much hath the Mercer for his 12 yards of ſatten: which is 17 ℔, 1 s̃, 3 d̃ a yard. But I thinke you will buie none ſo deare.

Scholer. No ſir by the grace of God this yeare.*

Maſter. Then what ſay you to this queſtion: If I ſold vnto you a horſe hauing 4 ſhooes, and in euerie ſhooe 6 nailes, with this condition, that you ſhall pay for the firſt naile 1 ob: for the ſecond naile 2 ob: for the third naile 4 ob: and ſo foorth doubling vntill the ende of all the nailes, Now I aſke you how much would the price of the horſe come vnto?

Scholer. Firſt to know the number of the nailes, I muſt multiplie 6 by 4, and it maketh 24. Then will I do thus: I will write the number of the nayles euerie one in order from 1 to 24, and againſt each number of the nailes the ſumme of halfe pence duely, as the order of Duplation or Multiplication by 2 teacheth, and as in this next figure following appeareth.

Then doe I reſort to the rule of ſumming vp this Progreſſion, where I conſider that the increaſe of this ſumme proceedeth by the multiplication of two, as the laſt example dis. And therefore multiplying the laſt ſumme by 2 alſo, and it yeeldeth 16777216, from which I abate the firſt number which is 1, and then reſteth 16777215, which I ſhould

1	1	diuide by 1 leſſe than I
2	2	did multiplie: but ſeeing
4	3	that it is 1, I neede not
8	4	to diuide it, for 1 (as you
16	5	haue before ſayde) dooth
32	6	neither multiplie nor di-
64	7	uide, therefore I take
128	8	that number 16777215
256	9	for the whole ſumme of
512	10	the halfe pence, which by
1024	11	Reduction I finde to be
2048	12	699050 s̄, and 7 d̄ ob:
4096	13	that is, 34952 l̄, 10 s̄,
8192	14	7 d̄, ob.
16384	15	Maſter. That is well
32768	16	doone, but I thinke you
65536	17	will buie no horſe of the
131072	18	price.
262144	19	Scholer. No ſir if I be
524298	20	wiſe.
1048576	21	Maſter. Well then
2097152	22	anſwere me to this que-
4194304	23	ſtion.
8388608	24	A Lord deliuered to a

Bricklaier a certaine nũ-ber of loades of Bricke, whereof hee willed him to make 12 walles, of ſuch ſort, that the firſt wall ſhould receyue 2 thirdeles of the whole number: and the ſecond 2 thirdeles of that that was left. And ſo euerie other 2 thirdeles of that that remained: and ſo did the bricke-

Bzickelayer : and when the 12 walles were made, there remaineth 1 load of Bzicke.

Now I aſke you, how many load went to euerie wall, and how many loade was in the whole?

Scholer. Why ſir it is vnpoſſible for mee to tell.

Maſter. Nay it is verie eaſie if you marke it well. Marke well that I ſayd, that euerie wall ſhould receiue 2 thirdeles of the ſumme that was left. Now take away 2 thirdeles from any ſumme, and you muſt needes grant that that which remaineth is 1 thirdele of the ſumme laſt before : example of 9, from which if you take 2 thirdeles, there will remaine 3, which is one thirdele of 9. Likewiſe from 3 bate 2 thirdeles: and there will remaine 1.

Scholer. This is true, and now I perceiue, that the leaſt wall had but two loade of Bzicke.

Maſter. And by the ſame reaſon may you know how manie loade euerie wall had, according as this figure following doth ſhew, and likewiſe what the whole ſumme of bzickes was: for if you make 12 ſummes, multiplying by 3, ſtill from the laſt remainer, as you may ſee here on the left ſide of the table, there will appeare all the remainers of the whole wall : and if you multiplie the laſt of thoſe 12 ſummes by 2 alſo, then will that be the ſumme of the loades which were deliuered to the

bzicke-

Brickelayer.

Againe, if you doe double euerie remainer, as you may see at the right side of this table, those numbers will shew the summe of loades that went to each wall: whereby you may perceyue, that each wall was 3 times so great as the next lesser.

Scholer. Loe, now it appeareth easie enough. Now surely I see that Arithmetike is a right excellent art.

Master. You will say so when you know more of the vse of it: For this is nothing in comparison to other poyntes that may bee wrought by it.

Scholer. Then I beseech you sir, cease not to instruct me further in this wonderfull cunning.

The remainer after euerie wall.			Loades due to each wall.
1	12	2	
3	11	6	
9	10	18	
27	9	54	
81	8	162	
243	7	486	
729	6	1458	
2187	5	4374	
6561	4	13122	
19683	3	39366	
59049	2	118098	
177147	1	354294	
Summe of the	531441		Loades deliuered.

The

The Golden Rule.

Maſter.



By order of the ſcience (as men haue taught it) there ſhould follow next the extraction of rootes of number, which becauſe it is ſomwhat hard for you yet, I will let it paſſe for a while & will teach you ỳ feate of the rule of Proportions, which for his excellencie is called the Golden rule. Whoſe vſe is, by three numbers knowne, to finde out another vnknowne, which you deſire to know, as thus. If you pay for your boorde for three monethes 16 s̄, how much ſhall you pay for 8 moneths.

To know this and all ſuch like queſtions, you ſhall conſider which two of your 3 numbers be of one denomination, and ſet thoſe two the one ouer the other, ſo that the vndermoſt bee it that the queſtion is aſked of: as in my queſtion 3 and 8 be both of one denomination, for they both be moneths, and becauſe 8 is the number that the queſtion is aſked off, I ſet them one ouer the other, and 8 vndermoſt, thus, with ſuch a crooked draught of lines. Then doe I ſet the

3
8

other

other number which is 16, against 3 at the right side of the line, thus.

3 ⊢ 16
8 ⊣

And now to know my question, this must I doe: I must multiplie the lowermost on the left side, by that on the right side, and the summe that amounteth, I must diuide by the highest on the left side. Or in plainer wordes thus: I shall multiplie the number, of which the question is asked (which is called the third number) by the number of another denomination, (which is called the second,) and the summe that amounteth must I diuide by the summe of like denomination, which is called the first. Then for the knowledge of this question, I multiplie 8 into 16, and there amounteth 128, which I diuide by 3, and it yeeldeth 42 shillings, and 2 s̄ remayneth, which I turne into pence, and they bee 24 d̄, of which the third part is 8 pence, so the third part of 128 s̄, is 42 s̄, 8 d̄: which summe I write at the right hande of the figure agaynst 8, thus.

The third number.
The second number.
The first number.

3 ⊢ 16 s̄.
8 ⊣ 42 s̄, 8 d̄.

Hereby I knowe, that if three monethes boording do come to 16 s̄, that 8 moneths boording will come to 42 s̄, 8 d̄: and likewise of any other like question.

But here must you marke, that the first number and the third bee of one denomination, and also the second and the fourth, the

which

which you seeke: or else be of such denominations, that you in working may bring them into one. As if a man should aske mee this question.

¶ Twelue weekes iourneying cost me 14 French Crownes at 6 s the peece, how many pounds is that in one yeare. Here you see no two numbers of one denomination: but yet in working you may turne them into like denominations, as thus: turne the one yeare into 52 weekes, and the fourth summe will be French Crownes, by the order of the working: Then to knowe this question, multiplie the third summe 52 by the second 14, and the summe will be 728: that diuide by your first number 12, and the quotient will bee 60 Crownes: and 8 crownes remayning: which if you turne into shillinges, they will bee 48 shillings, which if you doe diuide by your first number 12, the quotient will be 4: which signifieth 4 s, but these 60 French Crownes (which make 18 pounds, with the 4 shillings: for the summe that answereth to the question: and it is the iust expences of a yeare: And the sum will be thus.

13		14	
52		60	4 s

And take this euermore for a generall rule touching this whole Arte, that the doubtfull or vnknowne number that you would bee resolued of, shall alwayes bee set in the thirde place. Note also the first number and the third

must

must euer bee of one nature and denomination, or else must in working be brought to like denomination, and then of necessitie must the other number be in the second place.*

Remember also, that the place of the first number is the highest on the left side: and the place of the second right against it on the right side: the place of the third number is vnder the first, as by those examples you haue seene.

Scholer. This I trust I can doe.

Master. But and if the question bee asked thus: In 8 weekes I spende 40 s, how long will 105 shillings serue mee? Here you see that 8 weekes answeres himselfe, and sayth 40 shillings. But how long time 105 shillings will serue, you know not. Therefore you shall set 105 in the third place, according as I tolde you euen now. And the first place must alwayes bee of the same nature or Denomination, that the thirde is of, which here is 40. Then must 8 needes bee that other. Now multiplie 105 by 8, and it will be 840, which if you diuide by 40, it will yeelde 21, which is the fourth number, and sheweth how many weekes 105 shillings will serue, if you spend 40 s̄ in 8 weekes.

The figure of this question is this: as if you should say: If 40 s̄ serue for 8 weekes, 105

shillings.	*weekes.*
40	8
105	21

serue

ſerue for 21 weekes.

Other diuerſities there bee of working by this rule, but I had rather that you would learne this one well, then at the beginning to trouble your mind with many formes of working, ſith this way can do as much as all the other, and hereafter you ſhall learne the other more conueniently.

¶ And for your further aid and inſtruction to make you better acquainted with this Golden Rule, I haue here proponed 6 queſtions and there anſweres, which I thinke moſt conuenient and meete to preferre the deſirous to perfect vnderſtanding. The firſt foure are all braunches of one queſtion ſprung out of the beſt tree, (for a young learner to taſte of) that groweth in this Ground of Artes, for that no maner of queſtion in the Rule of 3 whatſoeuer it be, can be proponed, but it muſt be comprehended vnder the reaſon or ſtile of one of theſe foure.

The queſtions be theſe.

If 15 Elles of Cloth coſt 7 l, 10 s̃: what comes 27 elles to at that price: Anſwere 13 l, 10 s̃.

If 27 Elles coſt 13 l, 10 s̃: what are 15 Elles worth: Anſwere. 7 l, 10 s̃.

If 27 Elles coſt 13 l, 10 s̃: how many elles ſhall I haue for 7 l, 10 s̃. Anſwere. 15 elles.

If

If I sell 15 els for 7 ℔, 10 s̄: how many els are to be deliuered for 13 ℔, 10 s̄. Anſ. 27 elles.

If 8 pound of any thing coſt 16 s̄, 6 d̄: what money is to be receiued for 49 pound: Anſ. 5 ℔, 1 s̄, 0 d̄. $\frac{3}{4}$.

If 4 ℔ of any thing coſt 17 d̄: what money will 8765 pound of that commoditie coſt: Anſwere. 155 ℔, 4 s̄, 3 d̄, q̄.

Of all which queſtions I omit the woꝛke of purpoſe, ẏ you ſhould whet your wit thereby at conuenient leyſure, to climbe each bꝛaunch, and gather the fruit of them, and doe minde now, beforꝛe wee make an ende of this rule, to giue you ſome inſtruction of the Backer rule of 3, whoſe oꝛder is quite contrarie to this that you haue learned. *

Foꝛ in this Rule hitherto, euermoꝛe looke how much the thirde number is greater than the firſt, ſo much the fourth number is greater than ẏ ſecond. And contrarie waies: looke how much the firſt ſum is greater than the third, (if it doe chance ſo) ſo much is the ſecond ſum greater than the fourth. But in this Rule, there is a contrarie oꝛder, as this: That the greater the third ſumme is aboue the firſt, the leſſer the fourth ſumme is beneath the ſecond: and this rule theꝛefoꝛe you may call the Backer rule, as in exꝛample.

The backer Rule.

If I haue bought 30 yardes of cloth, of 2 yardes bꝛeadth, and would buie canuas of 3 yardes bꝛoade to line it withall, how manie yards

Queſtion of buying cloth.

yards should I neede?

Scholer. Why, there is none so broad.

Master. I do not care for that, I do put this example onely for your easie vnderstanding: For if I should put the example in other measures, it would bee harder to vnderstande. But nowe to the matter: if you would know this question, set your numbers as you did before: but you shall multiply now the first number by the second, and that ariseth thereof, you shall diuide by the third: which thing if you doe here, I meane if you multiplie 30 by 2, it will be 60: which summe if you diuide by 3, there will appeare 20: whereby I know, that if 30 yardes of cloth of two yardes broad, should be lined with canuas of three yards broad, 20 yards of canuas would suffice, as this figure sheweth.

Breadth. *Length.*
2 — 30
3 — 20

And now because yee found fault with my example, how say you, perceiue you this?

Scholer. Yes sir, I suppose.

Master. Then answere me to this question: how many elles of canuas of elle breadth, will serue to line 20 yards of Say, of three quarters broad.

Scholer. In good faith sir, I cannot tell, for I know not how to bring the summes to like denominations.

Master. Then will I tel you: sith there is mention here of quarters, and againe euerie one of the measures both elles and yards may be parted into quarters, doe you part them so both in the breadth and length, and then put forth the question by quarters.

Scholer. Then I shall say thus. How many quarters of canuas of fiue quarters broad, will line 80 of 3 quarters broad.

Master. Now answere to the question.

Scholer. First I will set them downe in their forme thus, for 5 is ioyned with the question, and is therfore the third number: then is 3 the number of the same denomination, I meane because they be both referred to breadth. Now I multiplie 80 by 3, and it is 240, which I diuide by 5, and it yeeldeth 48. Then say I, that 48 quarters of 5 quarters broad, will suffice to line 80 quarters of 3 quarters broad.

	Breadth.	*Length.*
	3	80
	5	

Master. Turne the quarters againe into elles and yards.

Scholer. Then I say, that 9 elles & three quarters of a yard of elle broad, will serue to line 20 yardes of three quarters broad, as this figure sheweth.

Breadth.	*Length.*
3	80
5	48

¶ Master. Nowe what say you to this que-

queſtion: I lent my friend 400 ℔ for 7 moneths, howe much money ought hee to lende me againe for 12 moneths, to recompence my courteſie ſhewed him. Can you anſwere to this?

Scholer. Yes Sir I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I diuide by 12, and it yeeldeth 233 ℔, and there is 4 ℔ remaining of my Diuiſion, what ſhall I do therwith?

Moneths. *Pounds.*
7 — 400
12 —

Maſter. Turne the ſame 4 ℔ into s̄, and then diuide it by 12 as you did before.

Scholer. Well ſir it ſhall be done, ſo haue I 6 s̄ for my quotient, and yet remaineth 8 s̄ vpon my diuiſion.

Maſter. You muſt alſo reduce that 8 s̄ into pence, which maketh 96, and diuide that alſo by your firſt diuiſor.

Scholer. So haue I done, and I finde 8 pence for my quotient, and nothing is left.

Maſter. This muſt you alwayes do, when any thing remaineth vpon your diuiſion: whether it be money, waight, meaſure, or any kinde of thing whatſoeuer. This rule is ſo profitable for all eſtates of men, that for this rule onely (if there were no more but it) all men were bound highly to eſteeme Arithmetike.

By this rule may a Captaine in warre worke many things, as Master Digges in his Stratiaticos doth declare: onely nowe in this my simple Addition, for a tast and incouragement, I will inlarge the Authour with a question or two more, wishing you, and euery my Countrymen or Gentlemen whatsoeuer, that by nature bee anie thing giuen to Militarie affayres, to bee familiar and acquainted with this excellent Art, the which he shall finde not onely at the Sea, but also in the Campe and fielde seruice, aboundantly to aid him, either in fortification, paying of souldiers wages, charges of ordinance, poulder, shot, munition, and instruments whatsoeuer, as for example.

Question of prouision touching an armie.

If it should chance a Captaine which hath 40000 souldiers to bee inclosed with his enemie, that hee could haue no fresh purueyance of victuals, and that the victuals which he hath, woulde serue that armie but onely 3 moneths, how many men shoulde hee dismisse, to make the victuall to suffice the residue 8 moneths?

Scholer. As you taught me, I set the numbers, thus, saying: If 3 moneths suffise 40000, to how many will 8 moneths suffise?

Moneths.	*Men.*
3	40000
8	

To know this, I multiplie the first num-

ber

ber 3 into the second 40000, and it yeeldeth 120000, which summe I diuide by 8, and there will be in the quotient 15000, which if I doe subtract from 40000, the remainer will declare that he must dismisse 25000 as this figure sheweth.

Moneths. Men.

3 — 40000

8 — 15000

¶ Master. Now answere me to this question: If 136 Masons in a moneth be able to build a fort to preserue the Souldiers from the enemie, and such expedition requireth that I would haue the same finished in eight daies, how many workemen say you is there to bee appoynted?

Schol. As you taught mee I set the numbers thus, saying: if 28 daies require 136 Masons, what number of men by the like proportion will 8 dayes require.

28 — 136

8 —

To know this, I multiply the first number 28 into 136: and it yeeldeth me 3808: which I diuide by 8. And my quotient is 476, which is the iust number of Masons that shall supplie this worke. And now mee thinke these questions are verie easie.

Master. Truely if you take delectation herein, you shall finde this Art not onely easie, but wonderfull pleasant and profitable. Now therefore one question more I will pro-

pose, and so leaue of this rule in whole numbers, vntill we come to the vse of it in broken numbers, for had you the vnderstandinge of broken numbers perfitely, not onely in this rule, but in all other, the question that in sight or apparance seemeth to be 100 times more harder to absolue, may thereby bee wrought as soone, or sooner than this.

Scholer. Your wordes doth greatly encourage mee to bee studious to attaine whole numbers, but might I once attaine to bee a practitioner in broken numbers, I shoulde thinke my selfe happie.

Master. What say you then to this question: If 48 Joiners in 2. daies make 200 light horsemen staues (esteeming they worke but 12 houres a day) and such neede requireth that 384 Joyners are set to the finishing of those 200 staues, in what time say you, will they make them vp.

Scholer. I see heere that I must turne my 2 dayes into houres. And so doing I set my numbers thus.

48 — 24
384 —

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded vppon an olde prouerbe, many handes make quicke speede.

I multiplie 48 into 24, and it amounteth to 1152, which I diuide by 384: and my quotient

quotient 3 houres, which is my desire.

I take this for a note worthy the marking, either in the rule of three, forward, or backeward, when the two ummbers are multiplied togither, the product is of the same nature and denomination that the seconde number is of. * Note.

Master. Well, sith you perceiue now the vse of this rule, I will shew other which ensue of the same, and first the double rule, which is so called, because there is in it double working, by which thing onely it differeth from this. The double rule.

Scholer. Then by an example I shall vnderstand it well enough.

Master. So shall you, and let this bee the example: If the cariage of 1 C waight (that is to wit 112 pound) 30 myles, doe cost 12 d, how much wil the cariage of 5 C waight cost, being caried 100 miles? Of cariage.

Scholer. I pray you shew me the working of it.

Master. You must make 2 workings of it: the first thus. If 1 C. waight cost 12 d, how much wil 5 C waight cost? Set your figure thus.

C. waight.	*Pence.*
1	12
5	

And multiply 5 by 12, and thereof amounteth 60, which if you diuide by 1, the quotient will be still 60, that is the price of 5 C

weight for 30 miles.

Then begin the second worke, saying: If 30 miles cost 60 s̃, how much will 100 miles cost? Set your figure thus.

Miles.	*Pence.*
30	60
100	

Then multiplie 100 by 60, whereof amounteth 6000, which being diuided by 30, will yeeld 200 d. Then you may say that so many pence shall cost the carriage of 500 pound weight 100 miles, after the rate of 12 pence for the 100, caried 30 myles.

Scholer. Now I perceyue it also.

¶ Master. These and such other like questions of the double rule of 3, are to bee answered much sooner, at one onely working by the rule of 3 composed of 5 numbers, which anon I will shewe you, and then when you haue the vse thereof, you may vse which way you thinke good.

Scholer. Sir I thanke you much for your courtesie, and I long now till this rule be ended, that I may see how I shall behaue myselfe with that newe rule of 5 numbers, for that I haue euer since you taught me hitherto, in the Golden Rule both forward and backeward wrought but with three numbers onely. *

Question of lowing.

Master. But yet a while wee will go on forward with this double rule of 3: therefore answere me to this question: 30 bushels of

wheate

wheate sowed, yeelded in one yeere 360: how many will 80 bushels yeelde in 7 yeere: I meane sowing euery yeere of those 7, still 80 bushels.

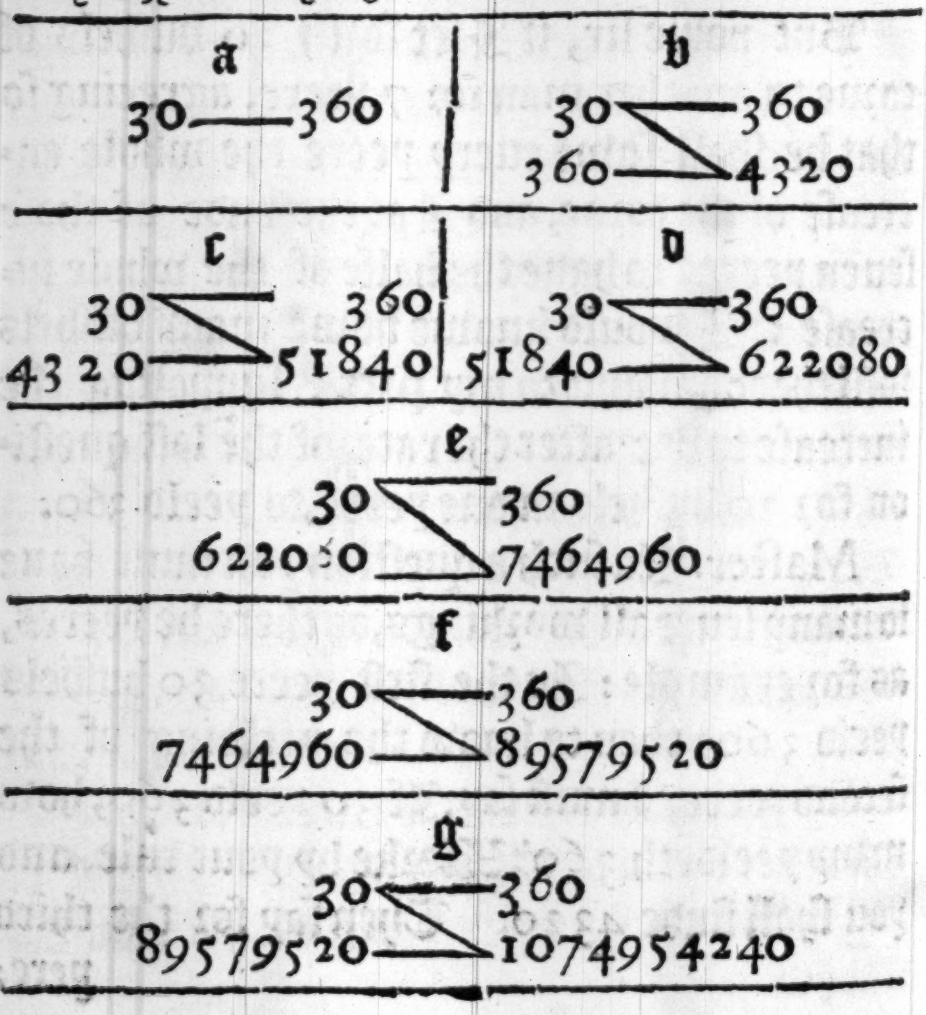
Scholer. First I say, that if 30 bushels will yeeld 360 in 1 yere, then 80 bushels will yeeld 960 in 1 yeere. Then for the seconde worke I say: If one yeere yeeld 960, then 7 yeere will yeeld 6720: as these two figures do shew.

Seede.	*Encrease.*	*Yeares.*	*Encrease.*
30	360	1	960
80	960	7	6720

But nowe sir, if I set forth 30 bushels of corne to another man for 7 yeere, agreeing so that he shall sowe euery yeere the whole encrease of the corne, and I at the ende of those seuen yeeres to haue the halfe of the whole increase: I would knowe howe many bushels will there amount to my part, supposing the increase to bee after the rate of the last question, for 30 bushels in one yeere, to yeeld 360. Question of corne.

Master. In such a question you must haue so many seuerall workings, as there be yeeres, as for example: In the first yeere 30 bushels yeeld 360: then to know the yeelding of the second yeere, I must say: If 30 yeeld 360, how many yeeldeth 360? Worke by your rule, and you shall finde 4320. Then say for the third yere:

yeere: If 30 yeeld 360, how many will 4320 yeelde? you shall haue 51840, and so euerie yeere multiplying the whole increase by 360, and diuiding it by 30, the increase of the next yere will amount, as these 7 figures following doe orderly declare: where I haue set 7 letters for the 7 yeres, of the which the first is set without art, because that is the increase which you doe presuppose: and the last number of each other doth shew the increase of the yeare that it standeth for, which the letters doe declare, so that the increase of the seuen yeere, is 1074954240 bushels: howe many quarters that is, and also howe manie wayes, you may by Reduction soone finde.



Now with one queſtion moꝛe I will pꝛoue you. If 6 mowers doe now 45 Acres in 5 dayes, howe many mowers will mowe 300 acres in 6 dayes?

Scholer. If 45 acres require 6 mowers, then 300 acres requireth 40. Now againe: if 5 dayes require 40 mowers, then 6 dayes needeth but 33 mowers.

Maſter. Why do you not make mention of the 2 that remaineth in the laſt Diuiſion? foꝛ the laſt part of the queſtion is wꝛought by the Backer rule, where the firſt number 5, is multiplied into the ſecond that is 40, whereof amounteth 200, which if you diuide by the third number 6, the quotient will be 33, as you ſayde, but then will there remaine 2, which cannot well be diuided into 6 partes: howbeit, you may vnderſtand by the firſt part of 2, the third part of one mans woꝛke, which you muſt put to the 33, oꝛ elſe you muſt ſay, that 33 woꝛkemen will ende all the 300 acres in 6 dayes, ſaue two mens woꝛke foꝛ one day, oꝛ 2 dayes woꝛke foꝛ one man. But ſuch bꝛoken numbers called Fractions, you ſhall hereafter moꝛe better perceyue, when I ſhall wholy inſtruct you of them.

¶ Maſter. Yet one queſtion moꝛe of field matters I will pꝛopone, and ſo I will make an end of this double Rule of 3.

Scholer. With all my heart ſir I thanke you, and I will diſpatch it as ſoone as I can

because I would faine see the order of the next rule of 5 nnmbers.

Master. If a Captaine ouer a band of men, did set 300 Pioners a worke, which in 8 houres did cast a trench of 200 Rods: I demaund howe many Labourers will bee able with a like trench in three houres to entrench a campe of 3500 Rods.

Scholer. I thinke I am now in the Backhouse ditch, for I knowe not well which way to goe about it: And besides that, truely I thinke I shall neuer come to preferment that way my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and seruice into the fauour of your Prince, for the auaile of your Countrey.

Example for Nauigation: Sir Francis Drake, a man greatly honoured for his knowledge, is not the tallest man, and yet hath made as great a venture for the honor of his Prince and Conntrey, as euer English man did.

Scholer. Sir, I thanke you for your good encouragement. My minde though I be little is as desirous of knowledge, as any other: I haue pondred now a litle of it, and thus I set forth the worke.

Rod.		*Men.*
200	—	300
Rod.		
3400		

Say-

Saying, if 200 rod require 300 men, what shall 3400 rod require? I multiply 3400 by 300: and it yeeldeth 1020000: which I diuide by 200, and my quotient is 5100 men.

Then I must say for my second worke, if in 8 houres 5100 men bee able to discharge it, howe many shall performe the same in 3 houres? Now if I would worke by the Golden Rule of proportion forward, I should find a lesse number of men, because 3 houres is lesse then 8 houres: but because reason teacheth mee that the lesser the time is, wherein the trench must be made, the more labourers I ought to haue, thereupon I vse nowe the Backer Rule, as in example. And I haue in my quotient 5000. So many Pioners must I haue to entrench the Campe in three houres.

Master. You haue answered the question verie artificially: And truly I commend you for your diligence and apt vnderstanding: and now according to my promise, I will (in whole numbers) giue you a little taste of the Rule of 3 compounded of 5 numbers.

The Rule of 3 compound of 5 Numbers.



This Rule of 3 composed, is distinct for most needful questiõs, into two seuerall partes or workings: and there belongeth vnto it alwaies 5 numbers, whereof in this rule being the first part, the second nũber and the fift are alwayes of one nature and like denomination, which rule is to bee wrought thus: you must multiplie the first number by the second, & that shalbe your diuisor: then againe, multiplie the other three numbers, the one by the other, and their product shalbe your diuidend.

And now according to my promise, we will first worke the question of weight and cariage which I deliuered you in the double rule of 3: to be absolued by this rule, which was this.

If the cariage of 1 C weight 30 miles cost 12 d, what will the caryage of 500 weight stand me in being caried 100 myles:,

C. weight. Miles. Pence. C. weight. Miles.

1 —— 30 —— 12 —— 5 —— 100

Now marke well how these fiue numbers stand

ſtand: Then multiply the firſt number by the ſecond, as 30 by 1, which maketh but 30, and that number keepe for your diuiſor. Then multiplie the other 3 numbers the one into the other: that is to witte 12 by 5, which maketh 60, laſtly 60 by 100, which as you ſee here in our Tables ariſeth to 6000, which 6000 you ſhall diuide by the product of the two firſt numbers, which here is 30. And as you ſee there is found 200 pence, which is the duetie that ye ought to pay for the carriage of 5 C weight 100 myles, after the rate of 12 d a C, and agreeth with the concluſion of the double rule of 3.

Scholer. Sir I thanke you, it is euen ſo.

Maſter. Yet note this for a generalitie in this rule, looke what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes your quotient. Note.

Scholer. Well now, and it pleaſe you by your patience, I will ſee how I can ende the queſtion then next following of 30 buſhels of wheat ſowed, which in one yeere yeelded 360, how many thē wil 80 buſhels yeeld in 7 yeere, ſowing euerie yere of thoſe 7 ſtil 80 buſhels and accordīg to your reaſōs I ſet my numbers thus.

Buſhels. Yere. Buſhels. Buſhels. Yeres.

30--1--360---80---7

80

28800

7

201600

Where I multiply 30 by 1, and it maketh 30 my diuisor: then multiplying the other 3 numbers the one into the other, as here appeareth in my tables: they make 201600: which I diuide by 30: and my quotient is 6720 bushels my desire, for so much also it came at two workings by the rule of 3.

Master. Yet one question more I will propound vnto you and so leaue this rule, till it please God hereafter, that I may make you worke it in broken numbers.

What comes the interest of 258 ℔ for fiue moneths to, after the rate of 8 ℔ taken in the 100 ℔ for 12 moneths.

Scholer. Sir this is yet within the compasse of some reasonable vsance. Therefore to minister equitie in this case, I will see how I can worke

℔	*moneths.*	℔	℔.	*moneths.*
100	12	8.	258	5

y same, which I set downe thus, praying you if I haue not done well to shew me mine errour.

Master. Proceede, you haue done very wel.

Scholer. Then I doubt not by the grace of God but to ende it: I multiply 100 by 12, it yeeldeth 1200: and the 3 other numbers multiplied togither, produceth 10320: which I diuide by 1200: and my quotient is 8 pounds. Then according as you haue taught me heretofore, I turne the 720 ℔ that is left,

into

into shillings: & diuiding it by my first number, my quotient is 12 s. So I answere that the lone of 258 l for 5 Monethes, after the rate of 8 l in the 100 l for a yeare, comes to 8 l, 12 s.

Master. You say true, I commend your diligence, now beholde the maner of the second part of this rule.

Master. In the second part of this rule of 3 compoſed: the third number is like vnto the first. And the rule is to be wrought as thus: you shall nowe contrarie to the last rule multiplie the thirde number and the fourth togither: and that product shall bee your diuisor. Then multiplie the fift by the second, and the product thereof by the first: and that is the number that shall be diuided. For example, I propound this question, for a proofe of my last question of interest. A Marchant hath receyued 8 l, 12 s, for interest of certaine money for 5 moneths terme, which hee receiued after the rate of 8 l in the 100 l, for a yeare. The question is nowe how much money was deliuered to rayse this interest: Beholde therefore the manner,

The second part of the rule of 3 compound.

The proofe of the last question.

how the que- l *moneths.* l *moneths.* l. s
stion is set 100—12 8——5——8.12
forth.

Scholer. Sir I perceiue it verie well: and according to ỹ doctrine which you prescribed for the working thereof: if it please you now

it is set downe, I thinke I can follow the worke.

Master. May stay a while, and afore you worke, marke well howe I deliuer a reason for the perfite vnderstanding of this Rule, which is thus: If 8 ℔ in 12 moneths do yeeld mee 100 ℔, to take 8 ℔, 12 s̄, for 5 moneths, must needes yeeld a great deale more.

So vpon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of 3 forward: And this latter part accordeth to the rule of 3 backeward.

Scholer. Sir, I yeeld you most heartie thankes for these your last instructions, they haue giuen mee great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter, when occasion shall require.

Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as here is mention made of shillings: turne all your money as you worke into shillings, for your more ease in worke,

Note.

Scholer. If it please you to behold me a little, I will quickly end it: for I haue but my first: my second: and my last number to bee multiplied together for my diuidend: And my third into my fourth for my diuisor.

100 ℔,

£	Moneths.	£	Moneths.	£	s
100	12	8	5	8	12
20		20		20	
2000		160		172	
12		5			
4000		800			
2000					
24000					
172					
48000					
168000					
24000		4128000			
4128000		800			

Which 4128000 I diuide by 800, and my quotient is 5160 shillings, which in pounds yealdeth 258, my desire.

Master. I will here for this time in whole numbers end this rule, and I will instruct you in the rules of Felowship. You may at your conuenient leisure for your exercise worke the same by the rule of 3 at twice. And for your aide and incoeragement therein, I set downe here a proper how to applie it.

Moneths.	A	£	s	Pound.	B	£
5		8	12	8		100
12		412	2/5 s:	412 4/5		258 £.

But nowe will I shew you of ỹ rule of Felowship or Companie, which hath sundry operations, according to the diuers number of the cõpanie. This rule is somtime without difference of time, & somtimes there is in it differẽce of time. First I wil speake of ỹ wõut difference of time, of which let this be an exãple.

Foure Marchants of one companie made a banke of money diuersly, for the first layde in 30 l. the second 50 l. the third 60 l. and the fourth 100 l. which stocke they occupied so long, till it was increased to 3000 l. Now I demaund of you, what should each man receyue at the parting of this money?

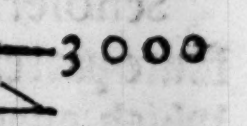
Scholer. I perceiue that this rule is like the other, but yet there is a difference, which I perceyue not.

Master. Then I will shew it to you. First by Addition you shall bring all the particular summes of the Marchants into one summe, which shall bee the first summe in your working by the Golden Rule, and the whole sum of the gaines by that stocke shall be the second summe. Now for the third summe, you shall

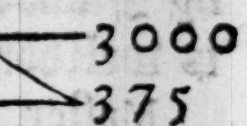
set

set the portion of each man one after another, and then worke by the Golden Rule, and the fourth sum will shew you each mans gaines: as in example.

The parcels of those foure Marchants make in one summe 240 l: set that in the first place, the gaines in the second, & the first mans portion of stocke in the 3 place thus.

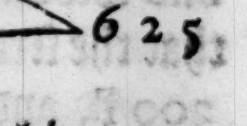


Now multiply the second by the third, and it will bee 90000, which you shall diuide by 240, and there will appeare 375 l, thus.

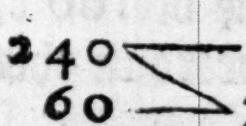


And that is the gaines for the first man.

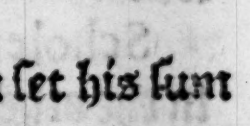
Now for the second man, set the 50 l, that he brought in the thirde place, and worke as before: and his part will be 625 l, as this figure sheweth.



Likewise for the third man set his money which was 60 l, and his part of gaines will be 750 l, as here appeareth.



And so for the fourth man, if you set his sum which is 100 l, his gaines wilbe 1250 l as the worke will declare.



Scholer. This I perceiue: but is their

any way to examine whether I haue wel done or no?

Master. For the tryall hereof adde together all ther 4 portions, and if their Addition make the whole summe of the gaines, then is the worke well done.

Scholer. That will I trie by and by, the foure parcels are these, which added together, make 3000, which is the iust summe of mony that they gained, whereby I know the worke is well done.

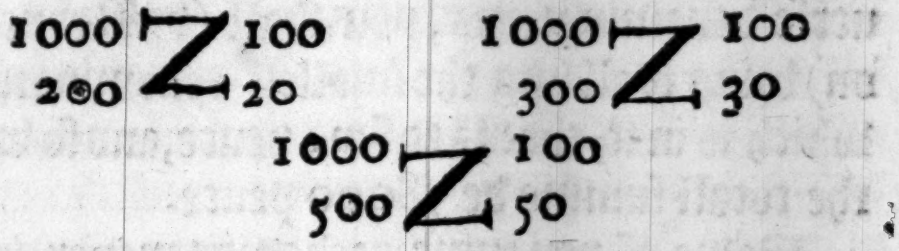
	375
	625
	750
	1250
	3000

Master. Well now another example will I put to you, not of gaines, but of losse: for one reason serueth for both.

If three Marchants in one shippe and of one felowship, had bought marchandise, so that the first had laid out 200 l, the seconde 300 l, and the thirde 500 l, and it chaunced by tempest that they did cast ouer boorde into the sea Marchandise of the value of 100 l, howe much shoulde each man beare in this losse?

Scholer. If I shall doe in this as you did in the other question, then must I ioyne their three portions together, 200, 300, and 500, which maketh 1000. Then say I, if 1000 leese 100, then shall 200 loose 20, and 300 shall loose 30, and 500, shall loose 50, as by these

these three figures it doth appeare plaine.



Master. Well, sith nowe you haue done these, I will propound a question of more importance, which shall make you not onely the abler to vnderstand this Rule, but also it will greatly ayde you in the next rule of Fellowship with time, if such neede be that your money be of diuerse denominations.

For this may not be forgotten in all such questions: if the number be of diuerse kindes you must by Reduction bring it into one kind, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of diuerse kinds, as some yeares, some moneths, weekes & daies, you shal make al moneths, weeks or daies, according as ẏ lest name of time in ẏ question is: As for example: Note.

First in diuersitie of money. Three companions bought 2000 sheepe, and paid for them 241 ℔, 13 s̄, 4 d̄, of which summe one paide 101, ℔, 10 s̄. The second paid 82 ℔, 17 s̄, 10 d̄. And the third paid 57 ℔, 5 s̄, 6 d̄: How many sheepe must each of them haue? Answere: The first shall haue 840. The second 686. And the third 474. And that must you worke Question of sheepe.

thus.

First considering that your money is of diuerse denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence, and so will the totall summe be 58000 pence.

Nowe if you turne each mans money into pence also, the first mans sum will bee 24360 pence: the second mans sum 19894 pence And the third mans money will bee 13746 pence.

Now to know howe manie sheepe euerie man shall haue, let the whole summe of money that is 58000 pence bee set in the first place, & in the second place set the number of sheepe, and then orderly in the third place set each mans money, and then multiplying the thirde and the second summes together, and diuiding that that amounteth by the first, there will appeare the number of sheepe that each man ought to haue: as these three figures do shew.

a

58000 — 2000
24360 — 840

b

58000 Z 2000
19894 Z 686

c

58000 Z 2000
13746 Z 474

Scholer. Why doe you set the money in the first place, seeing in the question you say,

2000

2000 sheepe cost 58000 d, and not thus, 58000 d, cost 2000 sheepe.

Master. You remember, I taught you at the beginning of this Golden Rule, that the first and third numbers must be of one name, and of like things: and euermore the number that the question is asked of, must be set in the third place. Now is the question plainely this: If foure men bought 2000 sheepe for 58000 pence, how many sheepe shall each man haue.

But seeing in this question there ought more respect to be had to the summe of money, than to the summe of the persons, (for in the summes of money is there proportion toward the sheepe, and not in the number of persons) therfore must we turne the question thus.

If 58000 pence bought 2000 sheepe, how many did 24360 buie? Againe, how many did 19894 d buie? and how manie bought 13746 pence?

Scholer. I perceiue it reasonable, and so shall I doe in all like questons.

Master. Euen so. But for easines of the worke marke this: whensoeuer the first and second numbers haue ciphers in the first places, you may both in the multiplication and in the diuision leaue out those ciphers, so that you leaue out like manie out of both summes, as in this question, the first number 58000 hath three ciphers, and so hath the second, that

is

is 2000: therefore cast away their ciphers, and so will the first number bee 58, and the second 2: set them in their places, and worke according to the rule, and you shall perceiue that it will bee all one, sauing that this is the shorter and easier way, as these three figures doe shew.

a

58 | 2
24360 | 840

b

58 | 2
19894 | 676

c

58 | 2
13746 | 474

And this you see is both easier, & also the more certaine way to knowe the answere to this question.

Scholer. Truth it is as you say: but sir, mee seemeth I might aske a further question here, not onely howe many sheepe each man should haue, but also what euerie sheepe cost.

Master. That question doth not onely belong to this Rule, but may also bee discussed by Diuision, especially if the questions number be one only: as thus. Diuide the totall sum 58000 pence by 2000 (or 58 by 2, omitting the ciphers) and the quotient will be 29 pence, that is 2 s. 5 d, howbeit, by this rule you may do it, and best when the number of the question doth exceede 1: as if I should aske this

this question, 2000 sheepe cost 58000 d, howe much did 20 cost? Then shall I set my figure thus.

2000 Z 58000
20

And doing after the rule, there wil amount 580 pence, that is 2 l, 8 s, 4 d: the price of one score: But if you will vse that easie way that I did teach you, you may chaunge the first and second number thus.

2 Z 58
20

Thus doe you perceyue the vse of the rule without time. And that you may as well perceyue the same with diuersitie of time, I propose this example. The rule of fellowship with time.

Foure Marchants made a common stocke, which at the yeares ende was increased to 35145 l. Now to know what shall be each mans portion of gaines, you must know each mans stocke and time of continuance. Question of a banke.

The first man of these foure laid in 669 l, which he did take from the stocke againe, at the end of 10 moneths. The second man laide in 810 l, for 8. moneths. The third layde in 900 l, for 7 moneths. And the fourth laid in 1040 l, for 12 moneths.

This question shall you examine as you did ẏ other before, sauing that where as in the third place of ẏ figure you did set ech mās sū alone, here you shall set the same being multiplied by the number of their time, & likewise in the first place of the figure you shal set ẏ number which Note. A generall rule.

moun-

mounteth of their whole summes so multiplyed by their time, and added into one whole summe, as thus.

The first mans summe is 669 l. which I multiple by 10 (that was the number of his time) and it maketh 6690. The seconde mans summe 810 l multiplied by 8, (which was his time) maketh 6480. The third mans summe 900 l, multiplied by 7 (for that was his time) yeeldeth 6300. The fourth mans summe was 1040 l, and his time 12 multiplie the one by the other, & it will be 12480.

These foure summes thus multiplied by their time, must bee set orderly in the third place of the figure: and in the first place must be set the whole summe of all foure, which is 31950, and the gaine must bee in the second place, which is 35145. Now to ende the question, I say first: If 31950 did get 35145, what did 6690 get? Answere, 7359 l, as by this figure appeareth.

a

31950 — 35145

6690 — 7359

Likewise the seconde man had to his part 7128 l, the third must haue 6930 l. And the fourth man shall haue for his part 13728 l, as these figures doe partly declare.

Scholer.

b

31950 — 35145
6480 — 7128

c

31950 — 35145
6300 — 6930

d

31950 — 35145
12480 — 13728

Scholer. This I like verie well: but what proofe is there of this worke?

Master. The same that I taught you for the other, to adde all the portions togither, and if they agree to the whole summe, then is your worke well done.

Scholer. That will I prooue in this example. The foure parcels are these, which if I adde togither, there will amount 35145, and that was the whole sum: whereby I perceiue the worke is well done.

	7359
	7128
	6930
	13728
	35145

Master. If it fall out otherwise, bee sure it is not well.

Scholer. Then doe I vnderstand this worke also verie well, but what haue I nowe to learne?

Master. There are many other excellent parts behind, of which I will not as nowe, make mention, because that without the knowledge of Fractions they cannot be duely taught, and much lesse vnderstanded. Therefore

fore will I propose to you two or three questions more, (that thereby you may better perceiue the vse of this rule and al other like) and so make an end for this time.

¶ Three partners by some ill aduenture sustained the losse of 160 l, whereof the first laide into the common stocke 200 l, for 10 moneths. The second laide in 350 l, and the third 100 l, but for how long the two latter, is vnknowne. But breaking of their partnership the first found himselfe a looser 80 l, the seconde 56 l, and the thirde 24 l. The question is, for how long time was the money of the two latter in companie.

For the solution hereof and of such other like, you must also multiplie the first mans 200 l that he put into the stocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme, if his money that lost 80 l, multiplied by his time, make 2000: what shall his money make that lost 56 l, and his that lost 24 l, which two numbers I commit to the triall of the rule of 3 at 2 workings thus: If 80 giue 2000, what giueth 56? And againe, if 80 giue 2000, what giueth 24?

80 Z 2000
56 Z 1400

10 Z 2000
24 Z 600

To conclude, if you now diuide 1400 the second mans portion by 350, which was his

his stocke that he laid into companie, you shall find in your quotient 4 monethes, and for so long time did the second man put his money into the common stocke.

Lastly if you diuide the thirde mans newe laying in, which was 600, by 100, which was his stocke that he put into companie: the quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolued.

Scholer. Sir I haue attentiuely beheld your working, and the more wee trauell herein, the more me thinke I am in loue with this excellent Art.

Master. Then what say you to this question?*

There is in a Cathedrall Church 20 Cannons, and 30 Uicars, those may spend by yeare 2600 l. but euerie Cannon must haue to his part 5 times so much as euerie Uicar hath: how much is euerie mans portion, say you? Question of Canons.

Scholer. I pray you make the aunswere your selfe also, so shall I perceiue best the meanes to answere to such other like.

Master. In this question you must doe as in those before saide, that haue diuersitie of time, for here is diuersitie of portions: Therfore shall you multiplie the number of the persons by their difference of portion: (as you did in the other by time.) Then must you multiplie the 20 (which is the number of Canons)

by

by 5, (for that is the number of their portion) so will it bee 100: Then 30 (that is the number of Vicars) by 1, (that is the number of their portion) and it will bee 30: put those two summes togither, and they make 130: then say thus: If 130 spend 26000 l, what may 100 spend? The rule sheweth 2000 l.

Againe for Vicars: If 130 spend 2600 pounds, what may 30 spend? Answere 600 l, as these figures shew.

130	2600		130	2600
100	2000		30	600

But if euery Canon should haue so often times 4 l, as the Vicar should haue 3 l, then should I multiplie 20 by 4, (that were 80) & 30 by three (that were 90) and then both were 170. Then should the figures be set thus,

	l	s	d			l	s, d
170	2600				170	2600	
80	1223,	10,	7,		90	1376,	9, 5

But this sort is too hard for you, by reason of the Fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend, which summe if you diuide by 20, you shall see each Canons portion: and so of the Vicars, if you diuide their summe by 30, the quotient will declare euery Vicars portion.

The

The ſecond Dialogue.

The accounting by Counters.

Maſter.



Dw ỳ you haue learned the common kinds of Arithmeticke with the pen, you ſhal ſee the ſame Arte in Counters: which feate doth not only ſerue foꝛ them that cannot wꝛite and read, but alſo foꝛ them that can doe both, but haue not at ſometime their pen oꝛ tables ready with them.

This ſoꝛt is in two foꝛmes commonly: The one by lines, and the other without lines. In that that hath lines, the lines do ſtand foꝛ the oꝛder of places: and in that that hath no lines, there muſt be ſet in their ſtead ſo many Counters as ſhall neede, foꝛ each line one, and they ſhall ſupplie the ſtead of the lines.

Scholer. By examples I ſhould better perceiue your meaning.

Maſter. Foꝛ example of the lines, loe here you ſee fiꝛe lines, whiche ſtand foꝛ fiꝛe places, ſo

100000
10000
1000
100
10
1

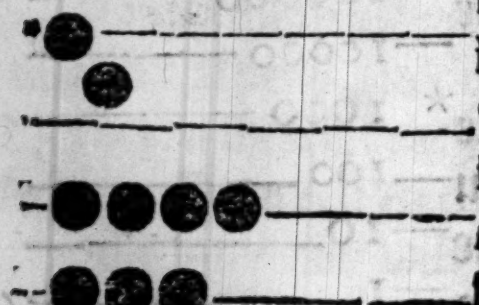
that ỹ neathermoſt ſtandeth foꝛ the firſt place, and the next aboue it foꝛ the ſecond, and ſo vpward, till you come to the higheſt, which is the ſixt line, and ſtandeth foꝛ the ſixt place. Now what is the value of euerie place oꝛ line you may perceiue the figures which I haue ſet on them, which is accoꝛding as you learned befoꝛe in Numeration of Figures by the pen: foꝛ, the firſt place is the place of vnites oꝛ ones: and euery counter ſet in that line betokeneth but one: and the ſecond line is the place of 10, foꝛ euery counter there ſtandeth foꝛ 10. The third line the place of hundꝛeds, the fourth of thouſands, and ſo foꝛth.

Scholer. Sir, I do perceiue that the ſame oꝛder is here of lines, as was in the other figures by places, ſo that you ſhall not need longer to ſtand about Numeration, except there be any other difference.

Maſter. If you doe vnderſtand it, then how will you ſet 1543?

Scholer. Thus as I ſuppoſe.

Maſter. You haue ſet the places truely, but your figures be not meete foꝛ this vſe: foꝛ the meeteſt figure in this behalfe, is the figure of a Counter round, as you ſee here, where I haue expꝛeſſed that ſame ſumme.



Scho-

Scholer. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many Digits as you haue, so many Counters you set in the lowest line: and for euery 10 you set one in the second line: and so of other. But I know not by what reason you set that one Counter for 500 betweene two lines.

Master. You shall remember this, that whensoeuer you neede to set downe 5, 50, or 500, or 5000, or so forth any number whose Numerator is 5, you shall set one counter for it in the nexte space aboue the line that it hath his denomination of: as in this example of that 500, because the numerator is 5, it must be set in a voide space: and because the denominator is hundred, I know that his place is the voyd space next aboue hundreds, that is to say, aboue the third line.

And further you shall marke, that in all working by this sorte, if you shall set downe any summe between 4, and 10, for the first part of that number you shall set downe 5, and then so many Counters more, as there rest numbers aboue 5. And this is true both of Digits and Articles. And for example I will set downe this summe 2 8 7 9 6 5, which sum if you marke well, you neede none other examples for to



learne the Numeration of this forme.

But this shall you marke, that as you did in the other kindes of Arithmeticke, set a pricke in the places of thousandes, in this worke you shall set a Starre, as you see before.

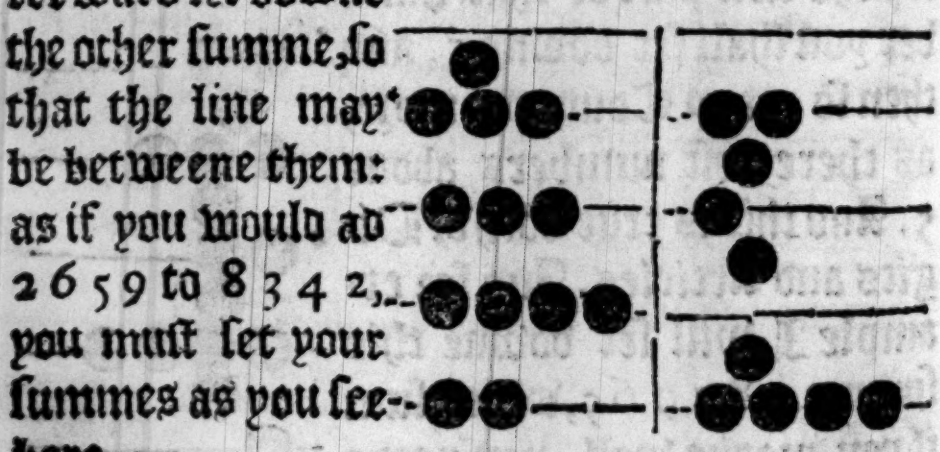
Scholer. Then I perceiue Numeration: but I pray you, how shall I doe in this Art to adde two summes and more together?

Addition.

Maister.

THe easiest way in this Art. is to adde but two summes at once together: howbeit, you may adde more, as I will tell you anon. Therefore whē you will adde two summes, you shall first set downe one of them, it forceth not which, & then by it draw a line crosse the other lines. And afterward set downe the other summe, so that the line may be betweene them: as if you would adde 2659 to 8342, you must set your summes as you see here.

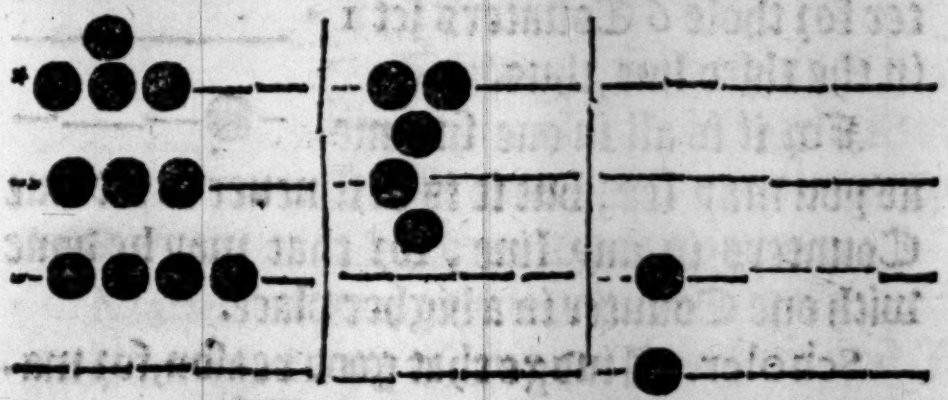
Addition of 2. sums.



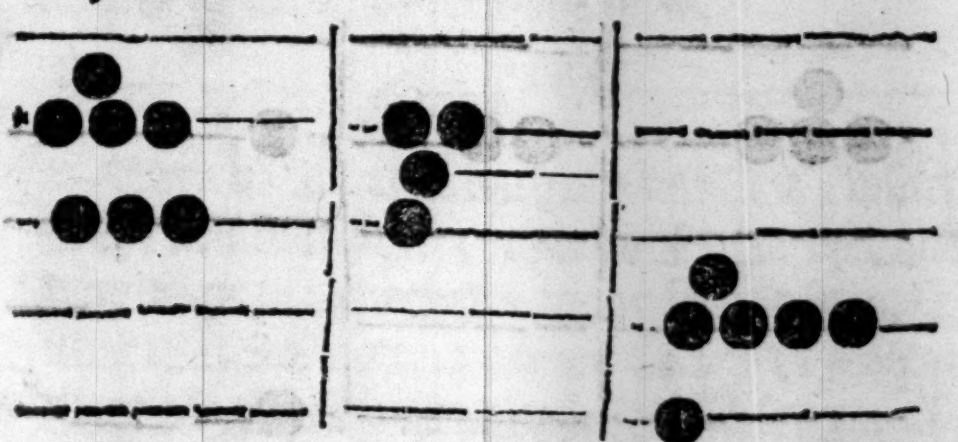
And then if you list, you may then adde the one

one to the other in the same place: or else you may adde them both together in a new place: which way because it is most plainest, I will shew you first.

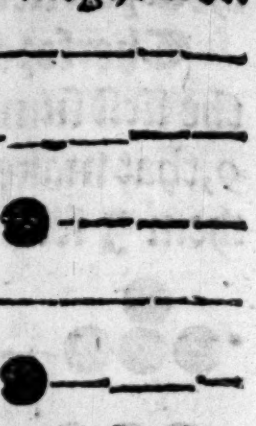
Therfore will I begin at ỹ vnites, which in the first summe is but 2, & in the second summe 9, that maketh 11. Those do I take vp, and for them I set 11 in the new roume, thus.



Then do I take vp all the Articles vnder a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90: or you may say better, that in the first summe there are foure articles of 10, and in the second summe 5, which maketh 9, but then take heed that you set them in their right lines as you see here.



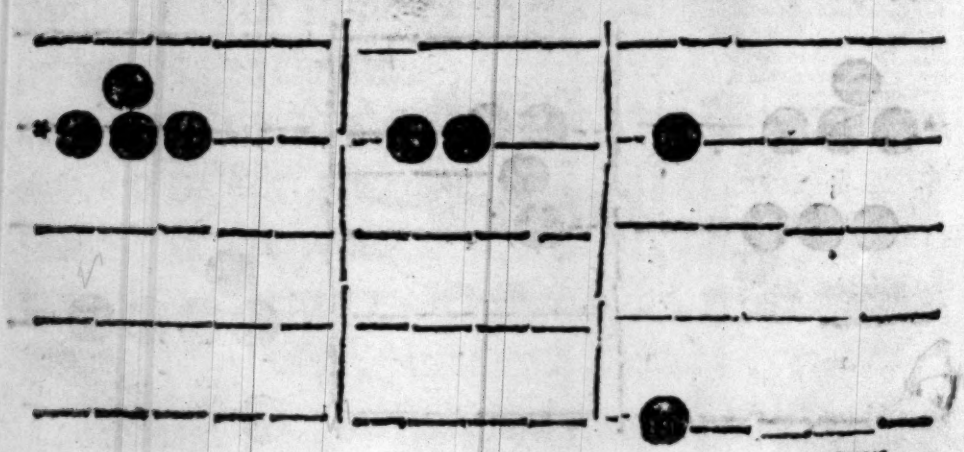
Where I haue taken away 40 from the first summe, and 50 from the second, and in their stead I haue set 90 in the third roome, which I haue set plainly that you might well perceiue it: howbeit, seeing that 90 with the 10 that was in the thirde roome alreadie doth make 100, I might better for those 6 Counters set 1 in the third line, thus.



For it is all in one summe as you may see, but it is best neuer to set fiue Counters in any line, for that may be done with one Counter in a higher place.

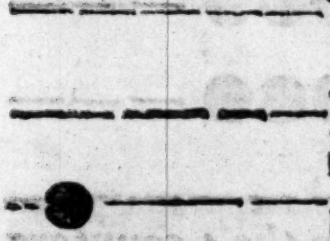
Scholer. I iudge that good reason, for many are vnnecessarie where one will serue.

Master. Well, then will I adde foorth of hundreds: I finde 3 in the first summe, and 6 in the second, which maketh 900, them do I take vp, and set in the third roome, where is 100 alreadie, to which I put 900, and it will be 1000: therefore I set one counter in the fourth line for them all, as you see here.



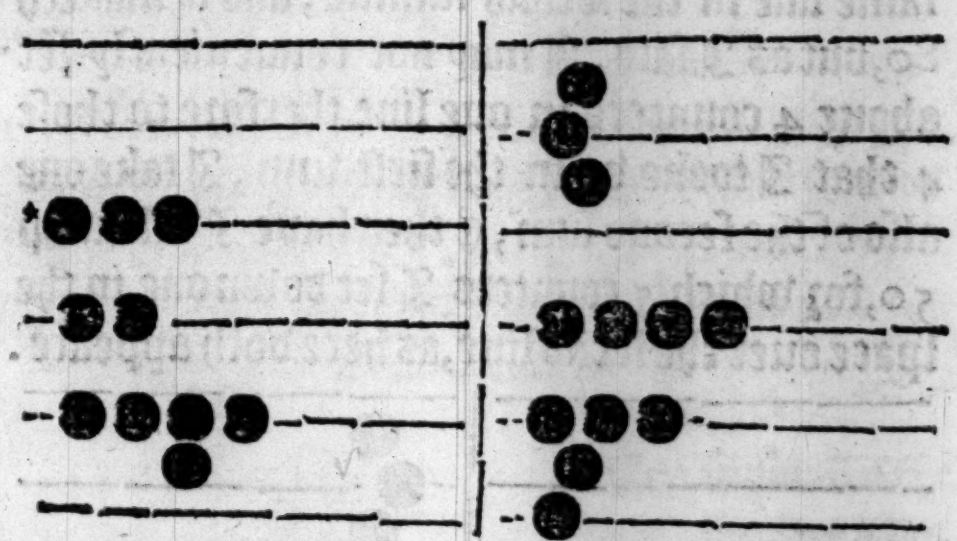
Then

Then adde I the thoulandes together, which in the firſt ſumme are 8000, and in the ſecond 2000, that maketh 10000: them do I take vp from thoſe two places, and for them I ſet one counter in the fift line, and then it appeareth as you ſee to bee 11001, for ſo manie doth amount of the Addition of 8342 to 2659.



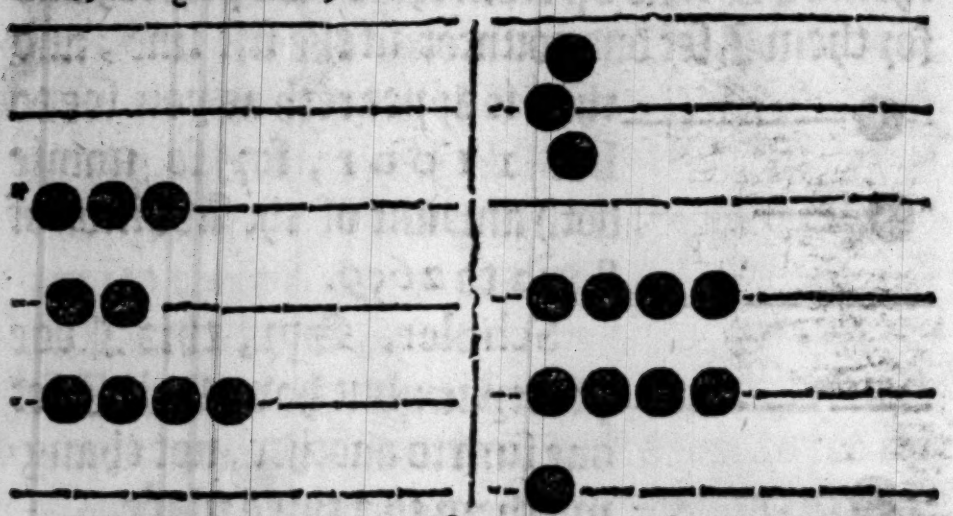
Scholer. Syr, this I doe perceyue: but how ſhall I ſet one ſum to another, not changing them to a third place. To adde ſummes togither.

Maſter. Marke well how I do it: I will adde togither 65436 and 3245, which firſt I ſet downe thus.

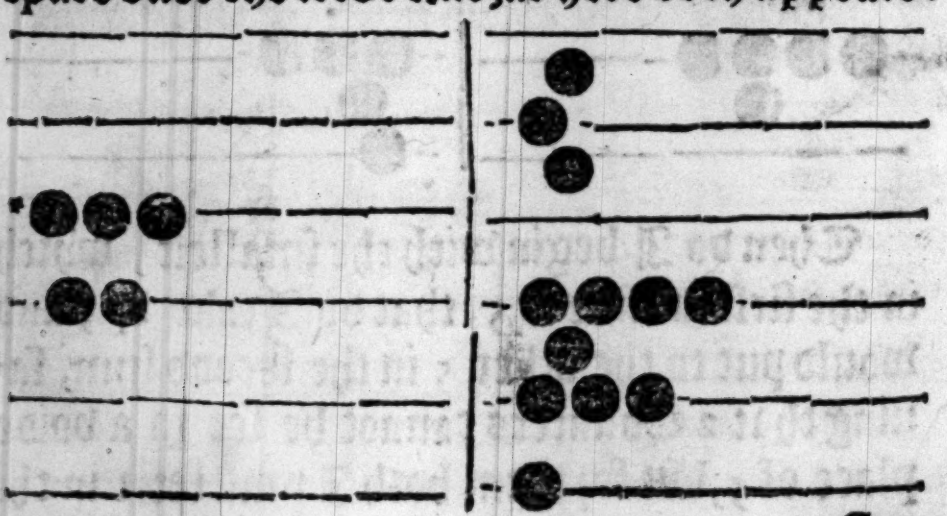


Then do I begin with the ſmalleſt, which in the firſt ſumme is 5, that do I take vp, and would put to the other 5 in the ſecond ſum, ſauing that 2 Counters cannot be ſet in a voide place of 5, but for them both I muſt ſet 1 in the

second line, which is the place of 10: therefore I take vp the fiue of the first summe, and the 5 of the second, and for them I set 1 in the second line, as you see here.



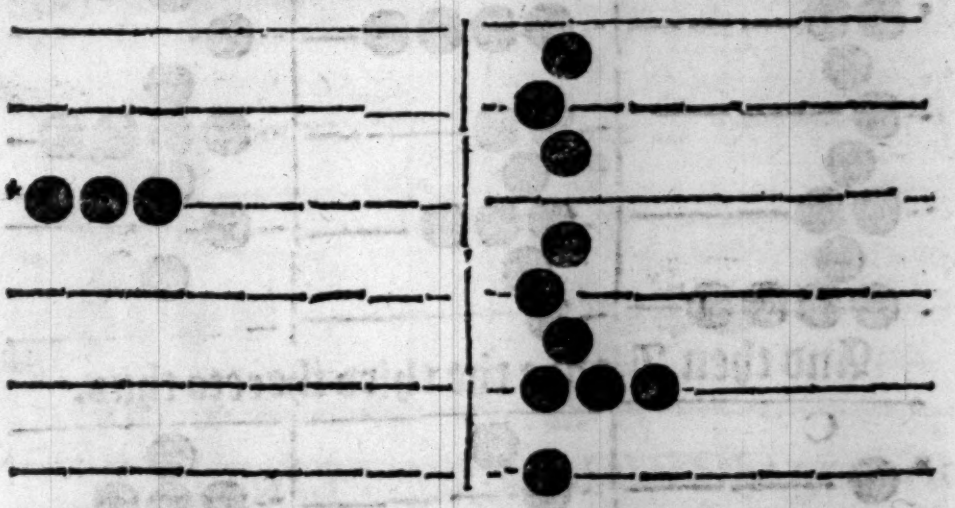
Then do I likewise take vp the 4 counters of the first summe and second line (which make 40) and adde them to the 4 Counters of the same line in the second summe, and it maketh 80, but as I said, I may not conueniently set aboue 4 counters in one line, therfore to those 4 that I tooke vp in the first sum, I take one also of the second sum, & then haue I taken vp 50, for which 5 counters I set down one in the space ouer the secõd line, as here doth appeare.



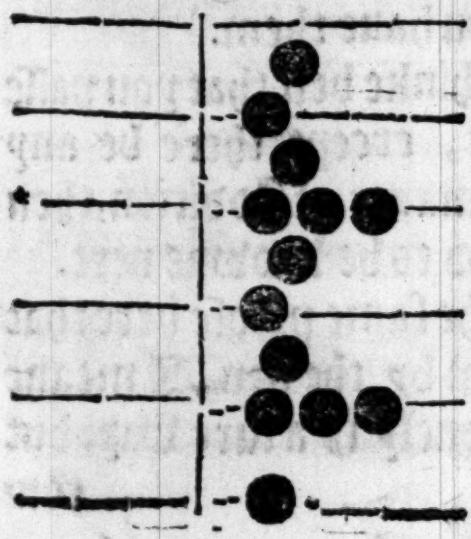
And

And then is there 80, as well with the [illegible]e-counters, as if you had set downe the oth[illegible]0 also.

Now do I take the 200 in the first summ[illegible] and adde them to the 400 in the second sum[illegible] and it maketh 600, therefore I take vp the [illegible] Counters in the first summe, and 3 of them in the second summe, and for them 5, I set 1 in the space aboue, thus.



Then I take the 3000 in the first summe, vnto which there are none in the second summe agreeing, therefore I do onely remoone those three Counters from the first summe into the second, as here doth appeare.

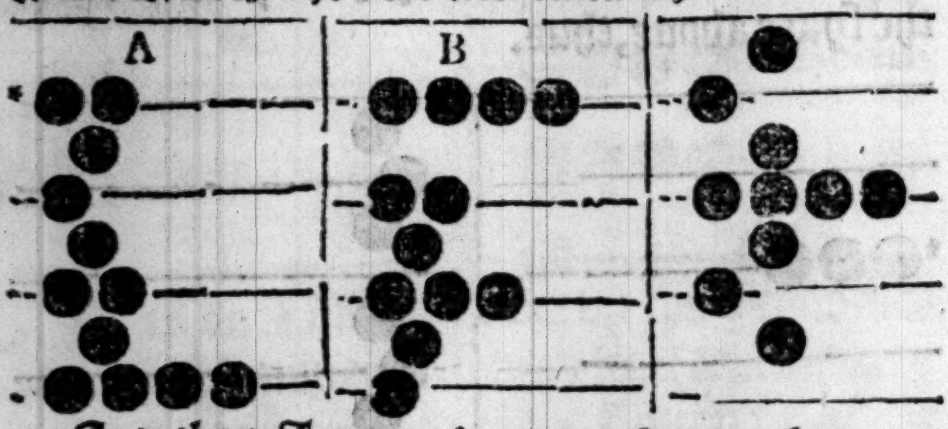


And so you see the whole summe that amounteth of the Addition of 65436 with 3245, to be 68681.

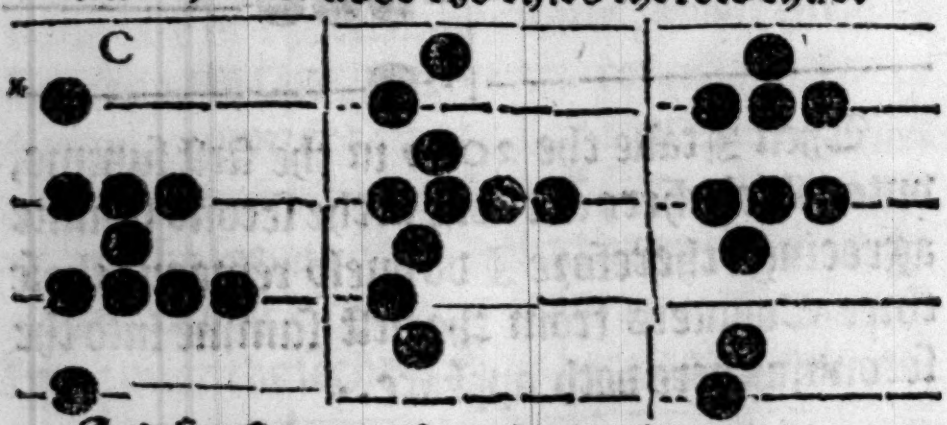
And if you haue marked

ked these two examples well, you neede
urther instruction in Addition of 2 onely
mmes: but if you haue moze than two sums
adde, you may adde them thus.

First adde two of them, and then adde the
third and fourth, oz moze if there be so many:
as if I would adde 2679 with 4286 & 1391.
First I adde the two first sums thus.



And then I adde the third thereto thus.



And so of moze, if you haue them.

Scholer. Now I thinke best that you passe
fooyth to Subtraction, except there be any
waies to examine this maner of Addition, then
I thinke that were good to be knowne next.

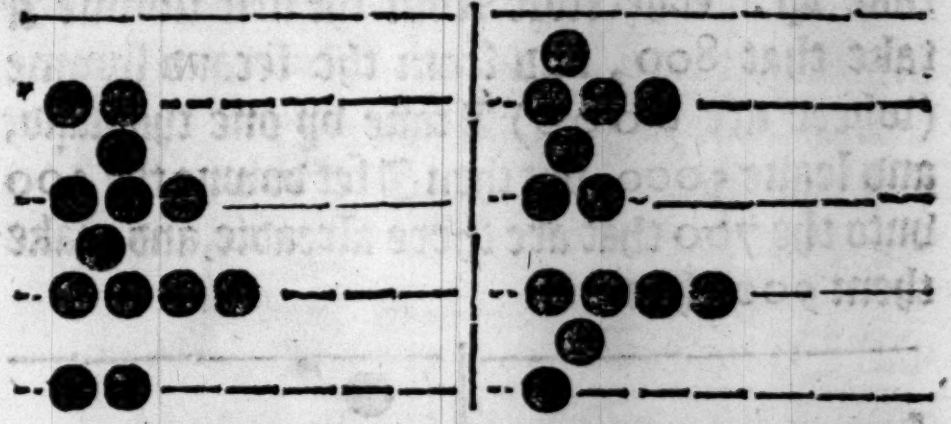
A proofe. Master. There is the same pzoofe here that
is in the other Addition by the pen, I meane
Subtraction, foz that onely is a sure way: but

con-

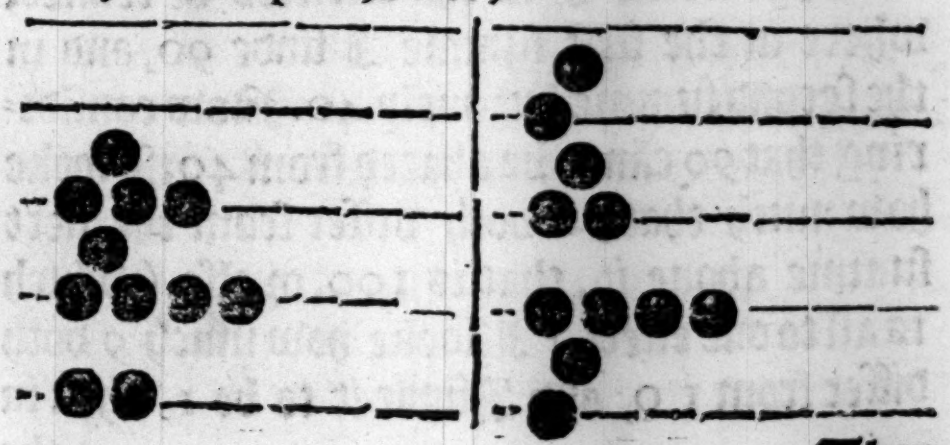
considering that Subtraction must bee le-
knowne, I will first teach you the Art of S o
traction, and that by this example.

Subtraction.

I Would Subtract 2892 out
8746. These summes must I se
downe as I did in Addition: but
here it is best to set the lesser num-
ber first, thus.

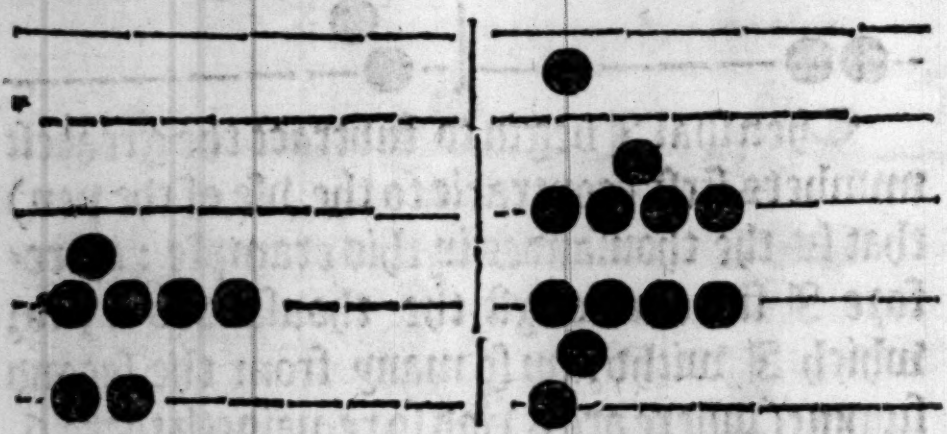


Then shal I begin to subtract the greatest
nnmbers first (contrarie to the vse of the pen)
that is the thousandes in this example: there-
fore I finde amongst the thousandes 2, for
which I withdraw so many from the second
summe (where are 8) and so remaineth there 6,
as this example sheweth.



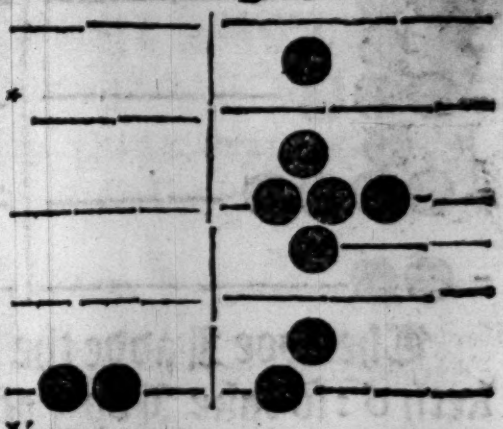
Then

Then doe I likewise with the hundreds, which in the first summe I finde 8, and in second summe but 7, out of which I can not take 8, therefore this must I do: I must make how much my summe differeth from 10, which I finde here to be 2, then must I bate for my summe of 800, one thousand, and set downe the excesse of hundreds, that is to say, 2, for so much as 1000 is more than I should take vp. Therefore from the first summe I take that 800, and from the second summe (where are 6000) I take vp one thousand, and leaue 5000, but then I set downe the 200 vnto the 700 that are there alreadie, and make them 900, thus.



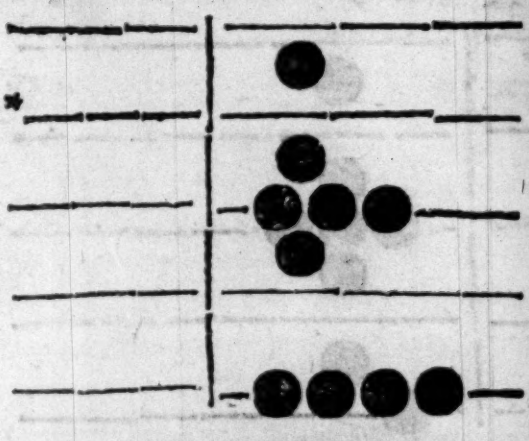
Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cānot be abated from 40, I looke how much that 90 doth differ from the next summe aboue it, that is 100, or else (which is all to one effect) I looke how much 9 doth differ from 10, and I finde it to be 1, then in the

the stead of that 90, I doe take from the second summe 100: but considering that is 10 too much I set down 1 in the next line beneath for it, as you see here.



Sauing that here I haue set one counter in the space, in stead of 5, in the next line:

And thus haue I subtract al, saue two which I must bate from 6 in the secõd summe, and there will remaine 4, thus.

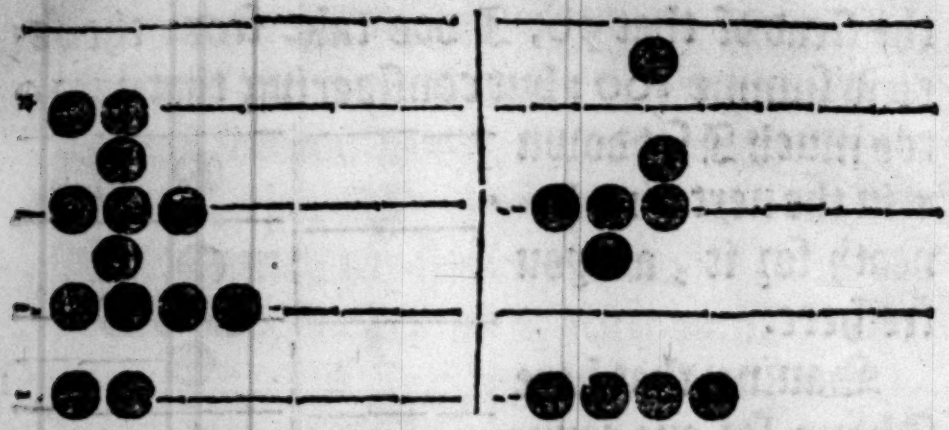


So that if I subtract 2842 from 8746, the remainer will be 5854.

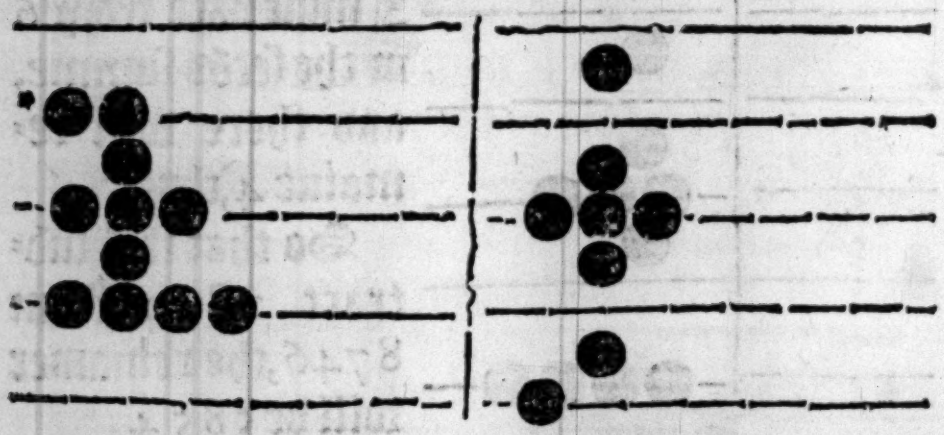
And that this is truely wrought, you may proue by Addition: for if you adde to this remainer the same summe that you did subtract, then will the furmer summe 8746 amount againe.

Scholer. That wil I proue: and first I set the summe that was subtracted, which was 2892, and then the remainer 5854, thus.

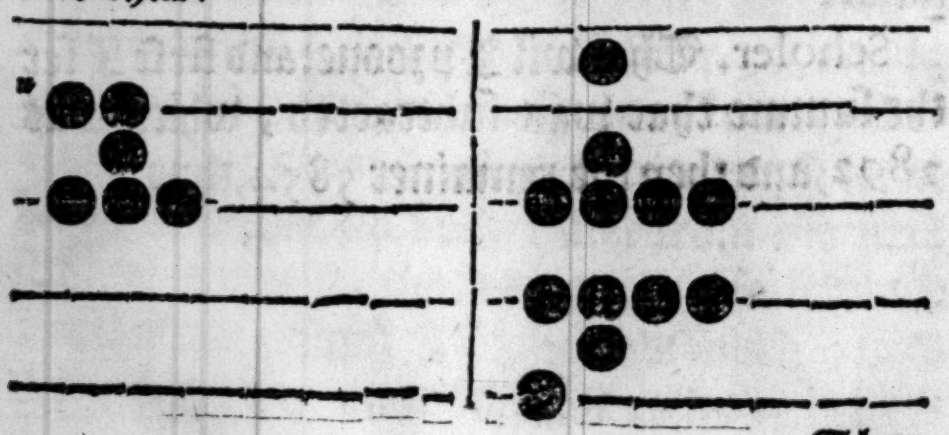




Then doe I adde the first 2 to 4, which maketh 6: so take I vp 5 of those Counters, and in their stead I set 1 in the space, and 1 in the lowest line, as here appeareth.

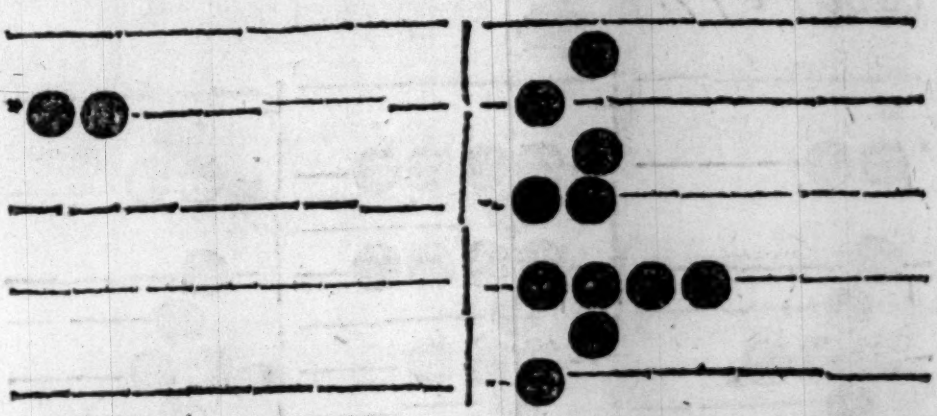


Then doe I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vp those 6 Counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line thus.

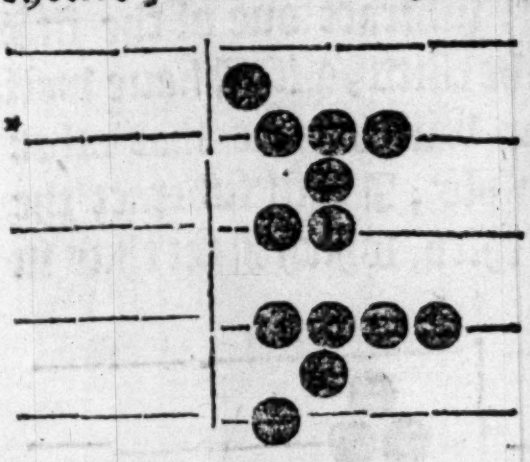


Then

Then do I come to the hundreds, of which I finde 8 in the first summe, & 9 in the second, that maketh 1700: therefore I take vp those 9 counters, and in their stead I set 1 in the fourth line, and 1 in the space next beneath, and 2 in the third line as you see here.



Then is there left in the first summe but onely 2000, which I shall take vp from thence, and set in the same line in the second summe, to the one that is there alreadie: and then will the whole summe appeare as you may wel see, to be 8746, which was the first grosse summe, and therefore I doe perceiue that I had well subtracted before.



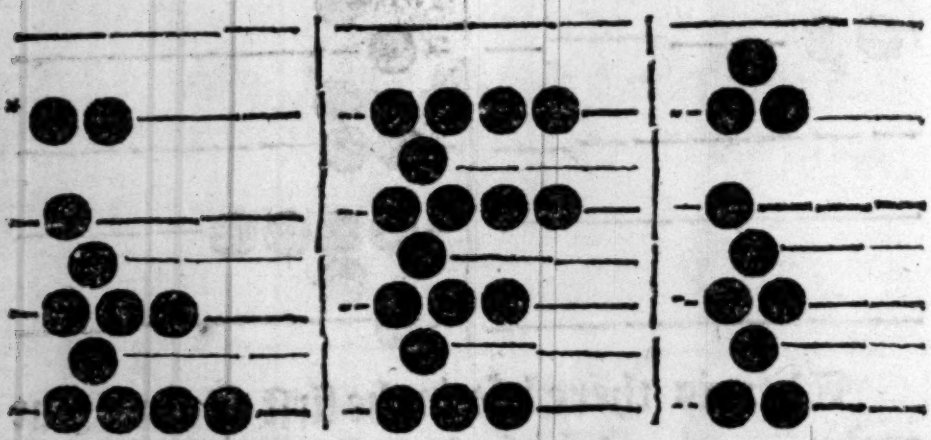
And thus may you see, how Subtraction may be tried by Addition.

Scholer. I perceiue the same order here with Counters, that I learned before in figures.

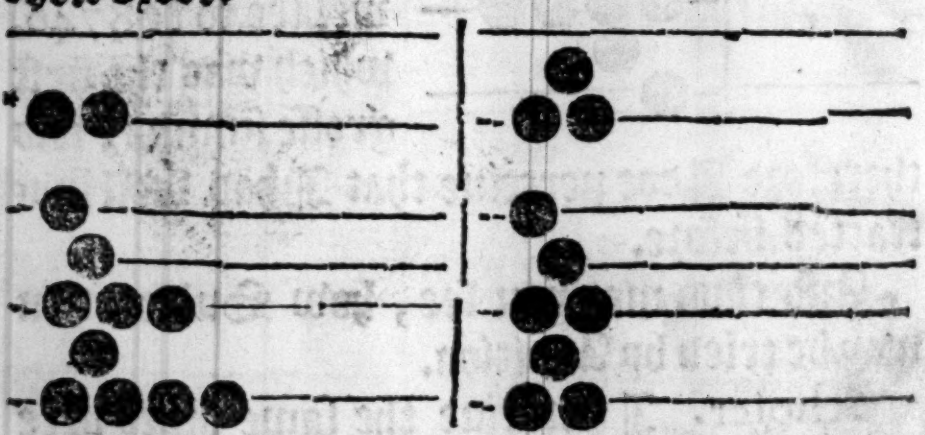
figures.

Maſter. Then let me ſee how you can trie Addition by Subtraction.

Scholer. Firſt I will ſet forth this example of Addition, where I haue added 2189, to 4988. And the whole ſumme appeareth, to be 7177.



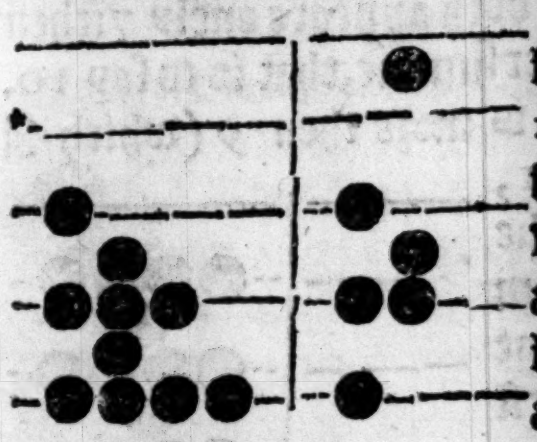
Now to trie whether that ſumme be well added or no, I will ſubtract one of the firſt two ſummes from the third, & if I haue well doone, the remainer will be like that other ſumme, as for example, I will ſubtract the firſt ſumme from the third, which I ſet thus in their order.



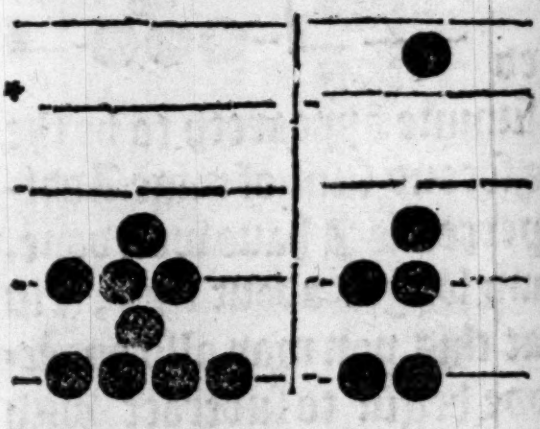
Then doe I ſubtract 2000 of the firſt

ſumme

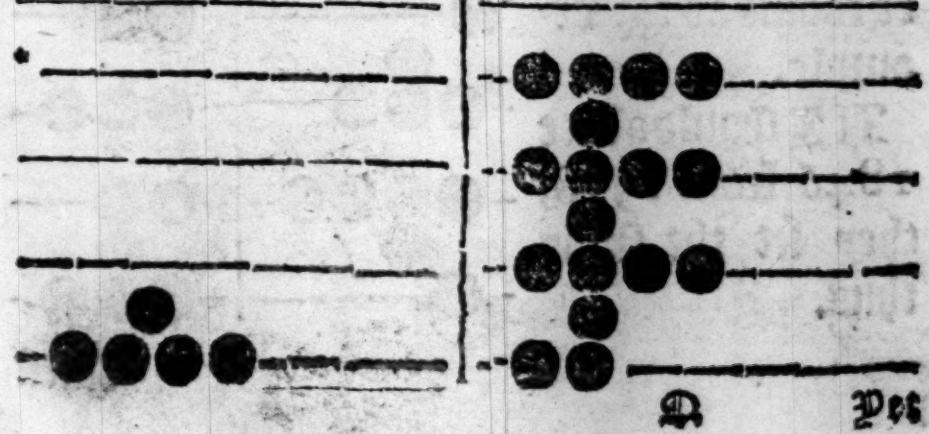
ſumme from the ſecond ſumme, and then remaineth there 5000, thus.



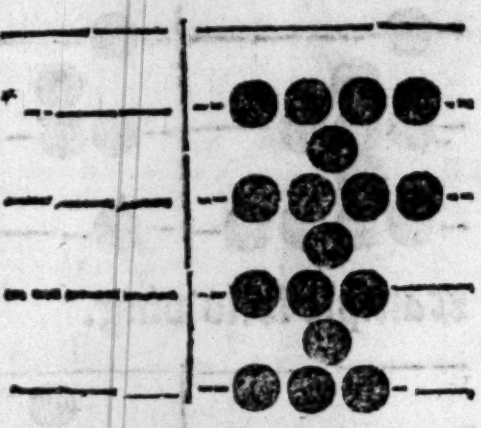
Then in the third line I ſubtract the 100 of the firſt ſum from the ſecond ſum where is onely 100 alſo: & then in ỹ third line reſteth nothing, as you may ſee in the example following.



Then in the ſecõd line with his ſpace ouer him, I find 80, which I ſhould ſubtract frõ the other ſumme: then ſeeing there are but onely 70, I muſt take it out of ſome higher ſumme, which is here onely 5000: therefore I take vp 5000: and ſeeing that is too much by 4920, I ſet downe ſo many in the ſecond roome, which with the 70 being there already, do make 4990, and then the ſummes do ſtand thus.



Yet remaineth there in the first ſumme, 9 to be abated from the ſecond ſumme, wherein that place of vnites doth appeare onely 7: then muſt I bate a higher ſumme, that is to ſay 10, but ſeeing that 10 is moze then 9 (which I ſhould abate) by 1, therefoze ſhal I take vp one counter from the ſecõd, & ſet downe the ſame in the firſt oz loweſt line, as you ſee here.

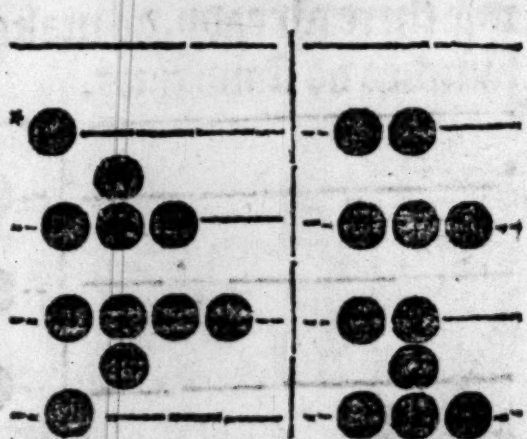


And ſo haue I ended this woꝛke, and the ſumme appeareth to be the ſame which was the ſecond ſum of mine Addition, & therefoze I perceaue I haue well done.

Another way of Addition.

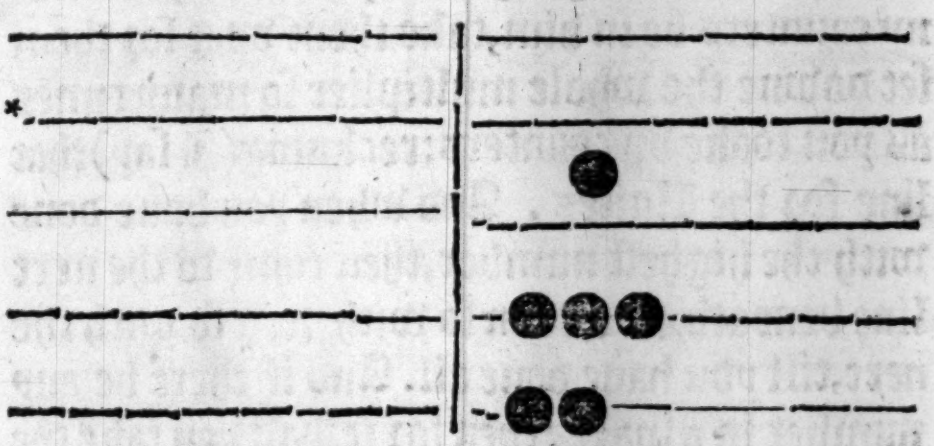
Maſter. To ſtand longer about this, it is but follie, except that this you may alſo vnderſtand, that many doe begin to ſubtract with counters, not at the higheſt iumme as I haue taught you, but at the neathermoſt, as they do vſe to adde: and when the ſumme to be abated in any line appeareth greater then the other, then do they boꝛow one of ỹ nert higher roome as foꝛ example.

If I ſhould abate 1846 from 2378, they ſet the ſums thus.



First

First they take 6, which is the lower line and his space, from 8 in the same roomes in the second summe, and yet there remaineth 2 counters in the lowest line. Then in the second line must 4 be subtracted from 7, and so remaineth there 3. Then 800 in the third line, and his space, from 200 of the second summe can not be, therfore do they bate it frō a higher roome, that is from 1000: and because that 1000 is too much by 200, therefore must I set downe 200 in the thirde line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the first sum, which if I withdraw from the second summe, then do all the figures stand in order, thus.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may vse which you list.

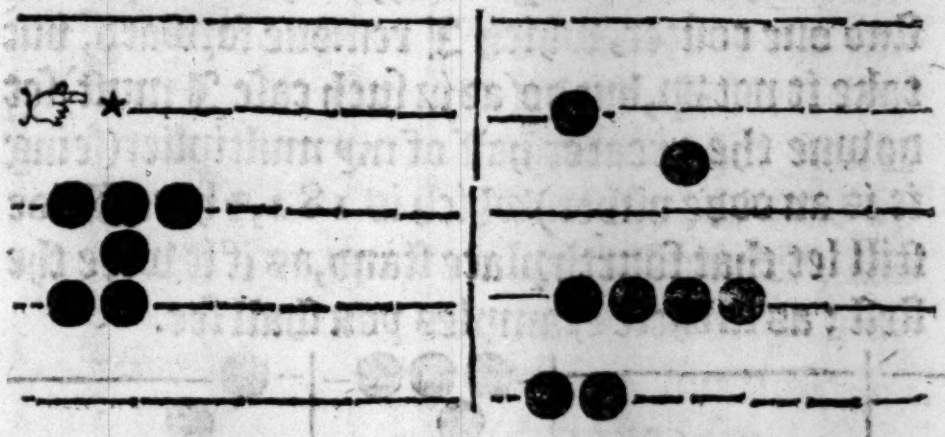
But now touching Multiplication: you shall set your numbers in two roomes (as you did in those other kinds) but so that the multiplier be set in ỹ first roome then shall you begin with the highest numbers of the second roome, and multiplie them first, after this sort. Take the ouermost line in your first working, as it were the lowest line, setting on it some moueable marke (as you list) & looke how many counters be in him, take them vp, & for them set downe the whole multiplier so many times as you tooke vp counters: reckning (I say) that line for the Unites. And when you haue done with the highest number, then come to the next line beneath, & do euen so with it, & so with the next, till you haue done all. And if there be any number in a space, then for it shall you take the multiplier 5 times: and then must you recken that line for the vnites, which is next beneath that space. Or els after a shorter way, you shall take onely halfe the multiplier, but then shall you take the line next aboue that space for the line of Unites. But in such working, if by chance your multiplier be an odde number, so

that

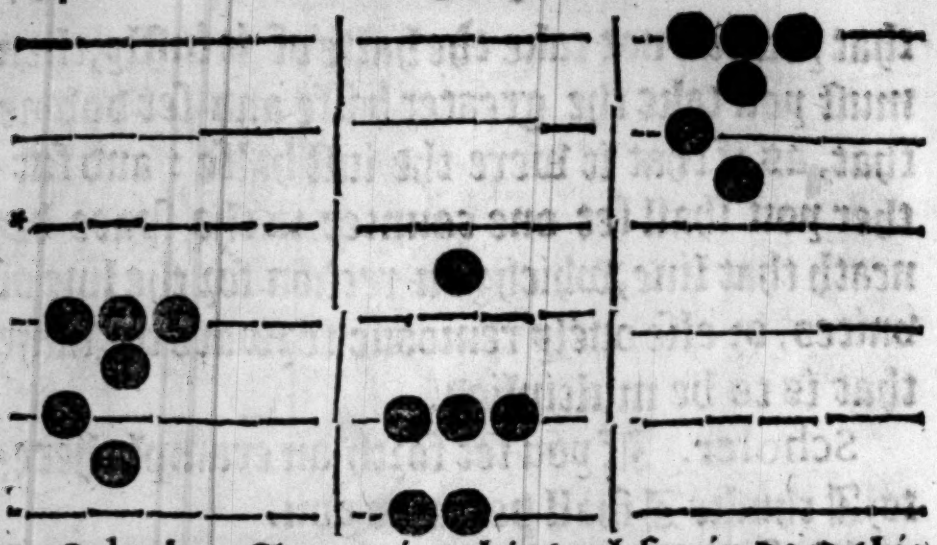
that you can not take the halfe of it iustly, then must you take the greater halfe and set downe that, as if that it were the iust halfe: and further you shall set one counter in the space beneath that line, which you reckon for the line of vnites, or else onely remoue forward the same that is to be multiplied.

Scholer. If you set forth an example hereto, I thinke I shall perceiue you.

Master. Take this example: I would multiplie 1542 by 365, therefore I set the numbers thus.



Then first I begin at the 1000 in the highest roome, as if it were the first place, and I take it vp, setting downe for it so often (that is once) the multiplier, which is 365, thus as you see here: where, for the one counter taken vp from the fourth line, I haue set downe other 6, which make the summe of the multiplier, reckoning the fourth line as if it were the first, which thing I haue marked by the hand set at the beginning of the same.



Scholer. I perceiue this wel, for in deed this sum that you haue set downe is 365000: for so much doth amoũt of 1000, multiplied by 365.

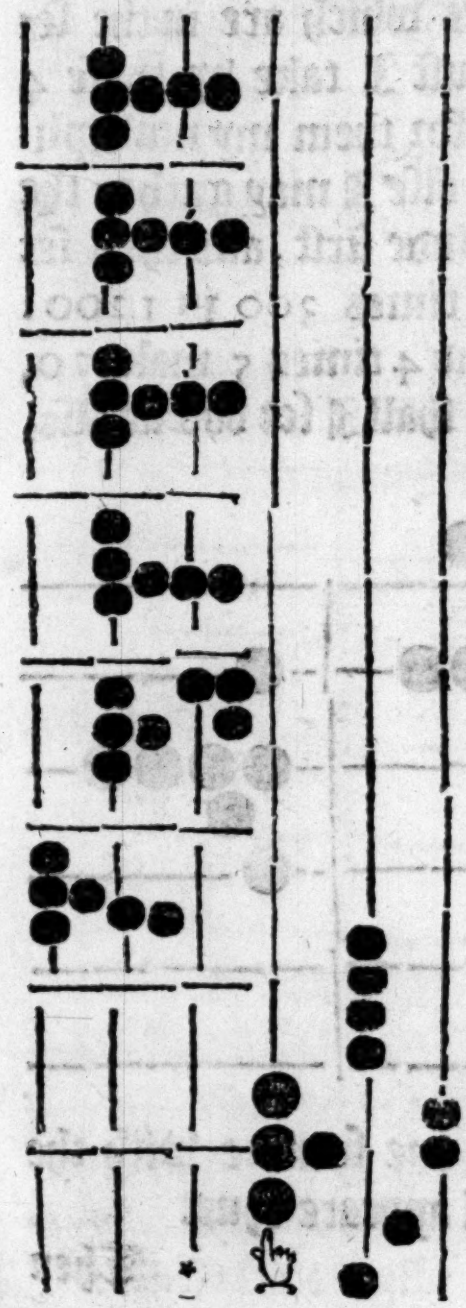
Mast. Well, then go forth, in the next space I find one coũter, which I remoue forward, but take it not vp, but do (as in such case I must) set downe the greater half of my multiplier (seing it is an odde nũber) which is 182, & here I doe still let that fourth place stand, as if it were the first: as in these examples you shall see.



Where I haue set this multiplication with other, but for the ease of your vnderstanding, I haue set a litle line betweene them. Now should they both in one summe stand thus.

How





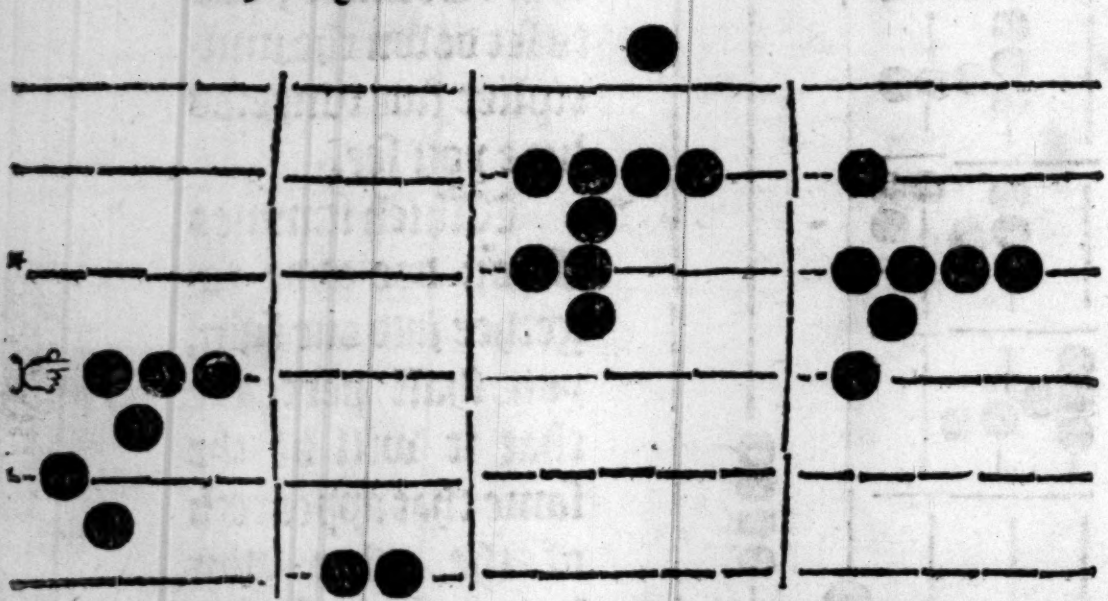
Howbeit, another fourme to multiplie such Counters in space, is this: First to remoove the finger to the next line beneath that space, and then to take vp that Counter, and to set down the multiplier fiue times: as here you see.

Which summes if you doe adde together into one sum, you shall perceaue that it will be the same that appeareth of the other working before, so that both sortes are to one intent: but as the other is shorter,

so this is plainer to reason, for such as haue had small exercise in this Art.

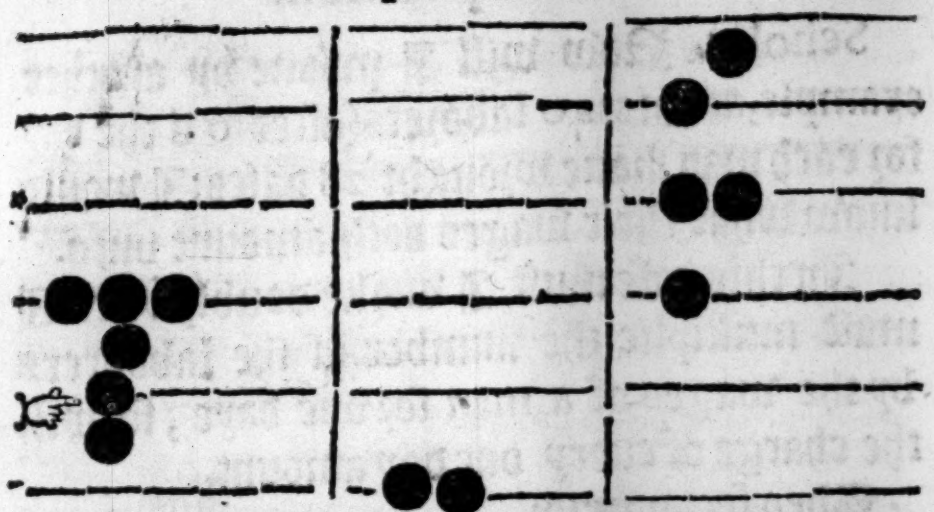
Notwithstanding you may adde thē in your mind before you set them downe: as in this example you might haue said, 5 times 300 is 1500, & 5 times 60 is 300: also 5 times 5 is 25, which al put togither, do make 1825, which you may at one time set downe if you list.

But now to go forth, I must remoue the hand to the next counters which are in the second line, and there must I take vp those 4 counters setting downe for them my multiplier 4 times seuerally, or else I may gather the whole summe in my minde first, and then set it downe: as to say, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460, that shall I set downe also, as here you see.

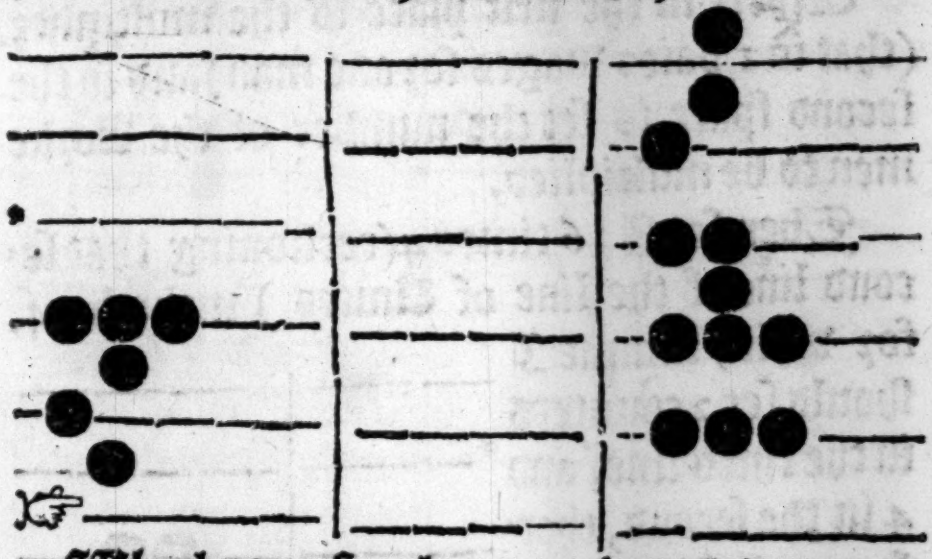


Which if I ioyne in one summe with the former numbers, it will appeare thus.

Then



Then to the end this Multiplication, I remoue the finger to ỹ lowest line, where are only 2, then do I take vp, & in their stead do I set downe twice 365, that is 730, for which I set one in the space aboue the third line for 500, & 2 more in the third line with that one that is there already, and the rest in their order, and so haue I ended the whole summe thus.
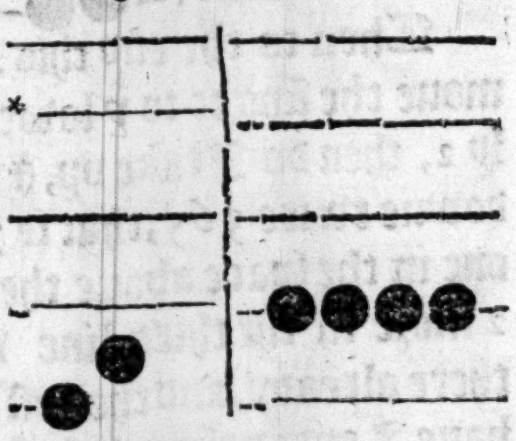


Wherby you see, that 1542 (which is the nũber of yeres sith Christ his incarnation) being multiplied by 365 (which is ỹ number of daies in one yeere) doth amoũt vnto 562830, which declareth ỹ number of daies sith Christs incarnatiõ vnto the end of 1542 yeares, (beside 385 daies and 12 houres for leape yeares.)

The sum of the daies sith Christs incarnatiõ.

Scholer. Now will I prooue by another example, as this: 40 labozers (after 6 d the day foz each man) haue wzought 28 daies: I would know what their wages doth amount vnto.

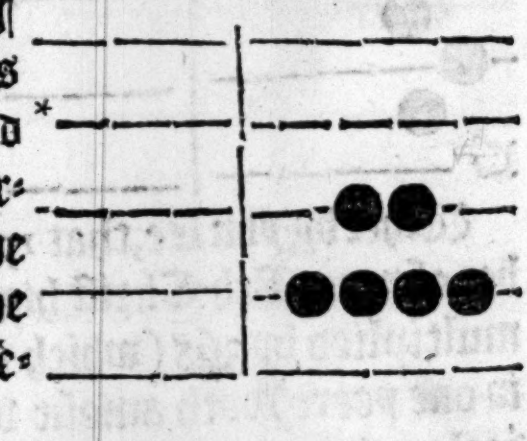
In this cale muſt I wozke doubly: firſt I muſt multiplie the number of the labourers by the wages of a man foz one daye, ſo will the charge of euery one day amount.

Then ſecondarily ſhall I multiply the charge of one day by the whole nũber of daies, and ſo wil the whole ſum appeare: firſt therfoze I ſhall ſet the ſums thus.



Where in the firſt place is the multiplier (that is 1 daies wages foz one man) and in the ſecond ſpace is ſet the number of the wozke men to be multiplied.

Then ſay I: 6 times 4 (reckoning that ſecond line of the line of Unites) maketh 24, foz which ſumme I ſhould ſet 2 counters in the third line, and 4 in the ſecond, therfoze do I ſet 2 in the third line, and let the 4 ſtand ſtill in the ſecond line thus.

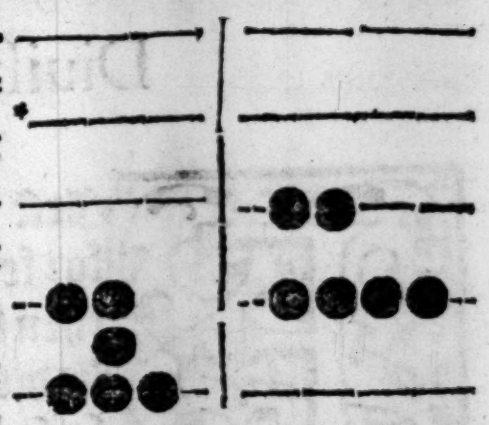


So appeareth the whole daies wages to

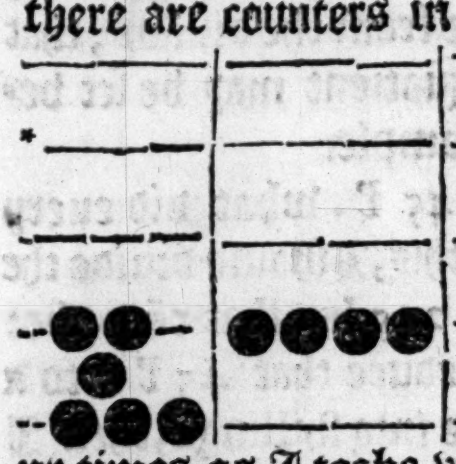
be

be 240 d, that is 20 s.

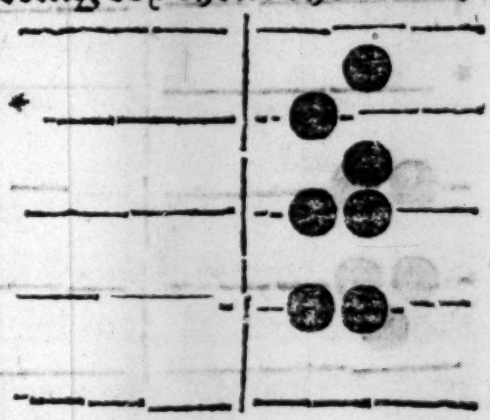
Then do I multiplie againe the same summe by the number of dayes, and first I set the numbers thus. Then because there are counters in



diuerse lines, I shall begin with ẏ highest, & take thē vp, setting for thē the multiplier so many times as I tooke vp counters, that is twise, then will the summe stand thus.



Then come I to the second line, and take vp those 4 counters, setting for them the multiplier foure times, so wil the whole summe appeare thus.



So is ẏ whole wages of 40 workemen for 28 daies (after 6 d each day for a man) 6720 d, that is, 560 s, or 28 l.

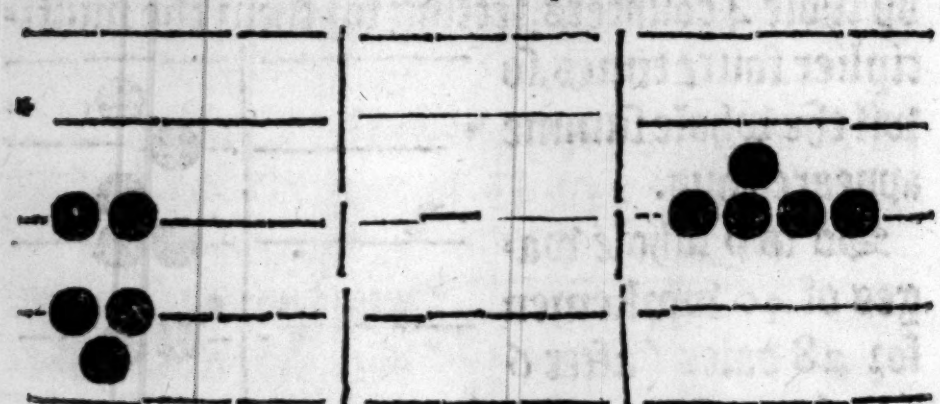
M. Now if you would proue Multiplication, the surest way is by Diuisiō, therefore wil I ouerpasse it, till I haue taught you the Art of Diuision, which you shall worke thus.

Diui-

Diuision.

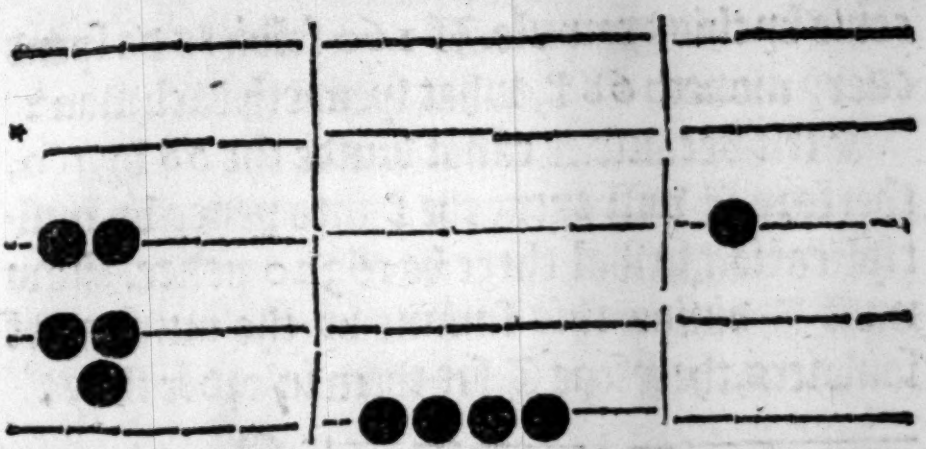
FIrst set downe the diuisor, for feare of forgetting, and then set the nũber that shalbe diuided, at the right side, so far from the diuisor, that the quotient may be set betweene them: as for example.

If 225 sheepe cost 45 ℔, what did euery sheepe cost? To know this, I should diuide the whole sum, that 45 ℔, by 225, but that cãnot be: therefore must I first reduce that 45 ℔ into a lesser denomination, as into shillings, then I multiplie 45 by 20, & it is 900: that sum shall I diuide by the nũber of sheepe, which is 225, these two numbers therefore I set thus.

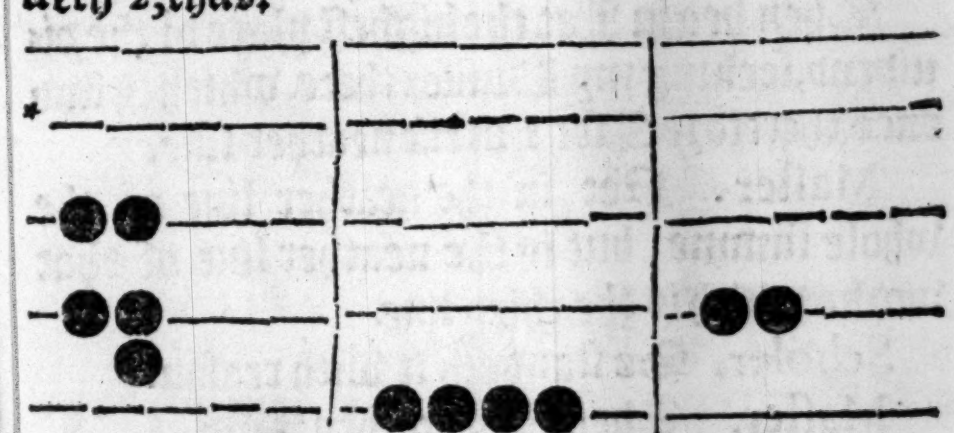


Then begin I at the highest line of the diuidend, and seeke how often I may haue the diuisor therein, and that may I do foure times: then say I foure times 2 are 8, which if I take from 9, there resteth but 1, thus.

And



And becauſe I found the diuiſor 4 times in the diuidend, I haue ſet as you ſee, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of the diuiſor as often out of the remainer, therefore come I to the ſecond line of the diuiſor, ſaying: 2 times 4 make 8, take 8 from 10, and there reſteth 2, thus.



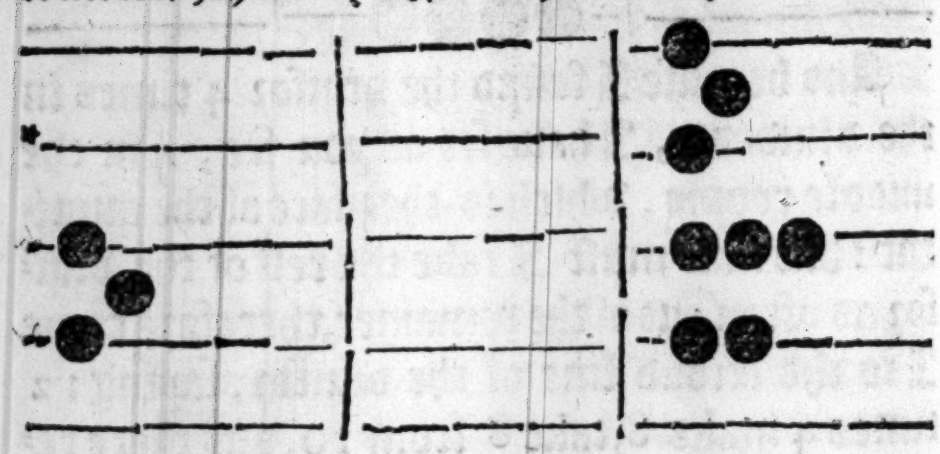
Then come I to the loweſt number which is 5, and multiplie it 4 times, ſo is it 20, that take I from 20, & there remaineth nothing, ſo that I ſee my quotient to be 4, which are in value ſhillings, for ſo was the diuidend: and thereby I know that if 225 ſheepe did coſt 45 ℔, euery ſheepe coſt 4 s̄.

Scholer. This can I doe, as you ſhall perceiue

Example of wages.

ceiue by this example. If 160 soldiers do spend euery moneth 68 ℔, what spendeth each man?

First because I cānot diuide the 68 by 160, therfore I will turne the ℔ into pence by multiplication, so shal there be 16320 pence. Now must I diuide this summe by the number of souldiers, therefore I set them in order thus.

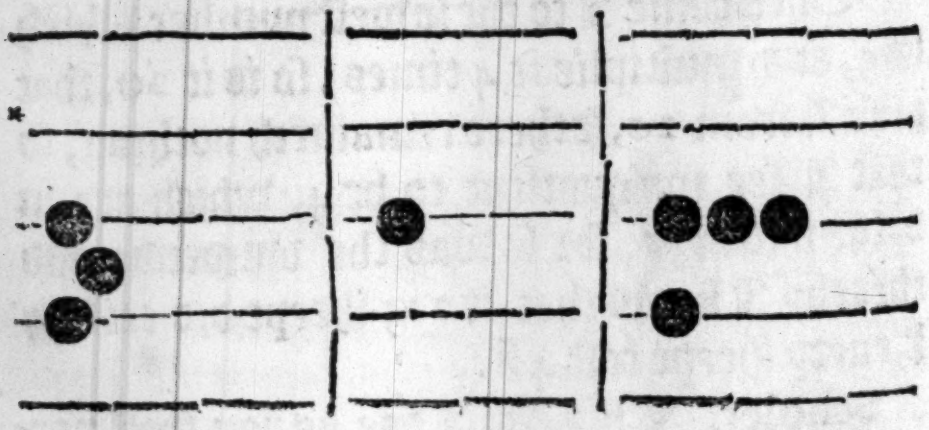


Then begin I at the highest place of the dividend, seeking my Diuisor there, which I find once, therefore I set 1 in the neather line.

Master. Not in the neather line of the whole summe, but in the neather line of that worke which is the third line.

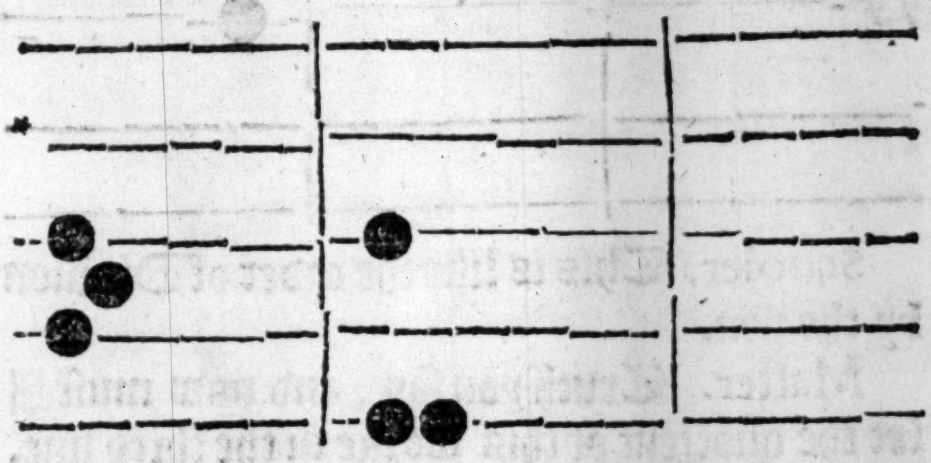
Scholer. So standeth it with reason.

Master. Then thus do they stand.



Then

Then seeke I againe the rest, how often I may find my diuisor: and I see that in 300 I might finde 100 three times, but then the 60 will not be so often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there resteth 100, out of which with the 20 (that maketh 120) I may take 60 also twice and then stand the numbers thus.



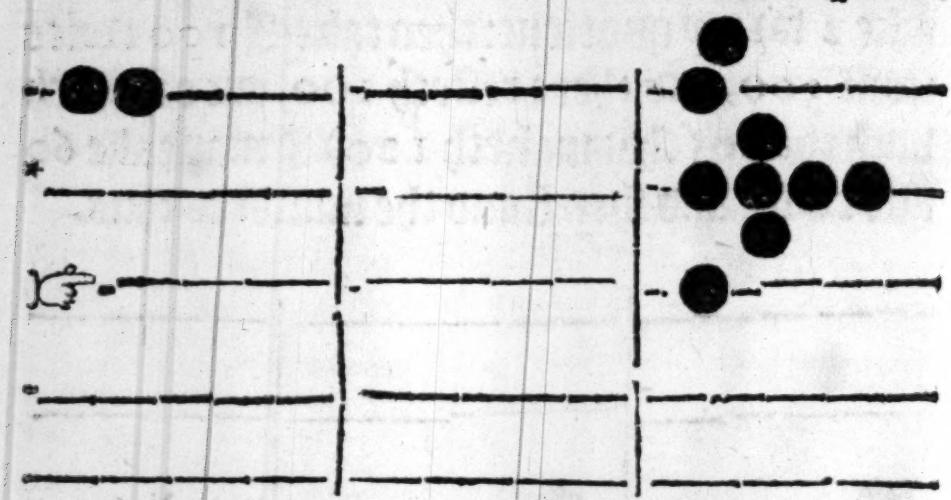
Where I haue set the quotient 2 in the lowest line: So is euerie Souldiers portion 102 d, that is 8 s, 6 d.

Master. But yet because you may iustly perceiue the reason of Diuision, it shall be good that you do set your Diuisor still against those numbers from which you doe take it, as by this example I will declare.

If the purchase of 200 acres of ground did cost 290 pound, what did one acre cost? Exãple of purchase.

First will I turne the poundes into pence, so will there be 69600 pence. Then in setting downe these numbers I shall doe thus. First set the diuidend on the right hand as it ought,

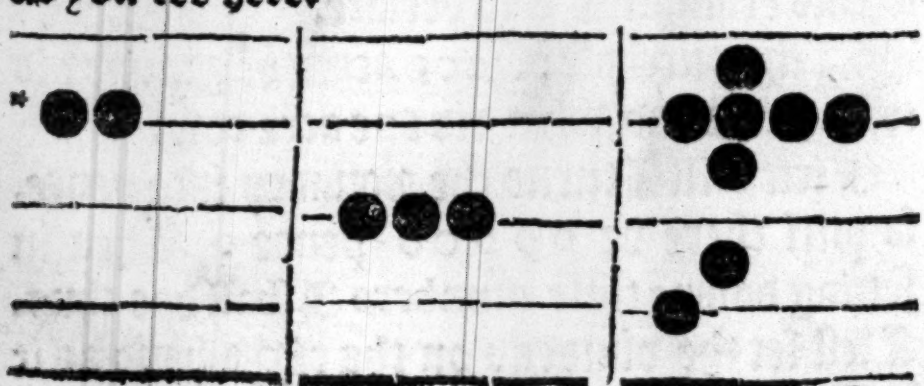
ought, & then the diuisor on the left hãd against those numbers from which I intend to take him first, as here you see, where I haue set the Diuisor two lines higher than his own place.



Scholer. This is like the order of Diuision by the pen.

Master. Truth you say, and now must I set the quotient of this worke in the third line, for that is the line of vnits in respect to the diuisor in this worke.

Then I seeke how often the deuisor may be found in the diuidend, and that I find 3 times, then set I 3 in the third line for the quotient, and take away that 60000 from the diuidend, and further I set the Diuisor one line lower, as you see here.



And

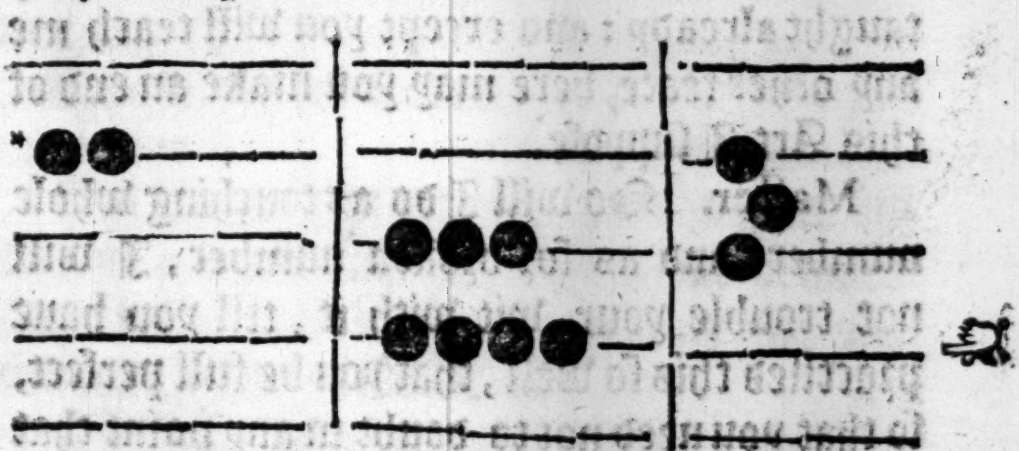
And then seeke I how often the diuisor will be taken from the number against it, which will be 4 times, and 1 remaining.

Scholer. But what if it chaunce that when the diuisor is so remooued, it cannot be once taken out of the diuident against it?

Master. Then must the diuisor be set in an other line lower.

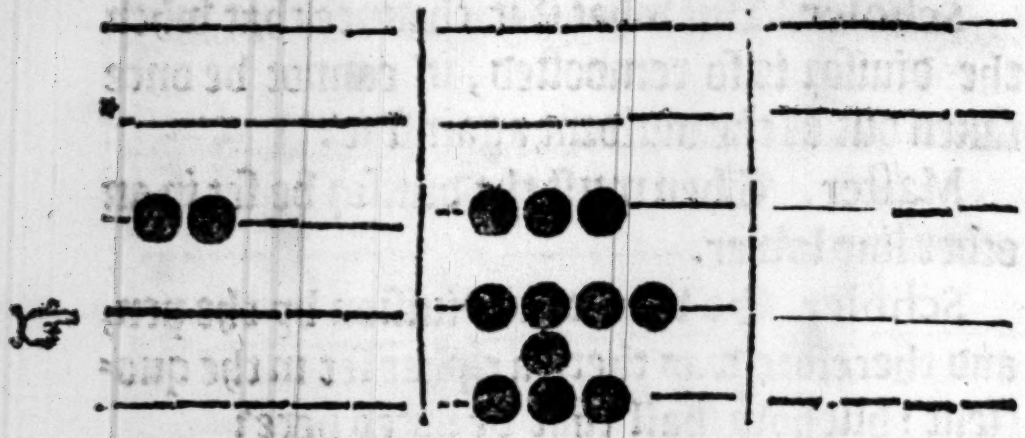
Scholer. So was it in Diuision by the pen, and therefore was there a cipher set in the quotient: but how shall that be noted here?

Master. Here needeth no token, for the lines do represent the places: onely looke that you set your quotient in that place which standeth for vnites in respect of the diuisor. But now to returne to the example. I finde the diuisor 4 times in the diuident, and 1 remayning: for 4 times 2 make 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the quotient I set 4 in the second line, which is in this worke the place of vnites.



Then remoone I the diuisor to the next

lower line, and seeke how often I may haue it in the diuidend, which I may do here 8 times iust, and nothing remaine, as in this forme.



Where you may see that the whole quotient is 348 d, that is 29 s, whereby I know that so much cost the purchase of one acre.

Scholer. Now resteth the proues of Multiplication, and also of Diuision.

Master. Their best proues are eche one by the other: for Multiplication is proued by Diuision, and Diuision by Multiplication, as in the worke by the pen you learned.

Scholer. If that be all, you shall not neede to repeate againe that which was sufficiently taught already: and except you will teach me any other feate, here may you make an end of this Art, I suppose.

Master. So will I do as touching whole number: and as for broken number, I will not trouble your wit with it, till you haue practised this so well, that you be full perfect, so that you need not to doubt in any point that I haue taught you, and then may I boldly in-

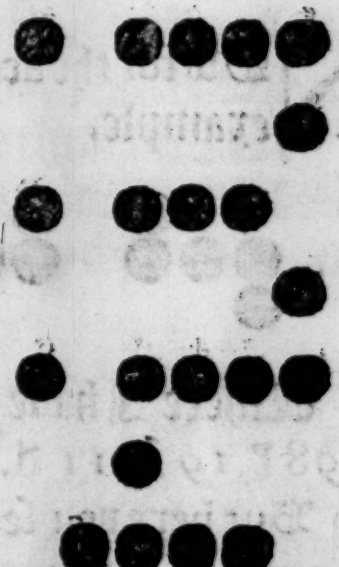
struct

ſtruct you in the Art of Fraction or Broken number: wherein I will alſo ſhew you the reaſons of all that you haue now learned. But yet before I make an end, I wil ſhew you the order of cõmon caſting, wherein are both pence, ſhillings, and pounds, proceeding by no grounded reaſon, but onely by a receiued forme, and that diuerſly of diuers men: for the Marchants vſe one forme, and Auditors an other.

Marchants vſe.



At firſte for Marchantes forme, marke this example here, in which I haue expreſſed this ſumme 198 l, 19 s, 11 d. So that you may ſee that the loweſt line ſerueth for pence, the nexte aboue for ſhillings, the third for poũds, and the fourth for ſcores of pounds.



And further you may ſee that the ſpace betweene d and s may receiue but one counter (as all other ſpaces likewiſe do) and that one ſtandeth in that place for 6 d.

Likewiſe betweene the ſhillings and the pounds, one counter ſtandeth for 10 s.

And betweene the pounds & 20 l, one counter ſtandeth for 10 l.

But beſide thoſe you may ſee at the left ſide of ſhillings, that one counter ſtandeth alone, and betokeneth 5 s.

So againſt the pounds, that one counter ſtandeth for 5 l. And againſt the 20 pounds, the one counter ſtandeth for 5 ſcore pounds, that is 100 pound, ſo that euery ſide counter is 5 times ſo much as one of them againſt which he ſtandeth.

Auditors account.

NOw for the account of Auditors, take this example.



Auditors account.

Where I haue expreſſed the ſame ſumme 198 l, 19 s, 11 d.

But here you ſee the pence ſtand towardes the right hand, and the other increaſing orderly toward the left hand.

Againe you may ſee that Auditors will make 2 lines (yea and more) for peuce, ſhillings, and all other values, if their ſummes extend thereto. Alſo you ſee that they ſet one counter at the right end of each rowe, which ſo ſet there, ſtandeth for 5 of that roome: and

on the left corner of the row it standeth for 10 of the same row.

But now if you would adde, either subtract after any of both those sorts, if you marke the order of the other feate which I taught you, you may easily doe the same here without much teaching: for in Addition you must first set downe one summe, and to the same set the other orderly. and in like maner if you haue many: but in Subtraction you must set downe first the greatest summe, and from it must you abate the other, euerie denomination from his due place.

Scholer. I doe not doubt but with a little practise I shall attaine these both: but how shall I multiplie and diuide after these formes?
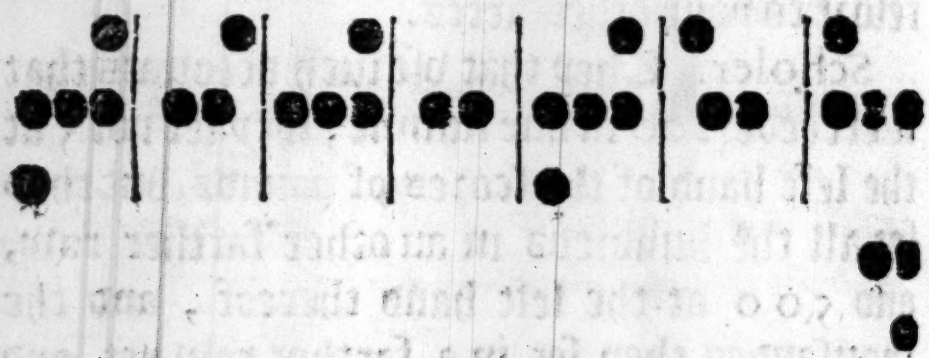
Master. You cannot duely do any of both by these sorts, therefore in such case you must resort to your other artes.

Scholer. They that vse such accounts that it exceede 200 in one summe, they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in an other farther row, and 500 at the left hand thereof, and the thousandes they set in a farther row yet, and at the left side thereof they set the 5000, and in the space ouer they set the 10000, and in a higher row 20000, which all I haue expressed in this example, which is 97869 l. 12 s. 9 d. ob. q. Ninetie seuen thousand, eight hundred, three score and nine pound,

twelue shillings and nine pence halfpenie farthing, for I had not told you before, where, neither how you should set downe farthings, which (as you see here) muste be set in a voide space sideling beneath ỹ pence: for farthing one counter, for ob. 2 coũters: for ob. farthing, 3 ☞ counters: and more there cannot be: for 4 farthings make 1 d, which must be set in his due place.



And if you desire ỹ same summe after Auditours manner: Lo here it is.



But in this thing you shall take this for sufficient, and the rest you shall obserue as you may see by the working of each sort: for the diuerse wits of men haue inuented diuers and sundrie wayes, almost vnnumerable.

The

The second part of Arithmetike touching Fractions, briefly set forth.

Scholer.

ALbeit I perceiue your manifold businesse doth so occupie, or rather oppresse you, that you cannot as yet completely end ỳ treatise of Fractions Arithmeticall, which you haue prepared, wherein not only sundry works of Geometrie, Musicke, & Astronomie be largly set forth, but also diuers conclusions & naturall works, touching mixtures of metals & cõpositions of medicines, with other strange exãples, yet in the meane season I cannot stay my earnest desire, but importunely craue of you some briefe preparation towarde the vse of Fractions, whereby at the least I may be able to vnderstand the common workes of them, and the vulgar vse of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will is good, you should not neede to vse any importunate crauing, for the attaining of that thing, whereby I may be perswaded that

I shall any waies profite the common wealth, or helpe the honest studies of any good members in the same: wherefore, while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

What a Fraction is.

Therfore first to begin with explication of this name Fraction, what take you it to be?

Scholer. Marie sir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to say, to be no whole number, but a part of a number.

Master. A Fraction in deed is a broken nũber, and so consequently, the part of an other number: but that must be vnderstanded of such an other number, as cannot be diuided into any other parts than fractiõs: for although I may take the third part of 60, or the fourth part of it, & so of other partes diuersly, yet these partes be not properly, nor ought not to be called tractions, because they may be expressed by whole nũbers: for the third part of it is 20: the fourth part is 15: the twelfth part is 5, and so forth of other parts, which all be whole numbers.

What a Fractiõ is properly.

Wherefore properly a Frãction expresseth the partes or part onely of an vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater than one: and therefore it followeth, that no one Fraction alone can be so great, that it shall make 1, as by examples I will declare, as soone as I haue taught you to know the

forme how a Fraction is expressed or represented in writing.

Numeration

BUt first to begin wt expressing of a fraction, which is the numeration of it, you must vnderstand that a fractiõ is represented by 2 numbers, set one ouer ỹ other, and a line drawne betweene them as thus, $\frac{1}{3}.\frac{3}{4}.\frac{2}{5}.\frac{10}{17}$. which foure Fractions you must pronounce thus: $\frac{1}{3}$ one third part: $\frac{3}{4}$ three quarters: $\frac{2}{5}$ two fift parts: $\frac{10}{17}$ ten seuenteene parts.

Scholer. I vnderstand the forme of their expression and pronuntiation, but their meaning or valuation seemeth more obscure: yet I thinke that by the two first Fractions I vnderstand the valuation of the two latter fractions, and so consequently of other.

Master. Value them then, that I may perceiue your taking of them.

Scholer. $\frac{2}{5}$ Betokeneth two fift partes, that is to say, if one be diuided into 5 partes, that Fraction doth expresse two of those fift partes: $\frac{10}{17}$ doth signifie, that if one be diuided into xvii.

parts,

parts, I must take ten of them: And this I gather of the two first examples: for $\frac{1}{3}$ that is one third part, doth easily declare, that if any one thing be diuided into three parts, I must take but one of them: so $\frac{3}{4}$ that is three quarters, doth declare that one being diuided into foure quarters, I must take (for this Fraction) three of those quarters.

If there be no more difficultie in their Numeration, then I pray you go forward to their Addition and Subtraction, and so to the other kinds of works, for I vnderstand that the same kindes of workes be in Fractions, that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is diuerse, as I will anon declare: but yet more in Numeration before we leaue it. You must vnderstãd, that those two numbers which expresse a Fraction, haue seuerall names. The ouermost which is aboue the line, is called the Numerator, and the other beneath the line is called the Denominator.

Numerator and Denominator.

Scholer. And what is the reason of their diuerse names? For in mine opinion both be Numerators, seeing both they do expresse the numeration of the Fraction.

Master. You are deceiued: for one onely (which is the ouermost) doth expresse the numeration: and the denominator doth declare the number of partes into which the vnite is diuided,

diuided, as in this example, when I say: Diuide a pound weight of gold betweene four men, so that the first man shall haue $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now doe you perceiue that by the Denominator (which is one in all foure Fractions) it is intended, that the pound weight should be diuided into so many partes, I meane 15, and by the foure seuerall numerators is limited the diuerse portion that each man should haue, that is, that when the whole is parted into 15, the first man shall haue 2 of those 15 parts: the second man three of them: the third man 4: and the fourth man 6. And so may you see the seuerall offices (as it were of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceiue, that a man can haue no more partes of any thing than it was diuided into, neyther yet aptly so many: so that it were vnaptly said: You shall haue $\frac{15}{15}$, that is 15 parts of any thing, seeing it were better said: You shall haue the whole thing.

Scholer. So doth it appeare reasonably, for the labour is vaine to diuide any thing, and then to applie the Diuision to no vse. And much lesse reasonable where it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts onely, it is not possible to take 16 of them, that is to say, more than altogither.

Master. This is true touching the proper

and

and apt vse of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes be called Fractions, because they be written like Fractions, although they be none in deede for $\frac{15}{15}$, and generally all such other: where the Numerator and Denominator be equall, are not Fractions: but the whole thing with all his parts. And so $\frac{16}{12}$ is not to be called a fraction, but a mixt number, of a whole number & a fraction: for it is as much, as $1\ \frac{4}{12}$, that is one whole one and 4 twelue parts, as shalbe declared in reduction. Therefore they do abuse the names, that call them fractions, where the Numerator is eyther equall or greater than the denominator.

Scholer. But is there any needefull cause why they should so abuse the name?

Master. There is cause why they shall sometimes, for easinesse in worke, write some numbers after that sorte, like fractions: but they needed not to call them fractions, but as they be, whole numbers or mixt numbers (that is whole numbers with fraction) expressed like fractions.

Nowe must you vnderstand, that as no fraction properly can be greater than 1, so in smalnesse vnder one the nature of fractions doth extend infinitly: as the nature of whole numbers is to increase aboue one infinitly, so that not onely one may be diuided into infinite

frac-

fractions or parts, but also euery fraction may be diuided into infinite fractions or partes, which commonly be called fraction of fractions, and they be expressed diuersly: As for example, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is three quarters of two third parts, of one halfe part. Whereby is signified, that if one be diuided into two halfes, and the one halfe into three partes, and two of those three parts be diuided ioyntly into foure quarters, this fraction of fractions doth represent three of those quarters.

Scholer. I pray you let me proue by an example in common money, whether I do rightly vnderstand you or no. One Crowne, which I take for an vnit, doth cõtaine 60 pence, therfore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fifteene pence, so then 15 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crowne. And so 3 pence is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceiue this well inough: yet this note I giue you by the way, that the forme of expressing these Fractions is voluntarie: and hath no other reason than the will of the Deuiser, which forme many follow: for some expresse them thus $\frac{3}{4}\frac{2}{3}\frac{1}{2}$ without any figure of distinctiõ betweene them, which forme many follow. Some other doe make lines betweene euery Fraction, and adde words of distinction, after this sorte, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{1}{2}$, which forme is most best.

Some

Some other expresse them thus in slope forme, to distinct them from seuerall Fractions of one whole number, for if they were set in one right line thus, $\frac{3}{4}\ \frac{2}{3}\ \frac{1}{2}$: then ought it to be pronounced, three quarters, and two third partes and an halfe, which maketh almost two whole vnites, lacking but one xii. part. And so is it nothing agreeable with the other fraction of fractions, wherefore it is a great ouersight in certaine learned men, which doe expresse them so confusedly with such seuerall fractions, that a man cannot know the one from the other.

Therefore some men (as Stifelius) doe expresse without a line numbers of proportion, being applied to Addition or Subtraction: because they must be taken as two, where the line in fractions maketh them to be taken for one: for of the Numerator and Denominator is made one number.

Three seuerall varieties.

Scholer. Then I perceiue there be three seuerall varieties in fractions: First when one onely fraction is set for one number, as $\frac{4}{5}$, that is foure fifth parts. The second is, when there be set two or more seuerall fractions of one number, as $\frac{4}{9}\ \frac{2}{5}$, that is foure ninth partes, and two fift partes. The third sort is fractions of fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is 4 ninth partes of two fifth partes.

Master. You haue said well, if you vnderstand

derstand well your owne wordes.

Scholer. If it shall please you, I will by an example in the parts of an old English Angell expresse my meaning.

Master. Let me heare you.

Scholer. The old English Angel did containe 7 shillings 6 d, that is 90 d. Now $\frac{4}{9}$ of it, is 72 d. And of the same 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth partes, and 2 fifth partes, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is 4 nine partes of two fifth partes, seeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yeeld but 16, for $\frac{1}{9}$ of 36, is but 4, and that taken 4 times maketh 16.

Master. This is plainely expressed, and truely, and hereby (I doubt not) but you doe perceiue, that as great a difference as is betweene 16 and 76, so much difference is betweene those two fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you vnderstand these varieties, I will proceede to the rest of the workes: first admonishing you, that there is an other order to be followed in fractions than there was in whole numbers, for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuision, and Reduction, but in fractions (to follow the same aptnesse in proceeding from the easiest workes to the harder) we must vse this order of the workes: Numeration, Multiplication, Diuision, Reduction, Addition, and Subtraction.

ction.

Scholer. That Multiplication and Diuision should goe together, and Subtraction to follow Addition, naturall order doeth perswade: but why Multiplication should be first in order here next to Numeration, and Reduction in the middle, I desire to vnderstand the reason.

Master. As in the Art of whole numbers order would reasonably begin with the easiest, and so goe forward by degrees to the hardest, euen so reason teacheth in Fractions the like order. And considering that Addition or Subtraction of fractions can very seldome be wrought without Multiplication and Reduction: and contrariwise, Multiplication and Reduction may bee wrought without this forme of Addition or Subtraction: Therefore was it orderly required, that Multiplication and Reduction should go before Addition and Subtraction. And the same reason serueth for the placing of Multiplication before Reduction.

Scholer. Then if Multiplication be the easiest, I pray you declare the forme of it first by rule, and then by example.

Master. Your example is good.

Multi-

Multiplication.

THerefore when any two Fractions be proponed to be multiplied together, the Numerator of the one must be multiplied by the numerator of the other: & the summe that amounteth therof, must be set for a new Numerator: likewise the Denominator of the one must be multiplied by the Denominator of the other, and that that amounteth, shall be set for the common Denominator: and this new third fraction expresseth the product of the Multiplication of the two first fractions proponed, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

Scholer. I perceiue then, that 3 being the numerator of the first fraction, is multiplied by 5, being the numerator of the second fraction, whereof amounteth 15, the numerator of the third fraction. And so likewise, 5 being denominator of the first fraction, is multiplied by 12 the denominator of the second fraction, whereof amounteth 60 the new denominator: so that I perceiue how the worke is doone, but I doe not perceiue how $\frac{15}{60}$ is greater than $\frac{3}{5}$: For I shall not vse my former maner of examination by the partes of some Coine, I see that $\frac{3}{5}$ of a Crowne is 36 d, and $\frac{5}{12}$ of a Crowne, is 25 d, whereof the one

multiplied by the other, doth make 900 ð, which is 15 Crownes: but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 ð, and that is much lesse than any of both the first fractions.

Master. That difference is betweene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers the summe that amounteth, is greater then both the other whereof it came: but in fractions it is contrariwise: for the summe that amounteth, is lesser then any of the other two fractions, whereof it came.

Scholer. I desire much to vnderstand the reason thereof.

Master. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect booke of Arithmetike, yet I will shew you this, because of the strangenesse of the worke.

You see in whole numbers, that of two numbers being multiplied together, is made the third number: which third number doth beare the same proportion to the number multiplied, that the multiplier doth beare to an vnite. And so in Fractions, the third number which amounteth of multiplication, beareth the same proportion to ech of the two first fractions, that the other of those two fractions doth beare to an vnite.

Scholer. Sir, I vnderstand your wordes thus:

thus: when 40 is multiplied by 12, there doeth amount 480, which 480 doeth containe 40 so many times in it, as twelue doth containe vnites, that is to say: twelue times. And so it appeareth that 480 doeth containe twelue so many times also, as 40 doeth containe vnites, that is 40 times. But now I see not how the thirde number in this example of Fractions can containe any of the two former (as it happened in whole numbers) seeing it is lesser than either of them.

Master. No maruaile if you cannot see that thing which is not possible to be seene of any man, how the third number in multiplication of Fractions should be greater than any of the two former Fractions, but yet this may you see (which I said) that the third number in Fractions so multiplied, doeth beare the same proportion to any of the two former fractions, that the other of those two Fractions doth beare to an vnite, as in your example $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$. Now say I, that $\frac{15}{60}$ doth beare the same proportion to $\frac{3}{5}$, that $\frac{5}{12}$ doth beare to an vnite, as you may in your owne forme of examination by coyne trie it. For in an old Angell (which in times past was currant for 7 s̃, 6 d̃) are 180 halfe pence, which I set for the intire vnite, whose partes (according to the fractions aforesaid) are these, for $\frac{15}{60}$ set 45 ob. for $\frac{3}{5}$ take 180 ob. and for $\frac{5}{12}$ put 75 ob. Now doth 45 beare the same pro-

portion to 108 that 75 doth beare 180: for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserued til another time, when the knowledge of proportions in due order shall be taught. Yet in the meane season I will shewe you how it commeth to passe that in Fractions the third summe must needes be lesser than any of the other two.

Consider thus, that when a Fraction is proponed, as in the former example $\frac{3}{5}$, if it be multiplied by more than 1, it will make more than one entire number. As if I multiplie $\frac{3}{5}$ by 5, that is to say, if I take it 5 times, it will make three entire vnites: exãple in a crowne, $\frac{3}{5}$ of it maketh 3 s̃, which if I take fiue times, it will amount to 15 s̃, that is three entire Crownes: so if I take the same $\frac{3}{5}$ but twise, it will yeeld 6 s̃, that is one entire Crowne and $\frac{1}{5}$. Now if I take it but once, it cannot be more than it was before, that is 5 s̃. And if I take it lesse then once, it cannot be so much as it was before. Then seeing that a fraction is lesse then one, if I multiplie a fraction by another fraction, it followeth that I do take that first fraction lesse than once, and therefore the summe that amounteth, must needes be lesse than the first fraction.

Scholer. Sir, I thanke you much for this reason. And I trust I doe perceiue the thing, as by example of this same fraction $\frac{3}{5}$ I will expresse.

yᵗᵉsse. If I take $\frac{1}{5}$ of a crowne once, that is to say, if I multiplie $\frac{1}{5}$ by 1, it will be as it was before, but 3 s: so if I doe multiplie it by $\frac{1}{2}$, that is, if I take but halfe one time, then will it be but halfe so much: likewise if I multiplie it by $\frac{1}{3}$, that is, if I take but the third part once, it will yeeld but 12 pence, that is the third part of the first fraction.

And so to make an ende. If I take it but the twelfth part once, that is, if I doe multiply it by $\frac{1}{12}$, it will yeeld but the twelfth part of the first fraction, which is but three pence. And it followeth that if $\frac{1}{12}$ make 3 pence, then $\frac{1}{12}$ must needes make fiue times so much, that is 15 pence, which was the summe that hath giuen the occasion of all this doubt.

Master. Then I perceiue you haue sufficient vnderstanding in this sorte of Multiplication for this time, wherefore I will omit that I might say more of Multiplication, till wee come to Reduction, and will passe to the other worke, and first to Diuision, whose place followeth Multiplication, both by natural order, and also in easinesse of worke.

Diuision.

WHensoeuer two Fractions be propoued, that the one should be diuided by the other, I must set downe first the Fraction that shall be diuided (which is called the Diuidend) and then after it the other, which is the Diuisor. Then shall I multiplie the numerator of the diuidend by the denominator of the diuisor, and that which amounteth, I must put for a new Numerator. Againe I shall multiplie the denominator of the diuidend by the numerator of the diuisor, and the number that amounteth thereof, I must put for the new denominator. And this third fraction is the quotient of the said Diuision.

Scholer. This seemeth easie in forme, as by example, thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{6}$, first I must multiplie 5 (being the numerator of the diuidend) by 6, which is denominator of the diuisor, and thereof riseth 30: then I multiplie 8 (being the denominator of the diuidend) by 2, being the numerator in the diuisor, and so riseth 16, the which I must make in a third fraction thus $\frac{30}{16}$.

Master. Me seemeth you are quicker in vnderstanding now, than you were when I taught you the Art of whole numbers: but that

that is no maruell, for the more knowledge that any man getteth, the readier shall he find his wit, and quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued here for ease of worke, and lightnesse of vnderstanding the nature of the Quotient.

Whensoeuer you diuide one Fraction by an other, either they be both equall togither: eyther else the one is greater than the other: if they be equall, their quotient shall be such that the numerator and the denominator of it shall be equall also. And if the two first fractions be vnequall, their quotient shall declare the same by the vnequalitie of the numerator and denominator, as in these examples following shall appeare.

First of equal Fractions: $\frac{4}{9}$ and $\frac{12}{27}$ be equal togither: and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforesaid.

Now in the vnequall Fractions, as $\frac{4}{9}$ and $\frac{3}{10}$ the quotient will be $\frac{40}{27}$: where the numerator is greater than the denominator.

Scholer. I see it is so, but I see not the reason why it should be so.

Master. The reason is this. When any Fraction is diuided by another, the quotient declareth what proportion the diuidend beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which must be founded, not two, but twice:

Note how to know the proportiõ between two numbers.

twice: declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the numerator in the quotient, representeth the diuidend, and the denominator representeth the diuisor. And this is alwaies true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne, till you haue learned the Art of proportions: notwithstanding somewhat of it I will declare in the nexte rule of Reduction. But now for the easie remembraunce of the quotient in Diuision, as soone as you haue set downe your two Fractions, the one against the other, then make a straight line for the quotient: and as soone as you haue multiplied the numerator of the diuidend, by the denominator of the diuisor, set the number that amounteth, ouer the said line, and then multiplie the other two numbers, and set their totall vnder the same line.

Scholer. I perceiue you would not haue me trust to memorie till I were better expert, least oftentimes I happen by misse remembrance to be abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other, (as here doeth appeare) and then make an other line for the quotient in some good distance, where I may set the numbers of the quotient, as soone as any of them

2 3
—by—
3 4

them is multiplied: So then as soone as I haue multiplied 2 by 4, which maketh 8, I shall set that 8 ouer that line thus $\frac{8}{\ }$. And then multiplie 3 by 3, which yeeldeth 9: and that 9 must be set vnder the same line, & then will the whole quotient appeare thus $\frac{8}{9}$. Whereby appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 8 is to 9: but how may I perceiue that? Note.

Master. Although you shall better perceiue it by the rule of Reduction, yet this example may be declared in common Coyne, as in a common shilling of xii d, of which $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 d, and so you may easily see that their proportions do agree. And if you had taken this example before, when you tooke the example of $\frac{5}{8}$ and $\frac{2}{6}$, your Quotient would appeare (as this doth) more easier to vnderstand, whereas that quotient being $\frac{30}{16}$, is not an easie proportion for you to perceiue, being yet litle acquainted with proportions: whereof to giue you some tast, I will enter to the rule of Reduction: in which also I will declare other workes, both of Multiplication and also of Diuision, which now I must for a time omit, as thinges that do need the helpe of Reduction.

Reduction.

Reduction.

Fiue varieties of Reduction.

THerefore will I now declare the diuersities of Reduction of Fractions, which commonly haue 5 varieties.

1 First, when there be sundrie fractions of one entire vnite, they must be reduced to one denomination, and also into one fraction.

2 Secondarily, when there be proponed fractions of fractions, they must be reduced likewise into one fraction, for otherwise they cannot be brought into one denomination.

3 Thirdly, when an improper fraction is proponed, that is to say, a fraction in forme, which in deed is greater than an vnite, it must be reduced into apt forme expressing the vnite or vnites of it, and the proper fraction distinctly. And sometimes also it shall be needefull to conuert such a mixt number of vnites, with fractions into the forme of a fraction, that is into an improper fraction: which 2 formes I esteeme but as one, because they worke on one kinde of number.

4 Fourthly, there happeneth sometimes Fractions to be written in great numbers, which might bee written in lesser numbers, therefore is there a meane to reduce such great numbers into their smallest termes.

5 Fiftly,

5 Fiftly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine parts, but none of like Denomination with that Fraction, then may you reduce the said Fraction into an other, whose denomination shal expresse the common parts of that whole thing.

Scholer. This distinction in doctrine delighteth me much, but more with hope than present fruite, for as yet I doe not vnderstand scarcely the varieties, and much lesse the practise and vse of their workes.

Master. Reduction is an orderly alteration of numbers out of one forme into another, which is neuer done orderly but for some needfull vse, as in euerie of the said 5 seuerall varieties I will distinctly declare.

First therefore, when two or more seuerall Fractions of any vnite be proponed, as for example $\frac{3}{16}$ and $\frac{4}{6}$: because it is hard to tell what proportion of the entire number those two fractions do expresse, therefore was Reduction diuised, to be a meane whereby these seuerall fractions might be brought into one denomination and fraction. The first sort of Reduction.

And in these Fractions this is the Art for bringing them to one denomination. How to reduce fractions of diuers denominations into one denomination.

Multiplie first the denominators togither, and the totall thereof you shall set twise downe vnder two seuerall lines for two new denominators, or rather for one common denominator:

nator: Then multiplie the numerator of the first fraction, by the denominator of the second, and set the totall thereof for the numerator ouer the first line. Likewise multiplie the numerator of the second fraction by the denominator of the first, and set that totall ouer the second line for the numerator of that fraction, and so are those two first fractions of seuerall denominations, brought to one denomination.

Scholer. If I vnderstand you, as I thinke I do, my example shall declare the same. The fractions which you proponed, were these, $\frac{2}{16}$ and $\frac{4}{6}$, whose denominators (being 16 and 6) I multiplie togither, and there amounteth 96, which I set vnder two lines, thus. $\frac{96}{96}$:

Then I multiplie the numerator of the first fraction by the denominator of the second, saying: 3 into 6 maketh 18, that set I ouer the first line for a new numerator, and it will be thus. $\frac{18}{96}$

Likewise I multiplie the numerator of the second fraction by the denominator of the first, saying: 4 times 16 maketh 64, that I set for the second numerator, and the fraction will appeare thus. $\frac{64}{96}$

So that both fractions brought to one denomination, must stand thus: $\frac{18}{96}$ and $\frac{64}{96}$

Master. You haue done well.

Scholer. I beseech you, let me examine it after my accustomed forme, by common parts

of

of coyne or other measure.

Master. Goe to.

Scholer. I haue a peece of gold which is accounted woorth 8 shillings, and containeth 96 pence, whereof $\frac{1}{16}$ that is the xvi part, is sixe pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Again $\frac{1}{6}$ of the same peece of gold, is 16 d. so that $\frac{4}{6}$ maketh 64 d, that is $\frac{64}{96}$. And so I finde the summes to agree with the other before.

Master. So haue you now the Arte to bring such two fractions into one denomination. And if there be more than two, then must you multiplie all the denominators together, and set the totall thereof so many times downe as there be fractions, and then to get for each one a new numerator, multiplie the numerator of the first, by the denominator of the second, and the totall thereof multiplie by the denominator of the third, and so foorth if there be more. Likewise multiplie the numerator of the second, by the denominator of the first, and the totall thereof by the denominator of the third. And in the same sorte multiplie the numerator of the third into the denominator of the first: and the totall thereof into the denominator of the second, and so foorth, if there were mo. So these 3 fractions $\frac{1}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ doth make by Reduction these other 3 fractions of one denomination $\frac{24}{60}$ $\frac{45}{60}$ $\frac{40}{60}$. All which you may bring into one fraction by adding the Numerators together, and putting that totall for

the

the common numerator, reseruing still that same common denominator. And those 3 fractions make one improper fraction thus. $\frac{102}{60}$

Scholer. All this I perceiue, and also that this last fraction is more than an vnite, and therefore you did call it an Improper Fraction.

Master. There be certaine other formes of working in this Reduction, which I will briefly touch also, to giue you an occasion to exercise your wit therein.

The first varietie of this Reduction.

The first varietie is this. When you haue made and written downe your common denominator (as I haue taught before) then to get a numerator for the first, doe thus. Deuide the common denominator by the denominator of the first fraction, and the quotient multiplied by the numerator of the same, yeeldeth a new numerator for the first new fraction. So likewise doe with the second and the third, and with all the residue, if there be more.

Scholer. That will I proue in your last example of these 3 fractions $\frac{2}{5} : \frac{3}{4} : \frac{2}{3}$. When the denominators be multiplied, they make 60, for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60, that I set downe three times, thus. 60 60 60: then to haue a numerator for the first, I must diuide 60 by 5, (the denominator of the first) and the quotient is 12, which I must multiplie by 2 (the numerator of the first) and that maketh 24, and so haue I for the first frac-

fraction $\frac{24}{60}$.

Likewise for the second fraction: I diuide 60 by 4, and there commeth 15: which I multiplie by 3, and so haue I 45 and the second fraction $\frac{45}{60}$. Then for the third in like sort wil come $\frac{40}{60}$. The second varietie.

Master. An other way is this. If it happen so that the lesser Denominator can by any multiplication make the greater, then note the multiplier, and by it multiplie the numerator ouer that lesser denominator, and for the lesser denominator put the greater, as thus in these two fractions, $\frac{3}{12}$ and $\frac{2}{3}$ three being the lesser denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4, I doe multiplie 2, which is the numerator ouer 3, and that maketh 8: vnder which I doe put 12 being the greater denominator, which is also made by multiplication of 4 into 3, and so haue I these 2 Fractions $\frac{3}{12}, \frac{7}{18}$: thus shortly reduced without altering the one fraction.

Scholer. This I vnderstand.

Master. Then marke this third way: If the denominators do not happen so, that one by multiplication may make the other, then looke whether they both may be partes of any other one number, as in $\frac{5}{12}$, and $\frac{8}{12}$, although the lesser taken but twise, be too great to make 18, yet they both may be parts vnto 36: therefore looke how many times twelue is in 36, and The third varietie.

that

that quotient beyng multiplied by the numerator ouer 12, the totall shall be put in stead of the numerator ouer 12, and for 12 put 36, thus, $\frac{15}{14}$. So likewise looke how often is 18 in 36, and because it is twise, therefore by 2 multiplie 7 which is ouer 18, and it will be 14, set that for the numerator, and in stead of 18 put 36, and then shall your fractions reduced, stand thus, $\frac{15}{36}$ $\frac{14}{36}$ in stead of $\frac{5}{12}$ and $\frac{7}{18}$.

And if you will proue whether you haue wrought well or no, that may be proued by Reduction of them againe to their former denominations, which Arte shall be taught in the fourth kinde of Reduction, where greater termes of fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction the same termes or numbers come againe that were before, then is the worke good, else not.

Scholer. Sir, I heare your wordes, but I do not vnderstand many of them, which it may please you to declare.

Master. With a good will, when conuenient place serueth, but that must be in the said fourth kinde of Reduction. In the meane season I will declare the second forme of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and so to one denomination.

When fractions of fractions be proponed, you shall multiplie the numerators of each

into

into other, and set the totall for the new numerator, and then multiplie all the denominators likewise, and take their totall for the new denominator, and so are they speedily reduced. Reduction of fractions into one denomination.

Scholer. If that be all then I understand it already, as by this example I will declare. These be the fractions, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$, which I would reduce to one denomination.

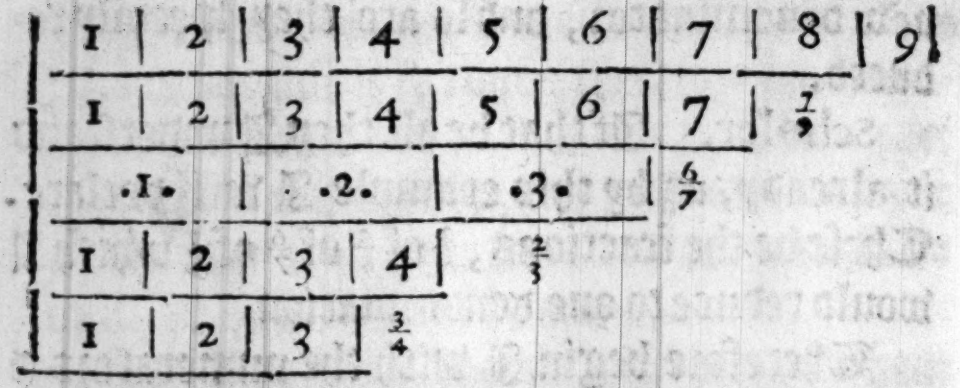
Therefore begin I with the numerators & multiplie them altogether, saying: 3 into 2 maketh 6, and 6 by 6 maketh 36, which multiplied by 7, yeeldeth 252, that I set ouer a line for the numerator, thus. $\frac{252}{}$

Then I multiplie the denominators, 4 by 3 maketh 12, and that by 7 bringeth 84, which multiplied by 9, yeeldeth 756, the new denominator. And so the whole reduced fraction is this, which is too hard a fraction for me to vnderstand yet. $\frac{252}{756}$

Master. You thinke so, and no maruell, but anon you shall learne to iudge it easily, for this fraction is no more in deede than $\frac{1}{3}$, although it be in greater termes, and therefore more stranger and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due vnderstanding of the Reduction.

The entire measure parted into 9



Here you see the longest measure, (which standeth for the whole and entire quantitie) first parted into Diuisions 9, whereof 7 are seuered by the second measure: and thereof againe are parted out 6. And that 6 being distinct into 3 parts, 2 of them are parted by the fourth measure, of which fourth measure, being diuided into 4 partes, the lowest measure doth containe $\frac{3}{4}$, so that the same $\frac{3}{4}$ must bee named, not $\frac{3}{4}$ of the whole measure, but in deede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

$\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$

Scholer. This example is so sensible, that I cannot choose but see it. And furthermore, I see also that the same fraction is equal to $\frac{4}{9}$ of the entire measure, as the lines which run vp and downe doe expressely set forth. Also I see here, that $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ is equall to $\frac{4}{9}$. And further yet that $\frac{2}{7}$ of $\frac{7}{9}$ is equall to $\frac{4}{9}$ or $\frac{2}{3}$.

$\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$

Master, I am glad that you see it so well, not doubting but you will gather greater light of knowledge hereby.

But

But now it is time that wee come to the third forme of Reduction, which teacheth of improper fractions, that is to say, mixt numbers of vnites and fractions, although they appeare like fractions, as this $\frac{26}{5}$, which doth include 5 vnites wholly, and $\frac{1}{5}$ ouer. Wherefore first you shall know them, by that the numerator is greater than the denominator. The third forme of Reduction of improper fractions.

Scholer. Indeed sir, that appeareth reasonable, that if the numerator doe expresse more parts to be taken of any vnite than the denominator doth signifie that vnite to be diuided into, it must needes follow that such a fraction importeth more than the whole, that is to say, the whole with certaine partes ouer. But what Reduction is there in it?

Master. There be two seuerall kindes of Reduction, concerning such fractions. Sometimes it shall be needfull to conuert those fractions into the vnites and the proper fraction that will remaine. And sometimes contrariwise, it shall be meete to reduce mixt numbers, that is vnites, written with fractions into the forme of one simple fraction, and so be there two waies.

Scholer. What is the meane of the first way to turne improper fractions into vnites with their proper fractions?

Master. That is thus. Your numerator being greater than the denominator, must be diuided by the same denominator, and the

Reduction of improper fractiõs into vnits with their proper fractions.

quotient thereof expresseth the vnites: the remainer shall be put for the numerator of the fraction that resteth, and the denominator must be the same that was before.

Scholer. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient will be 3, and there will remaine 2.

Master. That must you write thus, $3\ \frac{2}{5}$ where (you see) I haue written 3, without any line, as entire numbers ought to be written, and the 2 that remained I haue set ouer the former denominator with a line, as a proper fraction. And this number doth signifie now 3 vnites, and $\frac{2}{5}$ of one.

Scholer. Then if I would by vnites here vnderstand crownes, so it were 3 crownes, and $\frac{2}{5}$, that is 2 ß.

Master. Euen so, and therfore $\frac{17}{5}$ did signifie the same. But this happeneth sometimes, that when the Reduction is so wrought, there remaineth nothing. And then it is not a mixt number, but a simple entire number, represented like a fraction.

Reduction of whole numbers, either alone, or ioyned with fractions into fractions.

Scholer. As $\frac{15}{5}$ will make 3 iust, and $\frac{12}{5}$ wil make euen 6. This I will remember. But now, what is the second forme of Reduction, that you spake of for these sortes of fractions?

Master. Whensoeuer you haue any of these two sortes of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turne

them into the forme of a fraction, you must multiplie the whole number by that denominator, which you will haue to remaine still, and to the totall thereof adde the numerator which you haue already, and all that shall you set for the new numerator, keeping still the former denominator: as if you haue $6\frac{3}{4}$ which you would conuert into an improper fraction, you must multiplie 6 by 4, whereof commeth 24, and thereto adde the numerator which is 3, and so haue you 27 for the numerator, and 4 still for the denominator.

Scholer. Then is $\frac{27}{4}$ equall to $6\frac{3}{4}$.

Master. Euen iust, and so backward (as appeareth by the former reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the others worke.

Scholer. This I perceiue, but now if you would turne whole numbers without fractions into any fraction, I see not how that may be done, because there is no denominator to make the multiplication by.

Master. That was well marked: but this you know, that no man entendeth to turne any whole number into a fraction, but he hath in his minde that denominator by which the multiplication must be made: for the proofe whereof I set downe 7, which is a whole number. And if you will haue this number conuerted into any certaine fraction, will me to do it. Note.

Scholer. I pray you reduce 7 into a fraction.

Master. Then you care not what the fraction be, so it be some fraction.

Scholer. No, I passe not for the sort of the fraction.

Master. Then how can you thinke that you require mee to doe anie thing certaine, when you leaue me to doe as I list? And seeing you stand at that stay, whether thinke you that I must first intend in minde what fraction I will make of it, before I can doit indeede?

Scholer. Else you should do ignorantly.

Master. Then wil I limit my selfe (seeing you will not) to turne it into quarters. And therfore I multiplie 7 by 4 (which is ỹ denominations of quarters) and there amounteth 28 to be set for the numerator, and the 4 must be set for the denominator, and the fraction will be thus, $\frac{28}{4}$.

Scholer. Indeed I perceiue this to be reasonable, for without much triall I vnderstand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into fift parts, it will make $\frac{40}{5}$ which is all one with 8, for 8 crownes turned into fift parts (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

Master. Seeing you vnderstand now these three kindes of Reduction, I will declare vnto you the fourth kinde, that is when fractions

be

bee written in greater termes tham they neede, how they may bee brought to lesser termes.

Scholer. To write any thing in greater termes than needeth, seemeth to be a fault, and so this rule seemeth to amend that fault. The fourth kinde of Reduction.

Master. It were a fault to doe any thing without neede which after must be redressed: but in this case it is not so: neyther did I say absolutely (as you do) that it needeth not to expresse those fractions in so great termes, but that the fractions do not neede, I meane for their value to be understanded: but yet it may be needfull for the ease of those workes, whereto they be applied, as for example: In the first kind of Reduction this was your owne example: $\frac{3}{16}$ and $\frac{4}{6}$, which when you would reduce, you were faine to turne them first into one denomination, and so appeared they thus, $\frac{18}{96}$, and $\frac{64}{96}$, where the fractions (for their owne understanding) needed not to be turned out of smaller termes into greater, but yet the easinesse of working needed it.

Scholer. Sir, I understand now, not onely the difference of this neede (for the fractions might better be understanded as fractions severall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I understand what you meane by greater termes and lesser termes, whereof before I was in doubt, for I see you Termes of fractions.

call the numerator & denominator, the termes of the fraction.

Maſter. I am glad you vnderſtand it ſo wel. Now then when you would value any fractions (becauſe that may beſt be done when the termes are ſmalleſt) you ſhall reduce them to the ſmalleſt that you can, which thing you may doe thus: Diuide the greateſt of any ſuch two termes by the leſſer, and if any thing remaine, by that remainer diuide the laſt diuiſor: and if any thing remaine now, by that diuide the laſt diuiſor (which was before the remainer of the firſt Diuiſion) and ſo continue ſtill, till nothing do remaine in the Diuiſion: and then marke your laſt diuiſor, for it is the number that will eaſily reduce your fraction, if you diuide both the numerator and the denominator by the ſame number, and put for the numerator the quotient of his Diuiſion, and for the denominator alſo his quotient, that riſeth by his Diuiſion.

Reduction of fractions into their ſmalleſt termes or leaſt value.

Scholer. I take for example $\frac{18}{96}$, and becauſe 96 is the greateſt number, I diuide it by 18, and the quotient is 5, and there reſteth 6: what ſhall I do with this quotient?

Maſter. Nothing in this worke, but now ſeeing there remaineth ſomewhat, by that remainer muſt you diuide the laſt diuiſor.

Scholer. If I ſhall diuide 18 (which was the laſt diuiſor) by 6, that was the remainer, ſo is the quotient 3, and nothing reſteth.

Maſter.

Master. As for the quotient I omit him yet: but because there doth remayne nothing, therefore is 6 (which was your last diuisor) that number by which you may reduce the fraction proponed.

Scholer. Then as you taught me, I must diuide the numerator 18 by 6, and the quotient is 3, which I must put for the numerator ouer a line, thus. $\frac{3}{\ }$ And then by the said 6, must I diuide also the denominator 96, and the quotient will be 16, which I must take for the denominator. and so is the fraction $\frac{3}{16}$. And so me thinketh this rule doth proue the worke of the first Reduction.

Master. That is true, if the first Reduction were made of fractiõs in their least termes, and else not, without some helpe, as the second number in that place will declare.

Scholer. The second number was $\frac{4}{7}$, which was turned into $\frac{64}{96}$ by that rule. Nowe if I shal by this rule reduce it againe into the least termes, I must diuide 96 by 64, and there remaineth nothing, wherefore I must take that 32 for the diuisor, to reduce the sayd fraction. Then doe I diuide 64 by 32, and the quotient is 2, which I set for my numerator. Againe, I diuide 96 by 32, and the quotient will be 3, and so haue I but $\frac{2}{3}$.

Master. Muse not at the matter, for you haue done well enough: but you thinke you haue

haue not the fraction that you looked for, that is $\frac{4}{6}$, yet haue you one equall to it, as by the parts of a shilling you may proue.

Scholer. Truth it is for each of them will bring forth 8 pence, so that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, bee all three equall. And now I perceiue that because $\frac{4}{6}$ was not written in the least termes that it might be, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this rule well. But is there any other way to worke this Reduction?

Another way for to worke Reduction.

Master. Yes, but first note this, that if you find no such diuisor to reduce the fraction till you come to 1, because one doth make no diuision, therefore that fraction is already in his least termes, as by $\frac{71}{100}$ you may proove, and so of $\frac{81}{98}$ and many other like.

Note that to mediate any thing is to diuide the same by 2.

¶ But now for your better aid to finde the due proportion in least termes, with more ease for a young learner, you shall mediate or take the halfe of the numerator, and also the denominator as long as you may vpon a line, alwaies parting them with a right downe dash of your pen as your worke, which may easily be done, if the numbers be euen: as 2, 4, 6, 8, or 10: But if they be odde (though it be but in one of them) then must you abbreuiate them by 3, 5, 7, or 9, &c.

And because examples doe most instruct, I haue here set downe the manner of 2 or 3 whose

whose last number at the end of the line, sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least tearme or value: whereupon I set forth $\frac{288}{576}$, with a long line drawne frō it thus $\frac{288}{576}|$— And because both the numerator and the denominator end in euen numbers, I see this may be abbreuiated by 2, or 4, or 6, &c. therefore on the other side of the right downe dashe toward the right hand, I first take the halfe of the numerator: saying the halfe of 2 is 1, the halfe of 8 is 4, and againe the halfe of 8 is 4, which 144 is now a new numerator, & therefore I part it with a right downe dash as before.

Then do I also take the halfe of 576 in saying the halfe of 5 is 2, and the halfe 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new denominator.

Then beginning againe: saying the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to the last worke as appeareth here in the example, where the same is reduced to $\frac{1}{2}$, which is equall to $\frac{288}{576}$.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2
		28	14	7	1		
		112	56	28	4		

Abbre-

Abbreuiated first by 5, then by 293.

1465	293	1
4395	879	3

Scholer. Sir I thanke you much, this is very easie, and good for a young learner. *

Master. So it is, but yet notwithstanding, if you can without that diuision by memorie espie the greatest number that may diuide exactly both termes of your fraction proponed, than neede you not to vse that diuision, as in this fraction $\frac{60}{96}$ I see that 12 is the greatest number that can diuide them both: and therefore without any worke, by memorie onely, I turne that into $\frac{5}{8}$, but this abilitie in knowledge is gotten by exercise.

Yet one other way of easie Reduction in this kinde there is, when your fraction hath any Ciphers in the first places of both termes, then may you by casting away the ciphers, make a briefe Reduction, as thus $\frac{300}{400}$, here take away the ciphers, and it wil be $\frac{3}{4}$, which is the same in value with $\frac{300}{400}$.

Scholer. And so if I haue $\frac{400}{500}$, it will be $\frac{4}{5}$.

Master. You are deceiued, for you take away more ciphers from the Numerator, than you do take from the denominator, which you may not do.

Scholer. I confesse my fault, which came of too much hast, I was more gladder of the rule

rule than wise in vsing it: but now I vnderstand it I trust. The fift kinde of Reduction.

Master. Then may I go in hand with the fift or last kinde of Reduction, which teacheth how to turne any fraction proponed into any other denomination that you list: or into any partes of common coynes, waighes, or measures, or such like.

For declaration whereof, first you shall marke whether your fraction be a simple fraction, eyther elle a fraction of sundrie partes, I meane of more termes than 2. And if your fraction bee a fraction of fractions, or otherwise compounde, you must reduce it to one simple fraction. And then marke well the denomination of that other fraction, into which you would turne this, for by that denominator you must multiply the numerator of your first fraction, and the totall product thereof shall you diuide by the denominator of your first fraction, and that quotient shall be the numerator to the denominator proponed: as for example, I haue this fractiõ $\frac{3}{5}$, which I would turne into ten partes, therefore I multiplie this 10 by 3, that is the numerator of my fraction, and there riseth 30, which I diuide by 5, and the quotient is 6, which must be the numerator to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

Scholer. This is easie inough to do.

Master. Then shall you see an other example of the same fraction that is not so easie:

as if I would turne $\frac{3}{5}$ into viii. partes, proue you that worke.

Scholer. I must multiplie 8 by 3, and there amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Master. And see you nothing doubtfull in this worke?

Scholer. I see that when 24 was diuided by 5, there remained 4, which I did not passe of, because ye speake nothing of any remainer, but onely of the quotient.

Master. By likelyhoode you remember what I said to you in Diuision of whole numbers, that you should not passe of the remainer there, but onely note it as a summe that could not be diuided without knowledge of fractions. Wherefore now marke this, that in all diuisions of whole numbers, when there is any remainer, you shall set it ouer a line as a numerator, and set the Diuisor for the denominator, and that fraction doth make the Diuision complete, and is part of the quotient: as if I would diuide 48 by 5, the quotiēt will be $9\frac{3}{5}$: so in your former worke whē 24 was diuided by 5, the quotient should be $4\frac{4}{5}$, and so the new fraction should be thus: $\frac{4}{8}$ and $\frac{4}{5}$ of 8 that is $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{1}{8}$ part of any thing, which you may proue by example of some coyne.

Scholer. Then I take a crowne, whole $\frac{3}{5}$ is 3 s̄. Now if I would proue whether that 3 s̄ be

$\frac{4}{8}$ and

$\frac{4}{8}$ of $\frac{4}{5}$ of $\frac{1}{8}$ I shal haue a combrous worke to do.

Master. In deed for whole pence, your example is a little troublesome: yet turning the crowne into halfe pence, it is easie inough.

Scholer. That will I trie.

¶ First I see that $\frac{1}{5}$ of a crowne is 3 s̄: which is 36 d̄, or 72 halfe pence. Now if I can find that this fraction $\frac{4}{8}$ and $\frac{4}{5}$ of $\frac{1}{8}$ be equal vnto 3 s̄: then am I fully answered.

Because I cannot take $\frac{4}{8}$ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120, which I diuide by 8, my quotient is 15, which taken 4 times, makes 60 ob. Now resteth me to haue $\frac{4}{5}$ of the $\frac{1}{8}$ part of a crowne, wherof $\frac{1}{8}$ part is 15 ob. that 15 being parted into 5 parts, the quotient is 3, which takē 4 times maketh 12 ob. which with my 60 before amounteth to 72, which are then equall to $\frac{4}{5}$ my desire.

Master. I cōmend you for your diligence, you might haue wrought it thus either: $\frac{4}{8}$ being abbreuied as before I taught is $\frac{1}{2}$. Now the halfe of a crowne is 2 s̄, 6 d̄. Then $\frac{4}{5}$ of $\frac{1}{8}$ is a fraction of fractions, which if you doe reduce into one entire fraction as before you haue learned, in saying 5 times 8 is 40, for a new denominator and ouce 4 is 4, for a new numerator, it maketh $\frac{4}{40}$, and abbreuiated also make but $\frac{1}{10}$. Now the tenth part of a crowne is 6 d̄, which put to 2 s̄, 6 d̄ make also 3 s̄ your desire. *

But

But now one example more for this rule, & then shall we end it. If I haue $\frac{7}{15}$ of a Soueraigne (accounting the Soueraigne 20 shillings) how many shillings is that $\frac{7}{15}$?

Scholer. I must multiply 7 by 20, and that maketh 140, which I shal diuide by 15, and the quotient will be $9\ \frac{5}{15}$: or els in lesser termes, $\frac{1}{3}$.

Master. That is 9 s, and one third part of a shilling, that is 4 d, as by the same rule you may proue. And this for this time shall suffice for Reduction, saue that I must now repeate a little touching Multiplication and Diuision, and so go forward.

Multiplication.

IN Multiplication it happeneth sometime that there be whole numbers to be multiplied with fractions: And may be in two sortes, for either the whole number is seuerall from the fraction, and is the multiplier, or else, the whole number is ioyned with one, or both of the Fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no Reduction, but onely multiply the numerator of the fraction by that whole number, and the totall thereof set for the new numerator.

Reduction of whole numbers into fractions.

Scholer.

Scholer. I vnderstand you thus. If I haue $\frac{6}{23}$ to be multiplied by 16, then must I multiplie that 16 with 6. which is the Numerator, whereof commeth 96, and that must I set for the new Numerator, keeping still 23 for the Denominator, and so the Fraction will be $\frac{96}{23}$ that is $4\frac{4}{23}$.

Master. And in this sorte of worke you may abridge the labour, thus. If it happen the Denominator to be such a number, as may euenly be diuided by the said whole number proposed, then diuide it thereby, and set the quotient of that diuision for the former denominator: but reserue still the Numerator, and so is the Multiplication ended.

Scholer. Then I faine this example, $\frac{7}{20}$ to be multiplied by 5. And because 5 will iustly diuide 20, therefore I take the quotient of that diuision, which is 4, and set in stead of 20, and so the Fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$, that would haue followed of ỹ other sort of worke.

Scholer. I perceiue it very well.

How to multiplie mixt numbers.

Master. Now then for the other sort where the number is mixt, take this way: first to reduce the said whole number, and fraction into one fraction improper (as I shewed you in Reduction) and then multiplie them together, as if they were proper fractions.

Scholer. $13\frac{3}{5}$ beyng set to be multiplied by $\frac{5}{8}$, first I must reduce the mixt number, as

$13\frac{3}{5}$ / $\frac{68}{5}$

appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I must adde the Numerator 3, and so the fraction wil be $\frac{68}{5}$, which two fractions now I shall multiply after the accustomed forme, and it will be $\frac{340}{40}$.

Master. You haue done well: and so may you see, that although most part of the formes of Multiplication may be wrought without Reduction, yet some cānot, as namly mixed numbers

And yet one note more will I tell you of Multiplication, before we leaue it: That is, whensoeuer you would multiplie any Fraction by 2, which commonly is called Duplation, you may do it not onely by doubling the Numerator, but also by parting the Denominator into halfe, if it be euen.

Scholer. Then if I would double $\frac{5}{12}$, I may choose whether I wil make it, $\frac{10}{12}$, or els $\frac{5}{6}$. And in deede I see that all is one, but that the diuiding of the Denominator seemeth the better way to make smaller termes of the Fraction, and so they shall neede the lesse reduction.

Master. It is so: and now I shall not need to tell you that Multiplication is proued by Diuision, and Diuision likewise by Multiplication, but the like workes that I shewed you in Multiplication, will I shew you in Diuision also.

Diuision.

Diuision.

When any whole nũber shall be diuided by a fractiõ, you must multiply ẏ said whole number with the Denominator of the fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction. Diuision to diuide a whole nũber by a Fraction.

Scholer. Then 20 diuided by $\frac{3}{4}$ will make $\frac{80}{3}$, as here appeareth: $\frac{20}{1}$ by $\frac{3}{4}$ $\frac{80}{3}$

Master. Euen so. But if you would diuide the Fraction by the whole number, then multiplie the Denominator by the same whole number, & set the totall for the Denominator, without changing the Numerator. To diuide a Fraction by a whole number.

Scholer. Then to diuide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$. As here appeareth in this example. $\frac{20}{23}$ by $\frac{4}{1}$ $\frac{20}{92}$.

Master. You say well. And by the same example you giue me occasion to remember an other briefe way to doe the same: for if you had diuided the said Numerator by 4, & set the quotient for the Numerator, keeping still the old denominator, it would haue bene not onely as well done, but also in a fractiõ of lesser termes. Another briefe way.

Scholer. I gesse it to be euen so, by a like worke that you taught me in Multiplication. And for proofe thereof $\frac{20}{23}$ being the diuidend, & 4 the Diuisor, I diuide the Numerator 20

by 4, and the quotient is 5, which I set for 20 ouer 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by diuiding or abreuying both these termes by 4, and so reducing them to their least Denomination, I may easily proue: as appeareth in this example: $\frac{20}{92}|\frac{5}{23}$

Master. You conceiue it well. And if there be mixt numbers (either one or both) you must first reduce that mixt number into an improper fraction. And then worke as you haue learned.

Scholer. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

Master. Then take this note yet for diuision. If the Denominators be like, then diuide the Numerators as if it were in whole numbers, and the quotient, whether it be fraction, whole number, or mixt, is a good quotient for that Diuision. And generally if one of the Numerators may iustly diuide the other, by that quotient multiplie the Denominator of the lesser Numerator, and set it that doth amount in the roome of the same Denominator, and then for a Numerator to it, set the denominator of the other fraction.

Scholer. Then if I would diuide $\frac{3}{4}$ by $\frac{12}{17}$ I see that 3 will diuide 12, and the quotient will be 4, by which I must multiplie the other 4 that is the Denominator vnder 3, and then it is 16, which I set for the Denominator 4, and

ouer

ouer it in steed of the 3, I must set 17, the other Denominator, and so is it thus, $\frac{1}{17}\frac{7}{6}$.

Master. And so is $\frac{1}{17}\frac{7}{6}$ in steede of $\frac{5}{4}\frac{1}{8}$, which would haue risen by the common $\frac{3}{4}$ by $\frac{2}{17}$ worke: as here appeareth: $\frac{5}{4}\frac{1}{8}$

And now for Mediation (which is to diuide by 2) marke this: If the numerator be euen, set the halfe of it in his place without the Diuisor, & so haue you done: & if the Numerator be not euen, then double the Denominator.

Scholer. That is if I would mediate $\frac{6}{11}$, I may make the quotient $\frac{3}{11}$. And if I would mediate $\frac{7}{11}$, I must make it $\frac{7}{22}$.

Master. Now trust I that you haue sufficient knowledge in Reduction, Multiplication, and Diuision: and therefore will I go in hand with Addition and Subtraction, which now will appeare easie inough.

Addition.

WHensoeuer you haue any fractions to be added, you must consider whether they be of one Denomination or not. And if they be of one denomination, then adde the Numerators together, and set that that amounteth, for the Numerator ouer the common Denominator, and so you haue done. The reason is, because that such differ little in Addition or

To adde Fractions of one denomination.

Subtraction from the worke of vulgare denominations, where the Denominators be of no numbers: as 3 d & 5 d make 8 d, where the denomination is not altered. But and if the fractions be not of one denomination, or any of thē be mixt of whole numbers and fractions, then must you first reduce them to one denomination, and after adde them. And if they be many, then adde first two of them, & so the summe that doth amount of the Addition, ad the third, and then the fourth, & so forth, if you haue so many.

To ad fractions of diuers denominations.

Scholer. This seemeth easie inough, now that I haue already learned to multiplie & to reduce, without which two, I could neuer haue wrought this. And therefore now I see good reason, why you did place Multiplication and Reduction before Addition.

M. It is well cōsidered, but yet refuse not to expresse your vnderstāding of it, by an exāple.

Scholer. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$, and because the Denominators are like (and so needeth no Reduction) I adde 7 to 5: which maketh 12, & then is my summe $\frac{12}{18}$, that is in smaller numbers being abbreuiated $\frac{2}{3}$.

And if I haue many numbers to be added, as here $\frac{3}{8}$ $\frac{4}{5}$ $\frac{2}{10}$ (first I must reduce them, because they haue diuerse Denominators) into one denomination, and then will they be thus: $\frac{150}{400}$ $\frac{320}{400}$ $\frac{360}{400}$ or in lesse termes $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, which by Addition do make $\frac{83}{40}$ that is $2\frac{3}{40}$

Master. Now may we go to Subtraction.

Sub-

Subtraction.



Subtraction hath ỹ ſame preceptes that Additiõ had, for if the Denominators be like, thẽ muſt you ſubtract the one Numerator from the other, & the reſt is to be ſet ouer the cõmon denominator, & ſo your Subtraction is ended: but & if you haue many fractions to be ſubtracted out of many, then muſt you reduce them to one denomination, & into two ſeuerall fractions, that is all that muſt be ſubtracted into one fraction, and the reſidue into an other Fraction, and then worke as I ſaid before.

Scholer. For the firſt exãple I take $\frac{1}{12}$ to be ſubtracted out of $\frac{7}{12}$, & the reſt will be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other exãple I take $\frac{3}{4}$ to be ſubtracted out of $\frac{7}{8}$, which I muſt reduce, and it will be thus, $\frac{24}{32}$ and $\frac{28}{32}$.

Then do I ſubtract 24 out of 28, & there reſteth 4, which I ſet ouer the common denominator for a Remainer, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the third exãple, I take $\frac{1}{4}$ & $\frac{5}{6}$ to be ſubtracted from $\frac{7}{8}$ & $\frac{9}{10}$. And becauſe their denominators be diuers, I do reduce thẽ into one denomination thus: $\frac{1440}{1920}$ $\frac{1600}{1920}$ $\frac{1680}{1920}$ $\frac{1728}{1920}$.

Then do I adde the two first, & they make $\frac{3040}{1920}$. Also I adde the two last, and they yeeld $\frac{3408}{1620}$. Then do I subtract 3040 out of 3408, & there resteth 368, so is the remainer $\frac{368}{1920}$ that is in smaller Termes, $\frac{23}{120}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Master. Proue one example more of two Fractions of diuers denominations.

Scholer. I take these two Fractions $\frac{7}{8}$ to be subtracted frō $\frac{9}{24}$, which being reduced, will stand thus, $\frac{168}{192}$ and $\frac{72}{192}$. Now would I subtract 168 out of 72, but I cannot.

Master. Then may you perceiue that you mistooke the Fractions: for you can neuer subtract the greater out of the lesser, although you may adde, multiplie or diuide the greater with the lesser. And albeit that $\frac{7}{8}$ hath both his tearmes lesser than $\frac{9}{24}$, yet is $\frac{9}{24}$ the lesser Fraction: for generally if you multiplie the Numerators and denominators of two fractions crossewaies, that fraction is the greatest, of whose numerator commeth the greatest summe, as in this example: 7 multiplied by 24, maketh 168: and 9 being multiplied by 8, yeeldeth but 72 therefore is the first Fraction $\frac{7}{8}$ the greatest of these two, so can you not subtract it out of a lesser Fraction.

The greatest of two fractions.

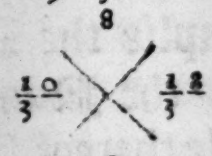
But if you should subtract a Fraction out of a whole number what would you do?

Scholer. Marrie I would reduce the whole number

number into a fraction of the same Denomination that my Fraction is, and then worke by Subtraction.

Master. So may you do, but it is easier much, if your fraction be a proper fraction, that is to say, lesse then an Unite, to take an Unite from the whole number, and then turne it into an Improper Fraction, and so worke your subtraction. As if I would subtract $\frac{3}{5}$ from 4, I may take one from 4, and turne it into $\frac{5}{5}$, from which if I abate $\frac{3}{5}$ there wil remaine $3\frac{2}{5}$. And if the first Fraction be an Improper fraction, then may I take so many Unites from the whole number, that they may make an improper fraction greater than that first, and then worke by Subtraction: As if there be proponed $\frac{10}{3}$ to be subtracted from 6, because $\frac{10}{3}$ is more than 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{3}$ then abate $\frac{10}{3}$, and there resteth $\frac{2}{3}$, so the whole remainer is $2\frac{2}{3}$. Or else you may at your pleasure take $3\frac{1}{3}$ which is $\frac{10}{3}$ from 6. whole: Then set one vnder 6. as thus $\frac{6}{1}$: And then to reduce those 2 fractions into one Denomination as here appeareth: $\frac{10}{3}$ from $\frac{6}{1}$.

Then $\frac{10}{3}$ from $\frac{18}{3}$ resteth $\frac{8}{3}$: which maketh $2\frac{2}{3}$ your desire:



And thus will I make an end of the workes of common fractions for this time, not doubting, but you can apply them both vnto the rule

rule of Progression, and also vnto the Golden rule, without any other teaching than you haue learned before, which might seeme tedious to repeat, saue that in some speciall diuersities, which be peculiar to fractions, I cannot ouerpasse, but instruct you somewhat by the way.

The Golden Rule.



Herfore as touching the Golden Rule for the placing of the 3 numbers proponed in the question, whereby to finde the third, and for the forme of their worke, with other like notes, I referre you to that which you haue already learned.

But this easie forme of working by fractions shall you note, that if your three numbers be fractions, for an apt worke & certaine, multiply the Numerator of the first number in the question, by the Denominator of the second: And all that againe multiplie by the denominator of the third number, and the totall thereof shall you keepe for to be the Diuisor. Then multiplie the Denominator of the first number by the Numerator of the second, and the whole thereof by the Numerator of the third, and the totall thereof shall be your diuidend. Now diuide this diuidend by the Diuisor which you found out before, and that number

shall

shall be the fourth number of the question, which you seeke for: As in this exãple. If $\frac{3}{4}$ of a yard of veluet cost $\frac{2}{3}$ of a Soueraigne (esteemed at 20 shillings,) what shall $\frac{5}{6}$ cost? A question of Veluet.

Scholer. If it please you to let me make the answere, I would first place these three numbers, as I learned in whole numbers thus: $\frac{3}{4}$ Z $\frac{2}{3}$ / $\frac{5}{6}$ L

And then according to your new rule, I must multiple 3 being Numerator in the first number, by 3 the Denominator of the second, and thereof commeth 9, which I multiplie agayne by 6, the Denominator of the thirde number, and so haue I 54, which I keepe for the diuisor, then multiplie I 4, the denominator of the first, by 2, the Numerator of the second, and there riseth 8, which againe I multiplie by 5 the Numerator of the third, and it maketh 40: then must I diuide 40, by 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in lesser termes, and then the figure $\frac{3}{4}$ Z $\frac{2}{3}$ / $\frac{5}{6}$ L $\frac{20}{27}$ will stand thus:

But what that is in money, I can not tell except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done here to mingle diuerse workes togither, where we do not seeke the value of ỹ thing in cõmon money, but in an apt number, which you haue well done. And therefore will I yet shew you

you an other like way of easinesse in worke, how you may change your 3 Fractions into 3 whole numbers, by which you shall worke as if the question were proponed in whole numbers. The first number you shall finde as I taught you: now to finde the diuisor of the second number, take the Numerator for the second fraction: and for the third number take that that riseth of Multiplication of the Denominator of the first, by the Numerator of the third, and then worke your question.

A question of siluer.

Scholer. For example hereof, I put this question, If $\frac{1}{1}\frac{1}{2}$ of 1 ℔ waight of Siluer, be worth $\frac{1}{4}\frac{2}{}$ of a Soueraigne, what is $\frac{1}{2}$ of 1 ℔ weight worth? For ỹ answere, first I place the Fractions in order thus:

$\frac{1}{1}\frac{1}{2}$ — $\frac{1}{4}\frac{2}{}$

$\frac{1}{2}$ —

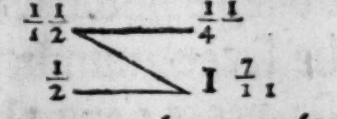
Then to turne these Fractions into whole numbers, I multiply 11 which is the Numerator of the first, by 4 (the Denominator of the second) and there commeth 44, which I multiplie by 2 the Denominator of the third, and so amounteth 88, which I set for the Diuisor, in the first place. Then in the second place I set 12, which is the Numerator in the second Fraction, and in the third place I set the summe that amounteth of 12, beyng the Denominator in the first number, multiplyed by 1, beyng Numerator in the thirde number, and so the figure will stand as here you see.

88 — 12

12 —

Then

Then to worke it forth, I multiplie 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient will be $\frac{56}{88}$, or in lesser termes, $1\ \frac{7}{11}$, and then the figure will stand thus.



Master. These two formes now you vnderstand well inough. And as for any other, at this time I will not repeate, onely this shall you marke for the proofe of this rule, whether your worke be well wrought or no. Multiply the first number by the fourth, and note what amounteth: then multiplie the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equall, then is your worke well doone, else you haue erred. And this shall suffise for the former rule: but in the backer rule, this shall you note for ease of worke, that you multiplie the Numerator of the first by the Numerator of the second, and the whole thereof by the Denominator of the thirde, and that amounteth thereof, shalbe the Diuidend. Then multiplie the Denominator of the first by the Denominator of the second, and that whole by the Numerator of the third, and that riseth thereof shall be the Diuisor. Example of this: I did lend my friend $\frac{1}{4}$ of a Portegnise seuen monethes, vpon promise that he should doe as much for me againe: and when I should borrow of him, he could lend me but $\frac{1}{12}$ of a Portegnise, now I demaund how long time must

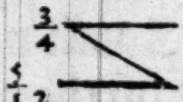
The proofe of the golden rule.

The Backer rule.

A question of loane.

I keepe his money, in iuſt recompence of my loane, accompting 13 moneths in the yeare.

Scholer. The firſt number muſt be the firſt money borowed, that is $\frac{3}{4}$ of the Portegueſe: the ſecond number the 7 monethes, that is $\frac{7}{13}$ of a yeare: and the thirde number the money that was lent in recompence, that is $\frac{5}{12}$ of a Portegueſe: then I ſet the number thus:



Then (as you taught mee) I multiplie three (being Numerator in the firſt number) by 7 the Numerator of the ſecond number, & it maketh 21, which I multiply by 12, the denominator of the third, & ſo haue I 252 for the deuidend: then I multiply 4 the denominator of the firſt, by 13 the Denominator of the ſecond, and it yeeldeth 52 which I multiply againe by 5, the Numerator of the third, and it will make 260, that is the diuiſor. Then muſt I diuide 252, by 260, ſo it wil be in the ſmaleſt fraction, $\frac{63}{65}$ of a yeare.

Statute of Aſſiſe of bread and Ale.

Maſter. And thus do you ſee ſome eaſe in working, better than to multiplie and diuide tediouſly ſo many Fractions. Another queſtion yet will I propoſe, to the intent you may ſee thereby the reaſon of the Statute of aſſiſe of bread and Ale, which in all Statute bookes in French, Latine and Engliſh, is much corrupted for want of knowledge in this Art: for the right vnderſtanding whereof I propone this queſtion.

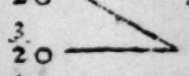
When

When the price of a quarter of Wheat is 2 s̄, the farthing white loafe shall waigh 68 s̄, thē I demaund, what shall such a loafe waigh, when a quarter of wheate is sold for 3 s̄ ? Question.

Scholer. The Question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You seeme to say resonably, how be it, in that statute of Assise, the rate is made by the proportion of partes in a pound waight Troie, else could it not be a Statute of any long cōtinuance, seing the shillings do change often, as all other moneys do: but this Statute being well vnderstanded, is a continuall rule for euer, as I will anon declare by a new table of Assise, converting the shillings into ounces and parts of ounces.

Therefore here by a shilling you must vnderstand $\frac{1}{20}$ of a pound waight, and so by pence $\frac{1}{20}$ of an ounce, wherefore although ye might worke this question proponed by whole number well inough, for that time when the Statute was made, yet to apply it to our time, & to make it serue for all times generally, it is best to worke it by fractions, setting for 2 shilling $\frac{2}{20}$: & for 68 shillings, $\frac{68}{20}$: & so for three shillings, $\frac{3}{20}$, & then will the figure of the question stand thus:



In which question because all the denominators be like, you shall worke onely with the Numerators,

Scho-

Scholer. Then I shall multiplie 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be 45 $\frac{1}{3}$: but how shall I make a Fraction of that to stand with the other?

Master. Haue you so soone forgotten what was taught you so lately? This is his forme.

Scholer. I remember it now and then it signifieth 45 twenty parts, and the third deale of one twenty part.

$$\frac{45\frac{1}{3}}{20}$$

Master. So is it, and that maketh in shillings, 45 s̄ 4 d̄: whereby you may note one great errour in the Statute bookes, which haue constantly 48 s̄ in that Assise. And by this rule, if you examine the Statute, you shall finde many summes false, wherefore for the true vnderstanding of that Statute and such like as I haue made mention of it, and somewhat recognised it, so do I wish that all Gentlemen and other studēts of the Lawes, would not neglect this Arte of Arithmetike, as vnneedfull to their studies. Wherefore, to encourage them thereto, and to gratifie both them & all other in generall, I will exhibite a Table of that part of the Statute in two columnes, & in a third columne I will adde the correction of those errours which haue crept into it.

Here followeth the Table.

The

The price of a quarter of wheate.		The waight of a farthing white loafe by the ſtatute bookes.			The correction by iuſt Aſſiſe.		
s	d	l	s	d	l	s	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 2/5	2	14	4 2/5
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10 2/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 2/3
5	0	1	8	2 1/2	1	7	2 2/5
5	6	1	4	8 1/4	1	4	8 8/11
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 3/5
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/3
9	6	0	14	4 1/4	0	14	3 5/9
10	0	0	13	7 1/2	0	13	7 1/5
10	6	0	12	11 1/4	0	12	11 3/7
11	0	0	12	4 1/4	0	12	4 4/11
11	6	0	11	10	0	11	9 2/23
12	0	0	11	4	0	11	4

In the cõmõ bookes there is no further rate of Assise made, than vnto 12 s the quarter of wheate: but in an auncient copy of 200 yeares old (which I haue) there is added the rate of assise vnto 20 s the quarter, but yet was that assise also either wrong cast at ẏ first penning, or els corrupt sith that time, for lacke of iust knowledge in the rule of proportion, which I will adde here also, to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of wheate.		The waight of a farthing white loafe by the Statute bookes.		The correction of the errours.	
s	d	s	d	s	d
12	6	11	0	10	10 $\frac{14}{15}$
13	0	15	0 $\frac{1}{2}$	10	5 $\frac{7}{23}$
13	6	10	1 $\frac{1}{2}$	10	0 $\frac{8}{9}$
14	0	9	7	9	8 $\frac{4}{7}$
14	6	9	2 $\frac{1}{2}$	9	4 $\frac{16}{29}$
15	0	9	1 $\frac{1}{2}$	9	0 $\frac{4}{5}$
15	6	9	1 $\frac{1}{4}$	8	9 $\frac{9}{31}$
16	0	9	0	8	6 0
16	6	8	6	8	2 $\frac{10}{13}$
17	0	8	3	8	0 0
17	6	7	10	7	10 $\frac{1}{7}$
18	0	7	6	7	6 $\frac{2}{5}$
18	6	7	3	7	4 $\frac{8}{37}$
19	0	7	2	7	1 $\frac{14}{19}$
19	6	5	10	6	11 $\frac{2}{13}$
20	0	5	6	6	9

These two tables I haue set seuerall, because no man should think that I would either adde or take away from any law those partes which might of right seeme either superfluous, eyther diminute, but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but euerie good subiects dutie with sobrietie to correct. And for auoyding of offence I haue rather done it in this priuate booke, rather than in any booke of the Statutes selfe, trusting that all men will take it in good part.

Scholer. I would wish so, but I dare not hope so, sith neuer good mā that would reforme errour, could escape the venemous tongues of enuious detractors, which because they either cannot or list not to do any good themselues, do delight to barke at the doinges of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answeres, but I will not seeme so hastie to make the answeres before I heare their obiections, but as I trust that men are of a better nature, and more gratefull now than some haue beene in time passed, as I haue done in the Statute of Assise for bread in rate of s̄, so wil I set forth the like table in pounds and ounces, and the parts thereof, that it may be easily applied to all times: but I meane not

by this to alter any word of the statute (being so good an ordinaunce, and of so great continuaunce) but onely to make it as a kind of exposition and declaration of the said Statute, trusting that thereby the Statute may be better vnderstood, and consequently better put in execution. And here you shall note, that I haue accounted the shillings after the rate of lx. s̄, to the pound waight, because I esteeme it the most apt rate for our time. Wherefore if in the first columne you finde the price of wheate directly against it in the second columne, you may finde the waight of the farthing white loafe, in this our time: and if you double the number (as I haue done in the third columne) then haue you the waight of the halfe penie white loafe, and so in the fourth columne is set the waight of the penie white loafe. It needeth not to tell you that, that the sight doth testifie, how that euery columne is parted into 3 smaller pillers, whereof the first columne hath these 3 titles, pounds, shillings, and pence: the other 3 columnes haue each of them these 3 titles, pounds, ounces, and penie waights. And as in the first columne xii d̄, make a s̄, and 20 s̄ make a pound, so in the other 3 columnes 20 pence waight maketh an ounce, and 12 ounces do make a pound.

A pound waight.

Scholer.

Scholer. Sir, I do thanke you most hartily for this, not onely in mine owne name and in the name of all Students, but also in the name of the whole Commons, to whom the restitution of this Assise (I trust) shall bring restitution of the weight in bread, which long time hath beene abused. And if you know any like thinges more, wherein you would vouchsafe to declare the errours, and set forth the truth, you cannot but obtaine great thanks of all good hearted men that loue the common wealth.

Master. I haue sundrie things to declare, but I haue reserued them for a priuate booke by it selfe, yet notwithstanding because the Statute of the rate of measuring of ground is so common that it toucheth all men, and yet no more common than needfull, but so much corrupt, that it is too farre out of all good rate, not onely in the English bookes of Statutes commonly printed, but also in the Latine bookes, and in the French also, for I haue read of each sorte, and conferred them diligently. I will giue you a Table for the restitution of those errours, as may suffice for this present time. And first I will propose one question to you touching the vse of that Statute, whereby you may perceiue the order how to examine the whole statute, and euerie parcell thereof, & the question is this.

A question of measure of ground.

When the Acre of ground doth containe

foure perches in breadth, then must it containe 40 perches in length: then doe I demaund of you, how much shall the length of an Acre be, when there is in the breadth of it 13 perches: but before you shall answere to this question, I will declare vnto you an other Statute, which is the ground of the former Statute. And that Statute is this. It is ordained that 3 Barlie cornes, drie and round, shall make vp the measure of an inch: 12 inches shall make a foote, and 3 foote shall make a yard (the common English bookes haue an elne) fiue yards and a halfe shall make a perch, and 40 perches in length, and 4 in breadth, shall make an Acre. This is that Statute: whereby you may perceiue, that the intent of the Statute is, that one Acre should containe 160 square perches. Now let me heare your answeare to the question.

A statute of measure.

An Acre.

Scholer. As I perceiue by the wordes of that Statute, a perch to be $\frac{1}{160}$ of an Acre, so could I make those numbers all in fractions, and so worke the question: but seeing I may do it also in whole numbers I take that forme for the most easie, therefore thus I set the question in forme. Then do I multiplie 40 by 4, and it maketh 160, which I diuide by 13, & the quotient is $12\frac{4}{13}$.

4 ⟍ 40
13 ⟍

Master. Now turne that $\frac{4}{13}$ into the common parts of a perch, as they be named in

the

the former statute: howbeit, it shall be best to take one of the least parts in Denomination for auoyding of much labour, as feete, wherof the perch containeth 16½.

Scholer. Then to turne ⅘ into feet, I multiplie 16½ by 4, and it maketh 66, which I must deuide by 13, and the quotient is 5 ⅓.

Master. So I find that if the acre hold in breadth xiii. perches, it shall containe in lenght 12 perches, 5 foote and ⅓ of a foote, which is not fully an ynch, for the ynch is ½ of a foote. But here all the Statute bookes in Latine and English (that I haue seene) do note it to be 13 perches, 5 foote and 1 ynch: which maketh aboue 13 perches too many in the acre, so that I would haue thought the error to haue crept into the printed bookes by the great negligence that Printers in our time doe vse, saue that in written Copies of great antiquitie, I doe finde the same. Yet haue I one French Copie, which hath 12 perches ¼, and one foote, and that misseth verie little of the truth. Note this error.

Scholer. Then I see it is true that I haue often heard say, that the truest copies of the statutes be the French copies.

Master. That is often true, but not generally, as I haue by conference tried diuersly: but in this statute the French booke is most corrupt in all other places lightly.

But now to performe my promise, I will

set foorth the Table for measuring of an Acre of ground onely by such partes as the statute doth mention, because at this time I doe of purpose write it for the better vnderstanding of the statute, and hereafter with other things I intend to set foorth this same more at large.

In this Table following, I haue not done as in the other statute before compared by restitution with the faultes crept into the statute, but onely haue written that true measure, which the equitie of the statute doth pretend. For it were vile to iudge of so noble Princes and worthy Counsellers, as haue authorised and set foorth this statute, that they would make one Acre in any forme greater than an other, but euerie one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needfull Arithmetike is to the students of the Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath in it no obscuritie, that I should need to declare.

The

The breadth.		The length of the acre.		
Perches.	Perches.	Feete.	Ynches.	Partes of an ynch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	$\frac{12}{13}$
14	11	7	0	$\frac{6}{7}$
15	10	11	0	0
16	10	0	0	0
17	9	6	9	$\frac{9}{17}$
18	8	14	8	0
19	8	6	11	$\frac{7}{19}$
20	8	0	0	0
21	7	10	2	$\frac{4}{7}$
22	7	4	6	0
23	6	15	9	$\frac{9}{23}$
24	6	11	0	0
25	6	6	7	$\frac{1}{5}$
26	6	2	6	$\frac{6}{13}$
27	5	15	3	$\frac{1}{3}$

The breadth.	The length of the acre.			
Perches.	Perches.	Feete.	Ynches.	Partes of an ynch.
28	5	11	9	3/7
29	5	8	6	12/29
30	5	5	6	0
31	5	2	7	29/31
32	5	0	0	0
33	4	14	0	0
34	4	11	7	13/17
35	4	9	5	1/7
36	4	7	4	0
37	4	5	4	8/37
38	4	3	5	13/19
39	4	1	8	4/13
40	4	0	0	0
41	3	14	10	28/41
42	3	13	4	2/7
43	3	11	10	24/43
44	3	10	6	0
45	3	9	2	0

Scholer. In deede Sir, I vnderstande the Table (as I thinke) by those other which you set forth before. For in the first Columne is set the perches of the breadth of any Acre, and then in the 2 Columnes following appeareth how many perches & how many foote the same Acre must haue for his length.

Master. You take it well: howbeit to speake exactly of breadth and length, the first Columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limite his length, and the shorter side doth betoken the breadth, yet it is no greater abuse in such tables, where a man cannot well change the title, to let the name remaine, although the proportions of the numbers doe change: for still by the first Columne, is expressed the measure of the one side, and by the two other pillers in one Columne, is set foorth the measure of the other side. And this shalbe sufficient now for the vse of the Golden Rule.

The rule of Felowship.

NOw somewhat wil I touch certaine other rules, which for their seueral names may seeme diuers rules & distinct from this, but in deede they are but braches of it: yet because

cause they haue not onely seuerall workings in appearance, but also pleasant in vse, I will giue you a taste of each of them. As for the rule of Felowship, both single and double, with time and without time, I shall neede to say little more than I haue alreadie saide in teaching the workes of whole numbers, yet an example or two will wee haue to refresh the remembrance of the same, and to declare certaine proper vses and applications of it, as this for one.

A question of vnequall societie.

Foure men got a bootie or prise in time of warre, the prise is in value of money 8190 l, and because the men be not of like degree, therefore their shares may not be equall, but the chiefest person will haue of the bootie the third part, and the tenth part ouer: the second will haue a quarter and the tenth parte ouer: the third will haue the sixt part: and so there is left for the fourth man a verie small portion but such is his lot (whether he be pleased or woth) he must be content with one 11. part of the pray. Now I demaund of you, what shall euery man haue to his share?

Scholer. You must be faine to answere to your owne question, else is it not like to be answered at this time.

Master. The forme to vnderstand the solution of this question, and all such like, is thus: Reduce all the denominators into one number by Multiplication, except that any of them

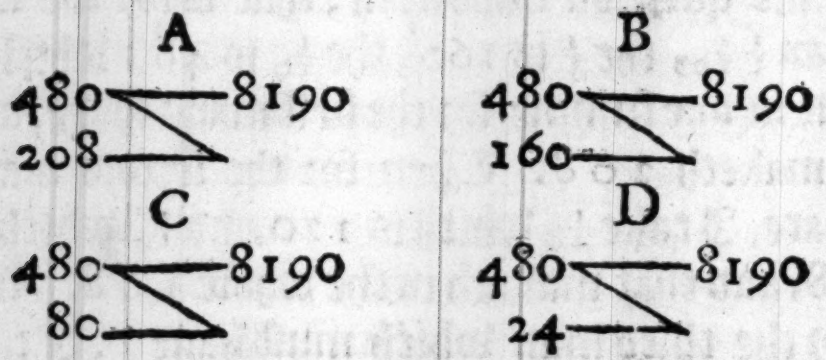
them be parts of ſome other of them, for all ſuch partes you may ouerpaſſe, and take for them: all thoſe numbers, whoſe partes they be: as in this example the ſhares be theſe, $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{6}$ $\frac{1}{20}$ if I multiply all the denominators together, beginning with 3, and ſo go on vnto 20, it will make 144000: but conſidering that 3 is a part of 6, I ſhall omit that 3, and likewiſe 10, which is a part of 20, I may ouerpaſſe alſo, and then is there but 3 denominators to multiplie, that is 4, 6, and 20, which make 480, which ſumme I take for my worke, becauſe all the denominators will be found in it. Then I take ſuch partes of it, as the queſtion importeth, that is for the firſt man $\frac{1}{3}$ $\frac{1}{10}$, the $\frac{1}{3}$ is 160, the $\frac{1}{10}$ is 48: which I put in one ſumme for the firſt mans ſhare, and it maketh 208. Then for the ſecond mans ſhare, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which muſt haue $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole ſumme: ſo that if the whole pray had beene but 480 l. then were the queſtion anſwered: but becauſe the ſumme was of greater value, by this meanes now ſhall I knowe the partition of it: I muſt ſet my numbers by the order of the Golden Rule, putting in the firſt place the number that I found by multiplying the denominators, and in the ſecond place the ſumme of

the

The reason of this rule.

the bootie. And looke what proportion is betweene the first number and that second, the same proportion shall be betweene the partes of that first number and the partes of the second, comparing each to his like. Therefore I must put in the third place, one of the parts or shares, and then worke by the former rule of proportion or Golden Rule. And because I haue foure seuerall parts of the first number, by which I would finde out foure like partes of the second number, therefore must I make foure seuerall figures.

Scholer. Now I trust I can answere to your questiõ, as by your fauour I will proue.



And to trie it, I set the 4 figures thus, marked with A, B, C, D, to shewe their order. And then in each of them I multiplie the second number by the third, and diuide their totall by the first, and so amounteth the fourth summe which I seeke for, if I doe multiplie 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the first mans portion.

And

And ſo woꝛking with the other thꝛee figures, I find foꝛ the ſecond man 2866½, and foꝛ the third man 1365: and then foꝛ the fourth man 409½. And ſo is euery mans ſhare ſet fooꝛth in the figure here annexed.

A

480 — 8190
208 — 3549

B

480 — 8190
160 — 2866½

C

480 — 8190
80 — 1365

D

480 — 8190
24 — 409½

And thus I thinke I haue done well.

Maſter. If you miſdoubt your woꝛking, and liſt to pꝛoue it, adde all the ſhares together, and if they make the totall, then ſeemeth it well done.

Scholer. I may ſet them thus: and then by Addition the iuſt ſumme doth amount, that is 8190, and theꝛefoꝛe (as you ſay) it ſeemeth to be well woꝛought.

The proofe by Addition.

3549
2866½
1365
409½
8190

But I beſeech you, is there any doubt in this triall, that you vſe that woꝛde, Seemeth? The iuſt proofe.

Maſter. You may eaſilie coniecture, that if you did aſſigne the firſt mans ſhare to the laſt, and ſo chaunge all the reſt, that one had an others ſhare, yet would the Addition appeare all

all one, and therfore is not the proofe exact.

But if you will make a iust proofe for the first mās part, take $\frac{1}{3}$ & $\frac{1}{10}$ of the whole summe, and if it agree with the number in the figure, then it is well done. And so do for the second, third, and fourth summes, and this proofe fayleth not. Now will I propound certaine other questions which haue beene set foorth by certaine learned men, albeit not without some ouersight, which questions I protest hartily, I doe not repeate to depraue those good men, whose labours and studies I much prayse and greatly delight in, but onely according to my profession, to seeke out truth in all thinges, and to remoue all occasions of errour, as much as in me lieth: and for that cause I will onely name the questions with out hurting the Authors name. The first question is this.

A question of building.

Foure men did builde a house. which coste them 3000 crownes, their shares were such, that one man should pay $\frac{1}{2}$ of the summe, and 6 crownes ouer: the second should pay $\frac{1}{3}$, and 12 crownes ouer: the third man must lay out $\frac{2}{5}$, abating 8 crownes, & the fourth man should pay $\frac{1}{4}$, and 20 crownes more: can you answere to this question?

Scholer. No in good faith sir, and that you know best of any man, for I know no more than you haue taught me.

Master. Then I dare say you cannot doe

it

it, neither yet the best learned man that euer did propose it, for the question is impossible: for declaration whereof I will be bold to vse first the representation of the numbers in their aptest forme, (although I haue not yet taught that maner of worke) because it may appeare plainely that the question is not possible, for here I haue set the partes, and added them, and they make the whole summe and ¾ and 30 more. Now how is it possible to diuide truly either gaynes, either charges, so that the particulars shall be more than the totall?

An impossible question.

½ +	6
⅓ +	12
⅔ +	8
¼ +	20
1 ¾ +	30

Scholer. It is against the forme of proofe by Addition of.

Master. You say truth. And because you shall perceaue it the better, I will trie it after the vulgare forme, as in this figure you see where the ½ with 6 ouer is 1506: for the totall is as you heard before 3000: the ⅓ and the 12 more, is 1012: the ⅔ would be 2000, but then abating 8, it is but 1992, and then last of all, the ¼ is 750, and the 20 more maketh 770: which all beyng added in one summe, do make 5280, where the totall summe should be but 3000, which summe of 3000 if you diuide by ⅔, so shall

1506
1012
1992
770
5280

you haue ¼ of it, that is 2250. and thereto adde 30 more, then will those three summes make 5280: whereby you may see how this forme as well as the other, doth declare that the particulars in that question would make more than the whole summe by ¾, and 30 more: and therefore can that questiō not be accepted as a possible thing, but yet doe certaine learned men propound such questions, and answere to them. Therefore somewhat to say to their excuse, rather of their good meaning than for their doing, I will anon declare what may bee said for their defence: but in the meane season I will propound the question as it may bee wrought by good possibilitie. As if foure men build a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doeth pay 6 crownes, so often shall the second man pay 4, the thirde man 8, and the fourth man 3. Or els thus: that the first man shall paie double so much as the fourth, and the second man shall pay ⅔ of the first mans charge: the thirde man shall pay double so much as the second: (and these two wayes are to one end) but further for their agreement it is appointed also, that the first man shall giue 6 crownes ouerplus, and the second 12, and the fourth shall giue 20, but the third man shall giue no ouer-

	3000
	2250
	30
	5280

plus,

plus, but ſhall haue 8 crownes abated of his charge. Now is the queſtion poſſible to be aſſoiled, and this is the way to do it. Marke the proportion of the ſeuerall charges, and ſet out ſmall numbers in that rate, by which you may reduce the worke to the Golden Rule, as here in the firſt forme, the numbers are already named, 6, 4, 8, 3: and in the ſecond forme, although they be not plainely named, yet they may be the ſame numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6: and againe 8 is double to 4. Now adde theſe together, and they make 21, which 21 muſt be ſet in the firſt number in the Golden rule: for if it with the ouerplus of each mans charge, would make the totall ſumme of the charges, then were thoſe ſeuerall ſummes the charges of each man, beſides his ouerplus: but now it is not ſo.

But yet this is true, (ſo excellent are concluſions Arithmeticall,) that looke what proportion each of theſe ſeuerall ſummes doth beare to 21, the ſame proportion doth the iuſt charges of euery man (beſides his ouerplus) beare to the totall of the charges, the ouerplus beyng deducted: Wherefore this may you note, that before you doe apply the totall of the charges to the Golden Rule: you muſt deduct the ouerplus which is 6, 12, and 20, that is in the whole 38: but then 8 muſt be reſtored for the abatement of the third man, and then remaineth to be deducted 30, Take 30 there- The Rule.

foꝛe out of 3 000, and there will reſt 2970, which I muſt ſet in the Golden rule foꝛ the ſecõd ſumme: & foꝛ the third ſumme I muſt put each of the ſmall numbers befoꝛe mentioned, which although they be not the ſeuerall charges, yet they repꝛeſent them in pꝛopoꝛtion. And ſo making foꝛ euery mans charge a ſeueral queſtiõ, ẏ figures wil be 4, which I marke with foure letters, A, B, C, D, thus.

A

21 — 2970
6 — 848 4/7

B

21 — 2970
4 — 565 5/7

C

21 — 2970
8 — 1131 3/7

D

21 — 2970
3 — 424 3/7

Where I hatte ſet foꝛ bꝛiefneſſe the ſumme of euery mans charge in the fourth place pꝛeſuppoſing that you can tell how to trie out that fourth ſumme by ſo many examples as ye haue had.

Scholer. As I truſt that I vnderſtand this fourme, ſo I deſire much to know what may be ſaide foꝛ them that miſtooke this queſtion.

Maſter. You ſeeme ſo deſirous to know this errour, that you haue foꝛgotten to examine whether this woꝛke be without fault.

Scholer. Me ſeemeth this woꝛke to be well done, becauſe the Addition of the 4 ſeuerall num-

numbers doth make the total summe of 2970, which was to bee diuided into such foure partes.

Master. But then haue you forgotten that the first man must pay 6 crownes more besides this share, and the second man twelue crownes more: the third man 8 crownes lesse: and the fourth man 20 crownes more, for without these, your first total of 3000 crowns will not be made.

Scholer. Then must I adde to the first mans summe 6 more, and it will be $854\frac{4}{7}$: and to the second summe I must adde 12, and it will be $577\frac{5}{7}$: from the thirde summe I must abate 8, and then will the summe bee $1123\frac{3}{7}$: then adding vnto the fourth summe 20, it will be $444\frac{3}{7}$: and these 4 summes will make 3000, which is the whole charge, as in this example it may appeare, where first I gather the $\frac{14}{7}$, that maketh 2, and so proceede I in the Addition to the end.

	$854\frac{4}{7}$
	$577\frac{5}{7}$
	$1123\frac{3}{7}$
	$444\frac{3}{7}$
	3000

Master. Now haue you well done, & this worke in the same summes is brought of other learned men for the true solution of the question as it was first proponed, which as (I said) was impossible: and now examine it by these seuerall summes, and see whether it do agree with the summes in the question proponed.

The first man must pay ½ and 6 ouer of the totall summe: how thinke you, is 845 4/7 the halfe and 6 more of 3000?

Scholer. No that it is not, for it would be 1506: and for the second man 1012: and for the third man 1992: and for the fourth man 770, whereof not one summe agreeth to this worke. But I marueil that so wise men could be so much ouerseene.

Master. It is commonly seene, that when men will receiue things from elder writers, & will not examine the thing, they seeme rather willing to erre with their auncients for companie, than to be bold to examine their workes or writings, which scrupulositie hath ingendered infinite errours in all kinds of knowledge, and in all ciuill administration, and in euerie kinde of Arte: but these learned men did not meane any other thing by this question, than to find such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to another: which thing you shall perceiue more plainely by another question of theirs, that is this.

A question of a testament.

A man lying vpon his death bedde, bequeatheth his goods (which were worth 3600 Crownes) in this sort. Because his wife was great with child, and he yet vncertaine whether the childe were male or female, he made his bequest conditionally, that if his wife bare a daughter, then should the wife haue halfe his goods

goods, and the daughter $\frac{1}{3}$, but if shee were deliuered of a sonne, then that sonne should haue $\frac{1}{2}$ of the goods, & his wife but $\frac{1}{3}$. Now it chanced her to bring forth both a sonne & a daughter, the question is: How shall they part the goods agreeably to the testator his will.

Scholer. If some cunning Lawiers had this matter in scanning, they would determine this Testament to be quite voide, and so the man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it, and also it might haue chanced that she should haue brought forth neyther Sonne nor Daughter, as often hath beene seene: so is the will vnsufficient in that point also.

Master. Such scanners should seeme too cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken fauorably, for the aid of the legatories, when there riseth such doubts. But let vs trie this worke, not by force of law, but by proportion Geometricall, seeing the testatour did minde to prouide for each sort of them.

Scholer. If the son shall haue $\frac{1}{2}$ by force of the Testament, so must the mother haue $\frac{1}{3}$. Againe, because she hath a daughter also, therfore ought she to haue $\frac{1}{2}$ & the daughter $\frac{1}{3}$: that is both wayes $\frac{1}{2}:\frac{1}{3}$, and $\frac{1}{2}:\frac{1}{3}$, which commeth to the whole goods, and $\frac{2}{3}$ more. Wherefore it seemeth also impossible.

Mast. In this matter the mind of the Testator is so to be vnderstood, that such proportion should be betweene the portion of the wife and the sonne, as is between $\frac{1}{3}$ and $\frac{1}{2}$, that is, the sonne must haue $\frac{3}{6}$ for $\frac{2}{6}$ to his mother, so shal he haue 3 to 2, that is as much as his mother, and halfe as much more: & the mother must haue the like rate in comparison to her daughter. Then must I find out 3 numbers in such proportion, that the first may be as much as the second, & halfe as much more (that is) in proportion sesquialtera, & the second to the third in the same proportion, such numbers be 6, 9, 4.

Scholer. I pray you sir, how shall I finde out those numbers?

Master. That will I gladly tell you.

To finde three numbers in any proportion.

Whatsoeuer the proportion be of any three numbers, multiplie the Termes of that proportion together and the number that amounteth, shalbe the middle number of the 3: then multiplie that middle number by the lesser terme, and diuide that totall by the greater, and the least number of the 3 will amount. So if you multiply that middle number by the greater extreame, and diuide that totall by the lesser extreame, then will the greatest number of that progression amount.

To find the proportion betweene two numbers.

Scholer. Then in this example, to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I must diuide (as you taught me in Diuision) $\frac{1}{2}$ by $\frac{1}{3}$ and the quotient will be $\frac{3}{2}$, that is $1\frac{1}{2}$, whereby I perceiue that

the

the proportion in this question is, as 3 to 2. Therefore (as you taught me euen nowe) I multiply 3 by 2, and the summe is 6, which must be the middle number: then I multiplie the middle number 6 by 2, which is the least tearme, and the summe is 12, that I doe diuide by 3, being of the greater Terme, and the quotient is 4, so is 4 the least number of the 3. Then I multiply 6 by 3, whereof commeth 18, and that I diuide by 2 and so haue I nine, which is the greatest number of the 3.

Master. Another way yet may you finde the third number in any progression, if you haue 2 of them: for if the middle number be one of them which you haue, then multiply it by it selfe (as in this example 6 by 6 maketh 36) and that totall diuide by the other number which you haue, and the third number will be the quotient.

Scholer. Then if I diuide 36 (which commeth of 6 multiplied by it selfe) by 4, the quotient will be 9, and if I diuide 36 by 9, the quotient will be 4. But what if I know the first number and the third, & would haue the middle number?

Master. Multiply the two numbers together, & in their totall you must seeke the roote of that number, & it shalbe the middle number: but because as yet you haue not learned how to extract rootes, therefore vse the first forme which I haue taught you, till I teach you to

extract

extract rootes. And now go forward with the answere to the same question.

Scholer. I perceyue then that the sonne must not haue $\frac{1}{2}$ of the goods, neither the mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{3}$ but yet must the goods be diuided into such proportion, that the sonne shall haue 9 crowns for 6 to his mother: and the mother shall haue 6 crownes for euery 4 to the daughter. Then I apply it to the golden rule in three exāples thus: where the first number is the Addition of those three numbers 9, 6, 4, & the third is one of them seuerally: the second is the totall of the goods in the testament: & then by the worke of the Golden rule I finde out the fourth number in euerie worke, that is for the sonne 1705 $\frac{5}{19}$: for the mother 1136 $\frac{16}{19}$: and for the daughter 757 $\frac{17}{19}$, which three summes added together do make the summe of the whole goods, as may be seene by this example.

19 — 3600
9

19 — 3600
6

19 — 3600
4

1705	$\frac{5}{19}$
1136	$\frac{16}{19}$
757	$\frac{17}{19}$
3600	

And this (me thinketh) I do perceiue, that because in this case there is a necessarie remedie diuised against an vrgent inconuenience, therfore those learned mē thought they might vse the like libertie in that other question.

Master. Your gesse is good, but they had so good reason for them in the one, as they haue

in

in the other: as in another example of theirs, it may better appeare, that is this:

A man left vnto his three sonnes 7851 crownes to be parted in this sort, that the first sonne should haue $\frac{1}{2}$, the second sonne $\frac{1}{3}$, and the third sonne $\frac{1}{4}$, which is not possible, for $\frac{1}{2}$, & $\frac{1}{3}$, and $\frac{1}{4}$, doth make $\frac{26}{24}$: or $\frac{13}{12}$, that is $1\frac{1}{12}$, so is it more than the whole: but reduce these fractions into one Denomination, the least that they will come to, and they will be $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$, & so may you part the goods in such proportion as these 3 Numerators beare togither: that is, the first to haue 6 for euerie 4 to the second: and the second to haue 4 as often as the third hath 3: and so their portions will be for the first, 2623 $\frac{7}{13}$: for the seconde 2415 $\frac{9}{13}$: and for the thirde 1811 $\frac{10}{13}$, and those three shares added together, will make the totall summe of the whole goods, as you may easily see in this example.

Another question is there proponed thus:

Another question of a testamẽt.

	3623 $\frac{7}{13}$
	2415 $\frac{9}{13}$
	1811 $\frac{10}{13}$
	7851

There is 450 Crownes to be diuided betweene 3 men, so that the first man must haue $\frac{1}{2}$ and $\frac{1}{3}$, the second man $\frac{1}{3}$ and $\frac{1}{4}$, the third man shall haue $\frac{1}{4}$ and $\frac{1}{5}$.

Scholer. I marueil that any man should be so ouerseene to propounde that question as a thing possible, sith $\frac{1}{2}\frac{1}{3}$, $\frac{1}{3}\frac{1}{4}$, $\frac{1}{4}\frac{1}{5}$, doe make $1\frac{2}{15}$ that is almost double the whole summe.

But I perceiue it might be thus propo-

ned,

ned, that as often as the first man did receiue 50 Crownes, so often the second man should receiue 35, and the third man 27, for $\frac{1}{2}$ & $\frac{1}{3}$, is equal to $\frac{50}{60}$, and so is $\frac{1}{3}$ & $\frac{1}{4}$ equal to $\frac{35}{60}$, and $\frac{1}{4}$ & $\frac{1}{5}$, is $\frac{27}{60}$, and so working the question, the 3 figures will appeare in this forme: whereby the first mans portion is found to be 200 $\frac{50}{58}$: the second mans part is 140 $\frac{15}{58}$: the third mans share is 108 $\frac{24}{58}$: which in the whole doth make 450 crownes that was ỹ whole summe to bee diuided betweene them.

112 — 450
50 — 200 $\frac{50}{58}$

112 — 450
35 — 140 $\frac{15}{58}$

112 — 450
27 — 180 $\frac{21}{58}$

Master. And thus you are (I thinke) sufficiently instructed in the Rule of Felowship:

The Rule of Alligation.

NOw will I go in hand with the Rule of Alligation, which hath his name, for that by it there are diuerse parcels of sundrie prices, and sundrie quantities alligate bound or mixed togither, whereby also it might be well called the rule of mixture,

and

and it hath great vse in composition of medicines, and also in mixtures of metals, and some vse it hath in mixtures of wines, but I wish it were lesse vsed therein than it is now a dayes. The order of this Rule is this: When any summes are proponed to bee mixed, set them in order one ouer another, and the common number whereunto you wil reduce them, set on the left hand, then marke what summes be lesser than that common number, and which be greater, and with a draught of your penne, euermore linke two numbers together, so that one be lesser than the common number, and the other greater than he, for two greater or two smaller cannot well be linked together, and the reason is this, that one greater and one smaller may be so mixed, that they will make the meane or common number very well, but two lesse can neuer make so many as the common number, beyng taken orderly: no more can two summes greater than the meane, neuer make the meane in due order, as it shal appeare better to you hereafter. And as it is of necessitie to linke euery smaller (once at the least) with one greater, and euery greater with one smaller: so it is at libertie to linke them oftener then once, and so may there be to one question many solutions. When you haue so linked them, then marke how much each of the lesser numbers is smaller then the meane or common number, and that difference

The reason of this Rule.

let

set against the greater numbers which be linked with those smaller, each with his match still on the right hand, and likewise the excesse of the greater numbers aboue the meane, you shall set before the lesser numbers which be combined with them. Then shall you by Addition bring all these differences into one summe, which shall be the first number in the Golden Rule: and the second number shall be the whole masse that you will haue of all those particulars: the thirde summe shall be each difference by it selfe, and then by them shall be found the fourth number, declaring the iust portion of euery particular in that mixture. As nowe by these examples I will make it plaine.

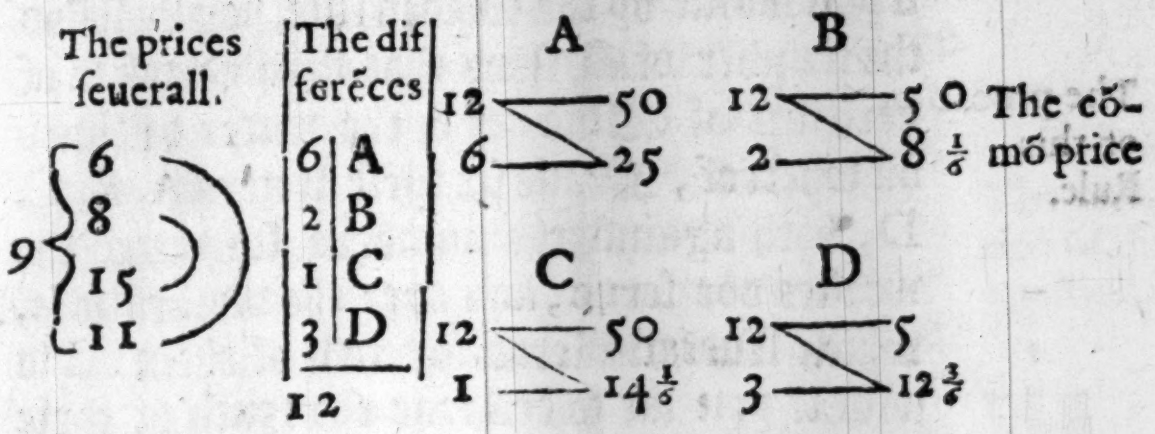
A question of mixing of wine.

There are foure sortes of Wine of seuerall prices, one of 6 d a gallon, another of 8 d, the third of 11 d, and the fourth of 15 d the gallon, of all these Wines would I haue a mixture made to the summe of fiftie gallons, so that the price of ech gallon may be 9 d. Now demaund I how much must be taken of euery sort of wine?

Scholer. If it shall please you to worke the first example, that I may marke the applying of it to the Rule, then I trust I shall be able not onely to do the like, but also to see the reason in the order of the worke.

Master. Marke then this forme & the placing of euery kind of number in it.

The



Here (you see) I haue set downe the seuerall prices which be 6, 8, 11, 15, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue set on the left side: And the difference betweene it, and euerie particular price, I haue set on the right hand not against the summe whose difference it is, but against the summe that it is linked withall: so the difference of 15 aboue 9, is 6, which I haue set not against 15, but against 6, that is linked with 15, and the difference betweene 6 and 9 that is 3) I haue set against 15. So likewise the difference betweene 8 and 9 is but 1, that haue I set against 11, and the difference of 11 aboue 9 (which is two) I haue set against 8. Then adde I all those foure differences, and they make 12, which I set for the first number in the Golden Rule: the second number I make 50, which is the summe of Gallons that I would haue, and the third summe is euery particular difference. Now

if

The proofe of this Rule.

if you worke by the Golden rule, you shall find the number of Gallons that shall be taken of each sort of Wine: For the better distinction whereof, I haue set these letters A, B, C, D, both against the numbers for which the workes doe serue, and ouer the workes also, which seuerally serue for each of them. And now if you list to examine the truth of these workes, adde those foure summes together, and they will make fiftie, that is the totall which I would haue, as by this crãple you may easily perceiue. And for to proove how the prices do agree, doe this. Multiply this totall summe 50, by the common price 9, & it will make 450: then keepe that summe by it selfe, and afterward multiplie euery seuerall summe of Gallons, by the price belonging to the same Gallons, and if that summe doe agree with this which you haue kepte firste, then is your worke well doone. As here, 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you shall set downe: then multiplie $8\frac{2}{6}$ by 88 which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: so againe $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And last of all $12\frac{3}{6}$ multiplied by 15, maketh $187\frac{3}{6}$. And these added togither doe

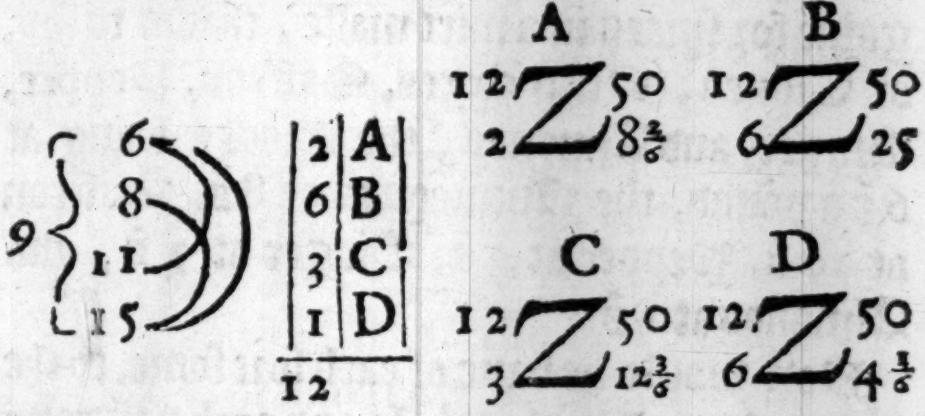
25	—
$8\frac{2}{6}$	
$4\frac{1}{6}$	
$12\frac{3}{6}$	
50	
150	
$66\frac{4}{6}$	
$45\frac{5}{6}$	
$187\frac{3}{6}$	
450	

do make 450, as in the example annexed you may see: wherfore seing it doth agree with the former sum of 50, multiplied by 9, I may iustly affirme this worke to be good & well done.

And now to proue how you can do the like, I propound the same question, onely willing you to vse some other forme of combining or thinking the summes.

The variation of this question.

Scholer. That shall I proue with your fauour, and therefore I combine 8 with 15, and 6 with 11, and then the forme will bee as followeth:



whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences bee altered, as the combination is chaunged, whereof I vnderstande the reason by your former worke. And therefore here appeareth no strange thing, but that now I must haue 8 2/6 gallons of sixe pence, and 25 gallons of 8 d, and 12 gallons and 3/6 of 11 d, & so consequently 4 gallons & 1/6 of 15 d, so that multiplying 8 2/6 by 6, it maketh 50, &

50	
200	
137 3/6	
62	
450	

then 25 multiplyed by 8, maketh 200: likewise 12 ⅔ multiplied by 11, yeldeth 137 ⅓, and 4 ⅓ multiplied by 15, maketh 62 ⅔, which 4 summes added in one, will yeeld in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of Gallons) by 9 the common or meane price.

A question of spices.

Master. Seeing you conceiue this worke so well, I will propound an other example vnto you of more varietie in the Alligations or combinins, as thus.

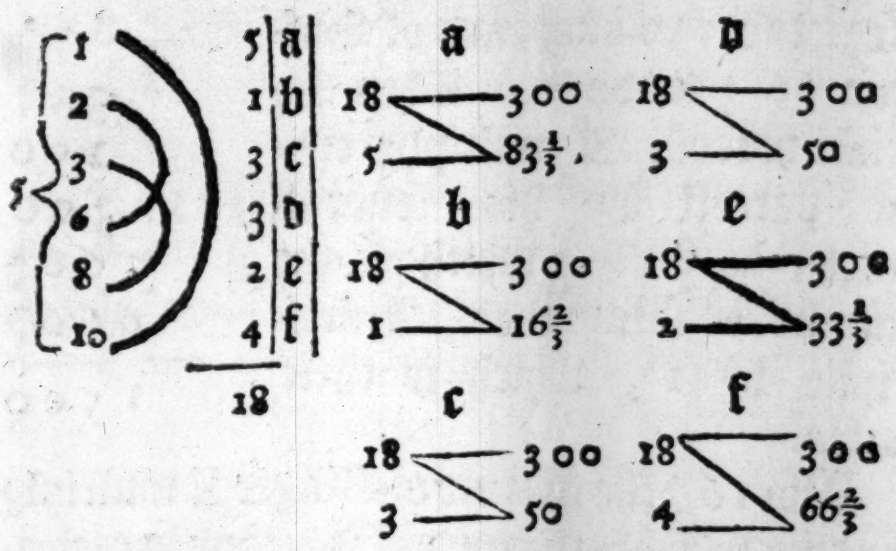
A Marchant being minded to make a bargaine for spices in a mixt masse, that is to say, of Cloues, Nutmegges, Saffron, Pepper, Ginger, and Aimonds, the Cloues being at 6 s̄ a pound, the Nutmegges at 8 s̄, Saffron at 10 s̄, Pepper at 3 s̄, Ginger at 2 s̄, and Almonds at 1 s̄.

Now would he haue of each sort some, to the value of 300 l̄ in the whole, and each poūd one with another to beare in price 5 s̄, how much shall he haue of each sort?

Scholer. That will I trie thus.

First I set dowee those sire seuerall prices and at the left hand I set the common price 5 s̄. Then I linke them thus, one with 10, two with 6, and three with 8, as in the example following.

Master.



Master. I had minded to haue combined them in more varietie, but I am content to see your owne worke first, and then more varieties in combination may follow anon.

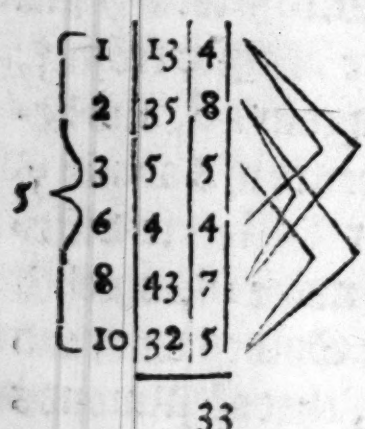
Scholer. Then to continue as I began, I seeke the difference betweene 1 and 5 (which is 4) and that I set agaynst 10: then agaynst 1 I set 5, which is the excesse of 10 aboue 5: so I gather the difference betweene 2 and 5, which is three, and that I set against 6, because it is combined with 2: and likewise the difference of 6 aboue fiue (which is 1) I set against 2. Then take I the difference of three from 5, which is two, and that I set against 8, and before that three I set the difference of 8 aboue 5 which is three. Then gather I all these differences by Addition, and they make 18, which I set for my first number in the Golden rule, and so appeareth by those works, that of Almonds I must take 83 l. 1/3, of Ginger 16 l. 2/3, of Pepper 50 pound, of Cloues 50 pound, of Nut-

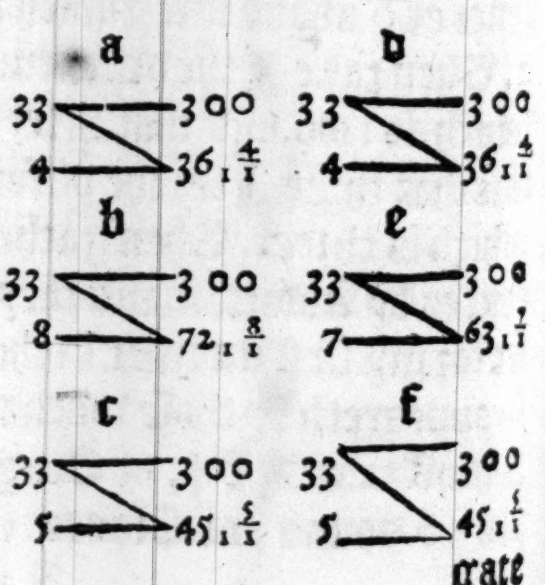
megges 33 pound $\frac{1}{3}$, and of Saffron 66 pounde $\frac{2}{3}$. Then for triall hereof, I multiplie euerie parcell by his seuerall price, as $8\,3\frac{1}{3}$, which is the summe of Almondes, I multiplie by 1, which is their price.

$8\,3\frac{1}{3}$	
$33\frac{1}{3}$	
150	
300	
$266\frac{2}{3}$	
$666\frac{2}{3}$	
1500	

Also $16\frac{2}{3}$ the summe of Ginger I multiply by two which is the price of it. And so each other in his kinde, as this Table annexed doth represent, and then adding them altogether, I find the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300 by the common price 5, wherefore it appeareth well wrought.

Master. Now will I make the Alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as in this example you may see where I haue Alli-





gate

gate 1 with 6 and 8, and therefore haue I set against 1 both their differences, that is 1 and 3 Likewise, because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I haue set onely 5, which is the difference of 10, with whom 3 is combined onely. Likewise 6 is onely alligate to 1, & therefore is the difference of 1 from 5: which is 4 onely set agaynst it: 8 is linked with 1 and 2, and therefore hath hee against him both their differences, 4 and 3, and 10 is ioyned with 2 and 3, therefore hath he their differences 3 and 2. And because of ease for you, in an other Collumne I haue set the differences reduced into one number, for euerie seuerall sort, and haue also added them togither, whereby appeareth that they make 33, and so consequently you see the workes of the Golden Rule set foorth for the sixe seuerall drugges, I haue added letters, a, b, c, &c. as before. But I woulde not wish you to cleaue still to these elementarie aydes, but accustome memorie to trust her selfe, so shall occasion of negligence bee best auoyded. And as for the proofe, trie it at more leysure, because the tyme nowe is short, and you sufficiently instructed in that proofe. And there resteth diuerse thinges behinde yet, of which I woulde gladly giue you some taste before our departure.

Scholer. But if it may please you to let me

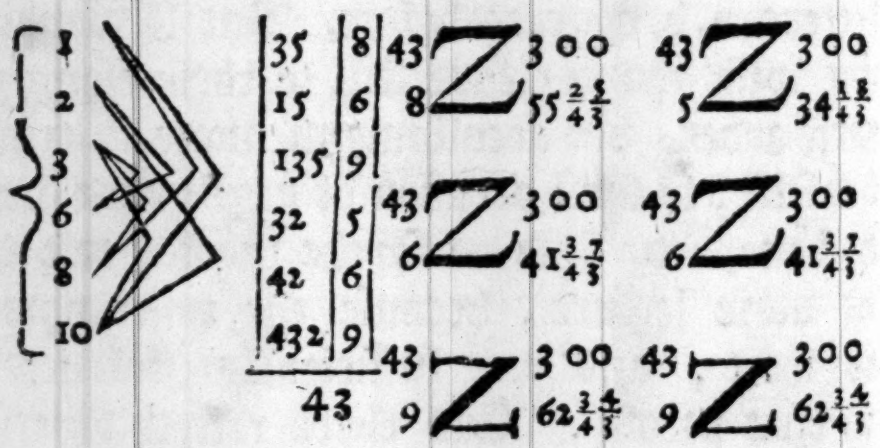
ſee all the variations of this queſtion, before you go from it, for me thinketh I could varie it two or three wayes more yet.

Maſter. I am content to ſee you make two or three variations, but I would be loth to ſtay to ſee all the variations, for it may bee varied aboue 300 wayes, although many of them would not ſerue well to this purpoſe.

Scholer. I thought it impoſſible to make ſo manie variations.

Maſter. Maruell not thereat, for ſome queſtions of this rule may bee varied aboue 1000 wayes, but I would haue you forget ſuch fantaſies, till a time of more leiſure. And now go forward with ſome variation of this queſtion.

Scholer. For the firſt variation, I linke the firſt number 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.

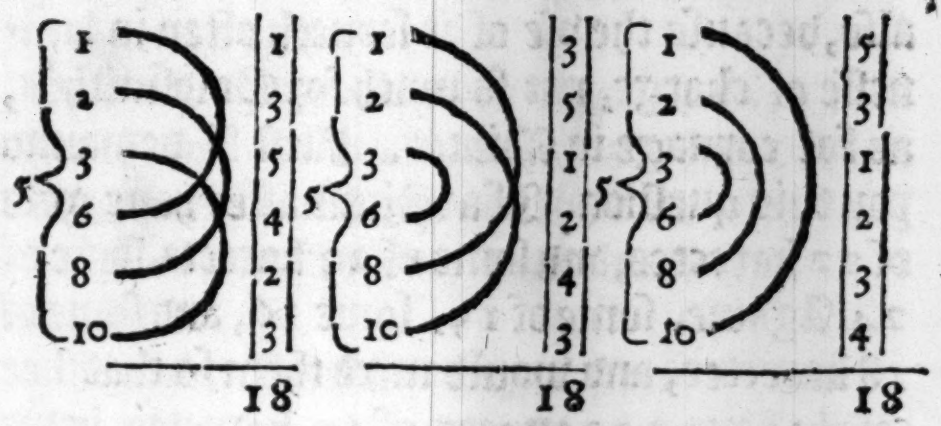


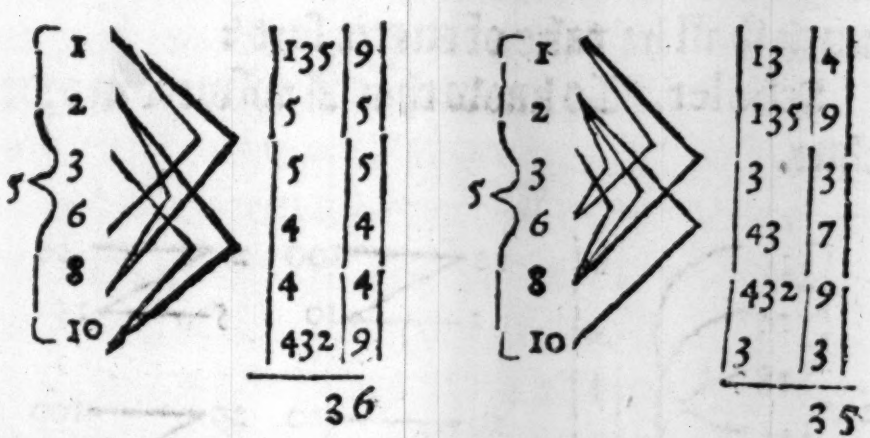
And ſo doth there appeare the portion of waight for euerie kind of drugge in this mixture. Now for the triall.

Maſter.

Maſter. May ſtay there, you ſhall not neede to make triall in one example ſo often, or if you liſt to doe it by your ſelfe, I am content. But now ſet foorth (for declaration that you conceiue the rule) two or three examples of ſeuerall combinations, and then will wee paſſe to ſome other example, and ſo ende this Rule.

Scholer. As it pleaſeth you, ſo will I doe. And theſe bee the varieties, in which as the



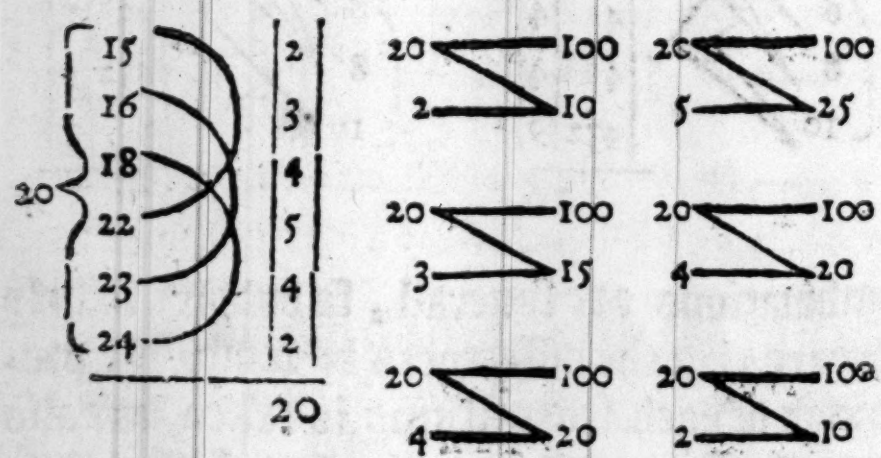


combinations are ſeuerall, ſo doth it plainely appeare, that the differences by which the proportion of each ſeuerall kind is taken, are alſo ſeuerall. And yet I ſee in the three firſt of theſe

fiue varieties, and in one other before, the totall summe of the differences to be one, that is to say 18, wherby I perceiue that the varietie of their mixture doth depend of the varietie of their differences seuerall, and not of the varietie of their totall summe.

Master. So is it. And seeing you conceiue it so well, I will make an ende of this Rule, onely exhibiting to you one question or two of the mixture of mettals, that by it you may deuise other like, and exercise your selfe therein also, because the vse of it serueth often in businesse of charge, not so much for Goldsmithes, as for coynage in Mintes. First I demaund you this question. If a Mintmaster haue gold of 22 karectes, and some of 23 karects, some of 24. Againe, some of 15, some 16, and some of 18 karectes, and would mixe them so that hee might haue 100 ounces of 20 karectes, howe much shall he take of euerie sort?

Scholer. To know that I answere in order thus.



Master.

Master. You haue wrought the question well, but how chanced it you made no doubt of that new name Karect?

Scholer. Because I thought it out of time to demaund such questions now, seeing you make so much hast to ende: and againe in this case the proportion of the number is sufficient for my purpose in this worke, trusting, that an other time you will instruct mee as well of this, as of sundrie other thinges which I haue heard you talke of, so I haue a great desire to know them.

Master. Your answere is reasonable, and your request and trust with Goods helpe I intend to satisfie. And to goe forward with this matter, let mee see your examination of this last worke.

Scholer. First for the one part I adde togither all the particular summes as they appeare in the worke, and they make 100, as here by their Addition doth appeare.

And so it seemeth, that the sums are well gathered, but for the further triall of them, I multiplie first 20, which is the common or meane sum of the karects by 100, which is the summe of the whole masse which I would haue, and it maketh 2000. Then I multiplie euerie particular summe by the karects

10	
15	
20	
25	
20	
10	
100	

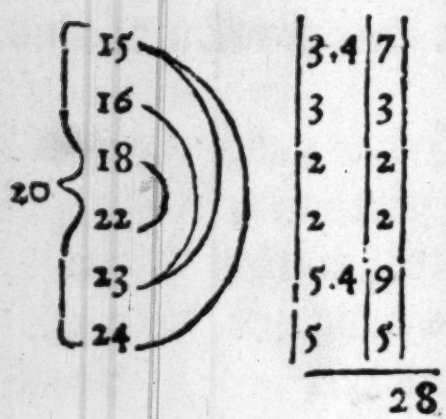
150	
240	
360	
550	
460	
240	
2000	

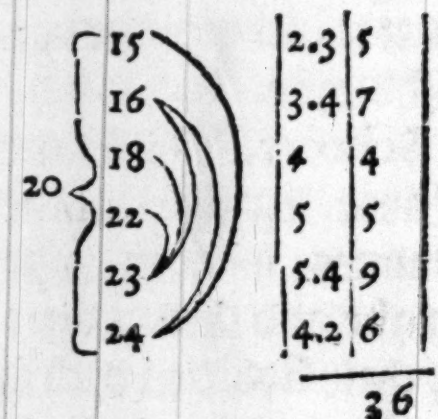
rects that it doth containe, as 10 by 15, and that maketh 150.

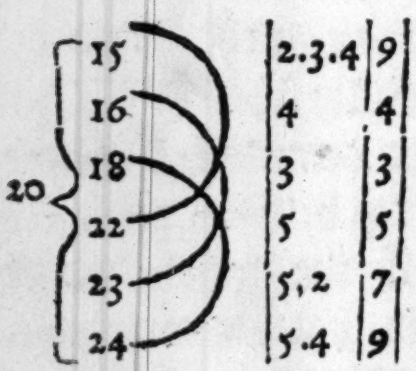
Likewise I multiplie 15 by 16, and it yeeldeth 240: so 20 by 18 maketh 360. And 25 by 22 yeeldeth 550: likewise 20 by 23 bringeth forth 460, and last of all 10 multiplied by 24 yeeldeth 240: which summes all ioyned together make 2000, that doth agree with the like summe before, wherefore I may well say, that the worke is good. And now if it please you I would set foorth some varieties of this question, to proue my wit.

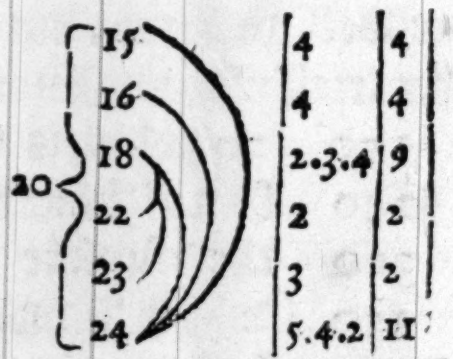
Master. Go to, let me see.

Scholer. Here be foure varieties.









And

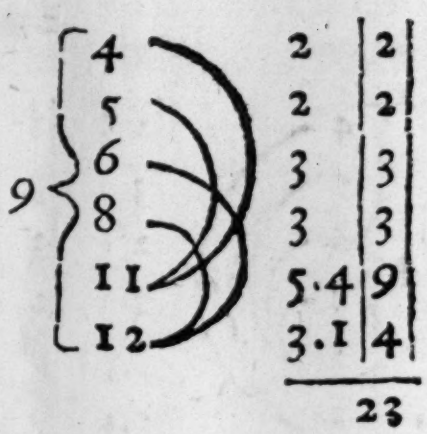
And more yet I could make, but not like to the number that you spake of in the variation of the other question.

Master. That will I teach you at more leysure, seeing it is a thing rather of pleasure than of any necessitie.

But nowe for your exercise in this Rule, one other question I will propose. A Mint-master hath sixe ingottes of siluer of sundrie fineneſſe, some of 4 ounces fine, and some of 5 ounces, some of 6, and other of 8, some of 11, and other of 12, and his desire is to mixe 500 pound waight, so that in the whole masse euery pound waight should beare 9 ounces of fine siluer, how much shall he take (say you) of euerie sort of siluer?

A question of mixing of siluer.

Scholer. To finde out that, I sette the numbers thus in order.

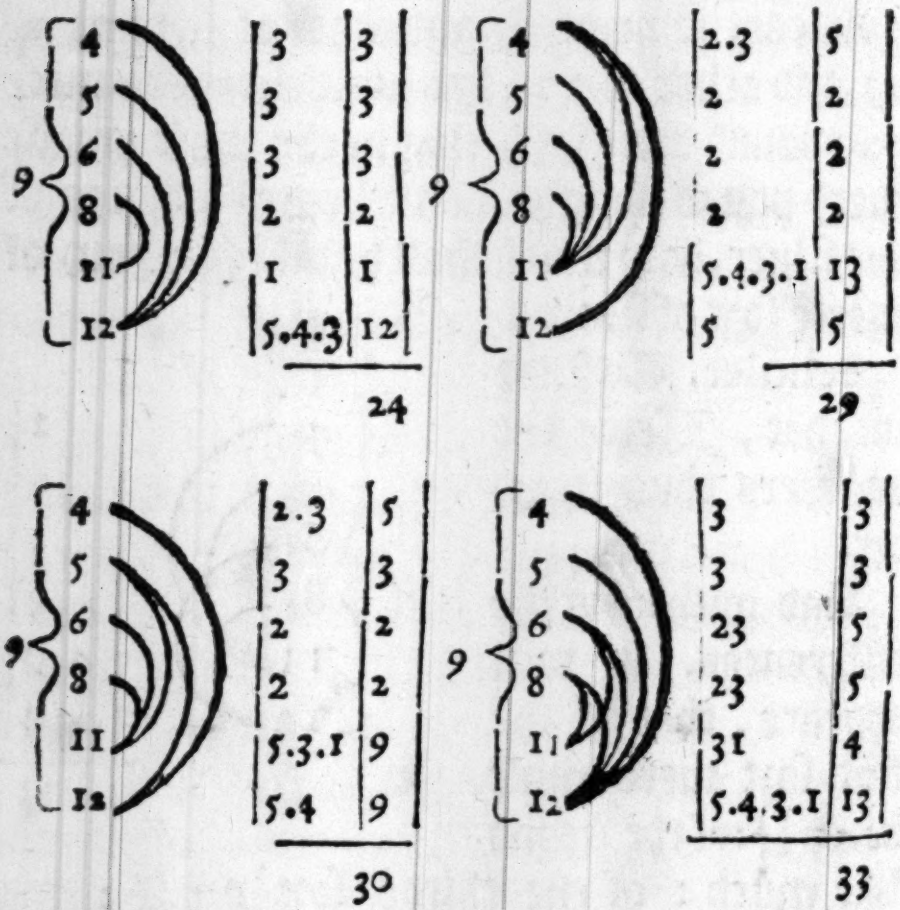


And gathering the differences, it will appeare, that of the first sort there must be 43 $\frac{1}{2}\frac{1}{3}$: of the second like much: of the thirde sort 65 $2\frac{5}{3}$: and of the fourth sort as much: of the fifth sort 195 $\frac{1}{2}\frac{5}{3}$: and of the sixth sort 86 $\frac{2}{2}\frac{2}{3}$, which in the whole will make 500 ℔ waight: and in ounces after 9 ounces fine 4500, that is of the first sort 173 $\frac{2}{2}\frac{1}{3}$: and of the second sort 217 $2\frac{2}{3}$:

217 $2\frac{2}{3}$: of the third sort 391 $2\frac{2}{3}$: of the fourth sort 521 $\frac{17}{3}$: of the fift sort 2152 $2\frac{4}{3}$, and of the sixt sort 1043 $\frac{11}{3}$, which altogether doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

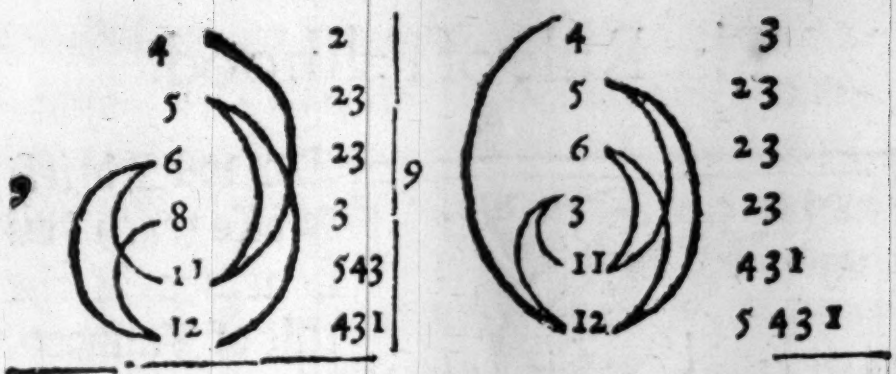
Master. This is well done of you, therefore now marke three or foure varieties, and so an end of this rule.

Sc. These 4 varieties I set for example.



Master. And by these it appeareth, that you can finde out more with which I will not now meddle, saue onely for to shewe you an easie helpe in drawing the lines of Combination, I will set forth two varieties here.

And



And this shall suffice now for the Rule of Alligation or mixture: for by these examples may you easily coniecture such other as doe appertaine to it, as well for the due working, as for varietie of drawing the lines of combination.

Scholer. Sir, albeit it pleased you ere while, to put me from my musing at the manifold varieties, that may fall in these combinations, and termed them fantasies, yet my fantasie giueth mee, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable. Therefore ye may well thinke, that at an other time conuenient I will request you to ayde me herein.

Master. Truth it is that this consideration may fall in practise as well Politike as Philosophicall, and sundry wayes in them bee applyed, therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The

The Rule of Falshood.

The occasion of the name.



Now wil I briefly also teach you somewhat of the rule of Falshood, which bereth his name, not for that it teacheth anie fraud or falshood, but for ỳ by false numbers takē at all aduentures, it teacheth how to finde those true numbers that you seeke for.

Scholer. So might any other Rule be called the Rule of Falshoode, for they worke by wrong numbers, and by them finde out the right numbers, so doth the rule of Alligation, the rule of Felowship, & the Goldē rule partly.

Master. In the Golden rule, the rule of Felowship, & the rule of Alligation, although the numbers that you worke by, bee not the true numbers that you seeke for, yet are they numbers in iust proportion, & are found by orderly worke: wheras in this rule ỳ numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therefore I sometimes being merrie with my friends, and talking of such questions, haue caused them that proponed such questiōs, to call vnto them such children or Idiots as happened

happened to bee in the place, and to take their answere, declaring that I would make them solue those questions, that seemed so doubtfull. And in deed I did answer to the question and worke the triall thereof also by those answeres which they happened at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this rule called the rule of false positions, and for briefnes the rule of Falshood, which rule for readinesse of remẽbrance, I haue comprised in these few Uerses following in forme of an obscure Riddle.

Gesse at this worke as hap doth leede,
By chance to truth you may proceede.
And first worke by the question,
Although no truth therein be done.
Such falshood is so good a ground,
That truth by it will soone be found.
From many bate too many moe,
From too fewe take too fewe also.
With too much ioyne to fewe againe,
To too fewe adde too many plaine.
In crossewise multiplie contrary kinde,
All truth by falshood for to finde.

The sence of these verses, and the summe of this Rule, is this.

When any question is proponed appertaining to this Rule, first imagine any number that you list, which you shall name the first

first position, and put it in steade of the true number, and then worke with it as the question importeth: and if you haue missed, then is the last number of that worke either too great or too little: that shall you note as hereafter shall bee taught you, and you shall call it the first errour.

Then beginne againe, and take an other number, which shall bee called the second position, and worke by the question: if you haue missed againe, note the excesse or default as it is, and call that the seconde errour. Then multiplie crosse wise the first position by the second errour, and againe the second position by the first errour, and note their totals seuerally by the names of totals. Then marke whether the two errours were both alike, that is to say, both too much, or both too little: or whether they bee vnlike, that is, the one too much, and the other too little. for if they bee like, then you shall subtract the one totall from the other (I meane the lesser from the greater) and the Remayner shall bee your diuidend, so must you abate the lesser errour out of the greater, and the residue shall bee the Diuisor. Now diuide the diuidend by that Diuisor, and the quotient will shew you the true number that you seeke for. But and if the errours bee vnlike, then must you adde both those totals (which you noted) togither, and take that whole number for the diuidend, so

shall

shall you adde both the errours together, and that whole number shall be the Diuisor, & the quotient of that Diuision shall giue you the true number that the question seeketh for, and this is the whole rule.

Scholer. This rule seemeth so vnlike any other, that without some example I shall not easily vnderstand it.

Master. With a good will propose halfe a score sundry questions & examples of varietie, for the better vnderstanding of the worke hereof: and for the first take this example: A Mason was bound to build a wall in 40 dayes, and it was couenanted so with him, that euery daie that he wrought, he should haue for his wages 2 s̄ 1 d̄, and euery day that he wrought not he should be amerced 2 s̄ 6 d̄, so that whē the wall was made, and the reckning taken of the daies that he wrought, and of the other that he wrought not, the Mason had clearely but 5 s̄, 5 d̄, for his worke. Now do I demaund, how many dayes did he worke of those 40, and how many did he not worke?

A question of Masonrie. The first example.

Scholer. I pray you expresse the order of the worke, that I may partly by imitation, & partly by comparing it with the rule, be able againe to do the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at aduenture, as for example, I say he played 12 daies and wrought 28 dayes. Now

caſt you the wages of euery day, and ſee whether it will agree with the ſumme of 5 s̄, 5 d̄.

Scholer. The 28 dayes that he wrought after 25 pence the day, yeeldeth 700 pence. Then the 12 dates that he wrought not, at 30 pence ech day, doth amount to 360 pence, which if I abate out of 700 pence, there reſteth 340: but you ſay he had not ſo much.

Maſter. He had but 65 pence, and by this ſuppoſition he ſhould haue had 340: therefore is this ſumme too much by 275, which ſumme I muſt ſet downe after this ſort, as you ſee here, where firſt I haue made a croſſe (commonly called S. Andrewes croſſe) at the ouer corner on the left hand

12
275+

I haue ſet the firſt poſition 12, and at the other corner vnder it, I haue ſet 275 which is the firſt errour, with this figure +, which betokeneth too much, as this line, —plain without a croſſe line, betookeneth too little. On the right hand of the croſſe I haue left two like rowmes for the ſecond poſition and his errour. Therefore to proſecute the worke, I ſuppoſe he played 16 dayes, and wrought 24.

Scholer. I was a while in doubt why you named the dayes of his working, ſeeing they be not ſet in the figure: and I doubted how you know them, or elſe whether that you did ſuppoſe them at all aduentures, as you did the dayes that he played: but now I ga-

ther

ther, that seeing 40 dayes is the whole time limited, then the dayes that he played beyng supposed the rest of 40, must needes be the dayes that he wrought, and therefore 28 followed 12 of necessitie, and 24 followeth 16 also of necessitie: but yet I scarce perceyue why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle: but now examine you this second position.

Scholer. If he played 16 dayes, then abating 16 times 30 d, the summe will bee 480 d. And for 24 dayes that he wrought, euerie day yeelding 25 d, the totall is 600 pence: so that abating 480 out of 600, there resteth 120: and as you say it should bee but 65, therefore it is too much by 55, that must be set on the right hand of the figure at the neather parte, and ouer it on the same side 16, which is the secõd position, thus.

And as I gather by your words it were all one if I did set 28 in stead of 12, & 24 in stead of 16.

12	16
275+	55+

Master. So were it. But this shall you marke, that of what nature soeuer the two positions be, of the same nature is the quotient. Therefore when the positions in this

Note.

queſtion are 12 and 16, which both beyng numbers of the playing dayes, the quotient ſhall declare the true numbers of playing dayes, whereas if the poſitions had beene 28 and 24, which are ſuppoſed to be the working dayes, then would the quotient declare the true number of the working dayes, and not of playing dayes, as it will doe now. And therefore to continue the worke of this queſtion, and to finde the true number of playing dayes, I muſt multiplie croſſewiſe the firſt poſition 12 by 55, that is the ſecond errour, and the totall will be 660, then I multiplie 275 by 16, and it yeeldeth 4400. Now becauſe the errours are like, that is to ſay, both too much, I muſt ſubtract 660, out of 4400, and ſo remaineth 3740, which is the diuidend. Againe I muſt ſubtract the leſſer errour 55 out of 275 that is the greater errour, and there will remaine 220, which ſhall be the Diuiſor, then diuiding 3740 by 220, the quotient will be 17. Wherefore I ſay now conſtantly, that 17 is the true number of daies that the Maſon plaid: and then it followeth, that he wrought 23 dayes, and ſo is the queſtion anſwered.

The proofe of this Rule.

Now for the order of triall of this worke there needeth none other proofe but onely this, to worke with this number according to the queſtion, and if it agree, then appeareth the number to be it that you would haue. As here

here now seeing he wrought 23 dayes, and must haue for euerie day 25 pence, the whole summe commeth 575. Then againe seeing he plaied 17 dayes, and must abate 30 pence for euery day, the whole summe of the abatement will be 510: therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 s. 5 d, the cleare wages of the Mason for his worke, according to the question.

Scholer. Nowe I trust I vnderstand the worke and the rule so well (and the better by this proofe) that I can be able to doe the like. And for a proofe I take the same question all saue the last number, where I wil suppose that he had 10 s for his wages cleare. And now to gesse at the nūber of the daies that he wrought, I suppose first that he wrought 20 dayes, then say I, if he wrought 20 dayes, his wages must be 500 d, then did he play other 20 daies, for which must be abated 600 d, and then hee looseth 100 d. And so am I at a stay, for it is not like vnto your former worke.

Master. You should haue required of me some question, and not haue taken a question of your owne fantasying, vntill you were more expert in this arte: for so might you as well happen on an impossible question as on a possible: but now to goe forward, cōsider that this number is too litle by 220, seing he should gayne by your supposition 120 d, and in this position he leeseth 100, those both make 220

which you shall set downe for the first error with this signe —, betokening too little, as here in this forme following doth appeare.

And now for the rest go forward your selfe once againe. 20

Scholer. As my error hath bttered my follie, so it hath procured me better vnderstanding. 220—



Now therefore considering this position not to solue the question, I take an other, supposing that he wrought 30 daies, then for his wages he must be allowed 750 d, and for the 10 daies which he wrought not, he must abate 300 d, and so remaineth cleare 450 d: but it should be onely 120 d, therefore is it too much by 330, which I set downe in the figure with the former position, and his error, and the figure appeareth thus. 20 30

Now must I multiplie in crosse wayes 220, by 30, and it will be 6600, 220— 330+



Then again I multiplie 330 by 20, and it will be also 6600. Wherefore if I shal subtract the one out of the other, there will remayne nothing to be the Diuidend.

Master. In this you forget your selfe againe: for in as much as the signes in the errors be vnlike, therefore must you worke by Addition, adding together those two totals to make

make the diuidend, and also adding the two errors to make the diuisor: And because you shall no more forget this part of the rule, take this briefe remembrance:

Vnlike require Addition,
And like desire Subtraction.

Scholer. You meane, that if the errors haue like signes, then must the diuidend and the Diuisor be made by Subtraction, as is taught before: And if those signes bee vnlike (as in this last example they be) then must I by Addition gather the diuidend and the diuisor. Therefore must I adde 6600 to 6600, and it will be 13200, which shall be the diuidend. Then againe I adde 220 to 330, and it will be 550, which must be the diuisor: wherfore diuiding 13200 by 550, the quotient will be 24, whereby I know that the Mason wrought 24 dayes, and then it followeth that he played 16 daies.

Master. Examine your worke whether it be agreeable to the question or no.

Scholer. For 24 dayes worke, the wages must be 600 d, and for 16 daies which the Mason wrought not, there must be abated 480, and then remaineth cleare to the Mason 120 pence, as the question importeth: wherefore it is euident, that 24 is the true number of daies that he wrought.

Master. Although you seeme now to vnderstand this worke, yet to acquaint your minde the better with the new trade of this rule, I thinke it good to propone to you fiue or sixe examples more, before I make an end of it.

Scholer. Sir I thanke you that you doe so consider my commoditie and profite in knowledge, for vndoubtedly it is practise and exercise that maketh men prompt and expert in euery kinde of knowledge.

Master. You say well so that they follow some certaine preceptes to gouerne and rule their practise by, else may practise procure custome of error, and a repugnance to exactnesse of knowledge, namely as long as the error is not plainely knowne to the vulgar sort. But to returne to our worke.

A question of wares.

There is a seruant that hath bought of veluet and damaske for his master 40 yardes, the veluet at 20 s̄ a yard, and the Damaske at 12 s̄, and when he commeth home, his maister demandeth of him how much he hath bought of each sort: I cannot tell (saith he) exactly, but this I know, that I paide for Damaske 48 s̄ more then I paide for veluet, now must you gesse how many yardes there is of eth sort.

Scholer. Although the gesse seemeth difficult, yet I will proue what I can doe: for I remember your saying, that it forceth not howe fond or false the gesse be, so it be some-

what

what to the question, and not an answere of a contrarie matter.

Therefore first I imagine that hee bought 20 yardes of Damaske, for which he should pay after the former price 240 shillings: then must he needes haue of veluet other 20 yards (to make vp the 40 yardes) and that would cost 400 s̄. So that the totall of the price of the Damaske is lesse then the summe paid for veluet 160 s̄, & should be more by 48. Therefore the first error is 208 too little. Then begin I againe, and suppose he bought of Damaske 30 yardes that cost 360 s̄, then had he but 10 yardes of veluet, which cost 200 s̄: and now the price of Damaske is greater than the price of the Veluet by 160 shillinges, and should be but 48, therefore is the second error 112 too much, which I set in forme of figure as heere doeth appeare.

Then doe I multiplie in crosse waies 208 by 30, and the summe will be 6240. Also I multiplie 112 by 20, and there will amount 2240. And in as much as the signes of the errors be vnlike, I knowe I must worke by Addition, therfore adde I those two totals together, and they make 8480. which is the diuidend: then adde I also the two errors together, 208 and 112, and they make 320, which is the diuisor. Where-

20 30

208 — 112 +

fore

foꝛe diuiding 8480, by 320, the quotient will be 26 ½, which is the true ſumme of yardes of Damaſke that he bought: and in Ueluet 13 yardes ½, and that appeared by examination thus: 26½ yardes of Damaſke at 12 s̄ the yard maketh 318 s̄, then in veluet he had but 13 yardes and ½ that coſt 270 s̄, at 20 s̄ the yard. Now ſubtract 270 out of 318, and there will remaine 48, which is the number of ſhillings that the Damaſke did coſt moꝛe than the Ueluet.

A queſtion of debt. The third example.

Maſter. Now ſhall you haue a queſtion of an other kind.

There are thꝛee men that doe owe money to me, and I haue foꝛgotten what the totall ſumme is, and what the particulars be.

Scholer. Why? then it is impoſſible to know the debt.

Maſter. Peace, ye are too haſtie: there is moꝛe helpe in it than you yet ſee: I haue thꝛee ſeuerall notes, whereby it appeareth that I did conferre their debtes togither, and found the debt of the firſt and the ſecond to amount to 47 l̄, the debt of the firſt man and the third man did make 71 l̄, and the ſecond man his debte with the third, did riſe to 88 l̄. Now can you tell what euerie man did owe, and what was the whole totall?

Scholer. Nay in good faith: but as I perceiue that it muſt be found by coniecture, ſo will I geſſe at it, ſuppoſing that the firſt man

did

did owe 20 l, & the second man 30, & the third.

Master. May stay there, for you are too far gone already, you may not suppose a seuerall summe for euerie man, for it is inough to suppose one summe for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to be 20 l, the second man cannot owe 30 l, for the declaration is, that their debts added togither did make 47 l, so must the second man his debt be but 27 l. Now this second debt with the third must make 88, therefore subtract 27 out of 88, and there will remaine 61, as the third man his debt. Then saith the declaration, that the first and third mens debts doe make 71: but by this supposition they make 81 that is 10 too much: which I must set for the first error. Now worke you the second position.

Scholer. I suppose the first mans debt to be 24 l, then must the second mans debt (by your declaration) be but 23 l, seeing both they make but 47 l. And the second man his debt with the third, doe make 88 l, and the second man oweth but 23, therefore the third man must owe 65 l. Now the third mans debt with the first, should make by the declaration 71 l, and they doe make 89 l: that is 18 l too much: and that is the second error, which I set downe with the first, and their position in this forme, and then doe I multi-

plie

plie in crosse wayes 20 by 18, and it is 360. Also 10 by 24 maketh 240. And because ỹ signes of the errors be like, I must worke by Subtraction: therfore I subtract 240 out of 360, and there resteth 120, which is the diuidend: then do I subtract 10 out of 18 by the same reason, and so is the diuisor 8, which is found 15 times in 120, therefore I say that the first man did owe 15 ℔, and then the second man must owe 32 ℔, for those 2 doe make 47 ℔, and the third man his debt is 56, for so much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

20 24
104 184

A question of Friendship. The third question.

Master. For the fourth example take this easie question for the varietie in worke. Two men hauing seuerall summes which I know not, do thus talke togither: the first saith to the second, If you giue me 2 s̄ of your money, then shall I haue 3 times so much money as you: the second answereth, It were more reason, that our summes were made equall, and so will it be, if you giue me 3 s̄ of your money. Now gesse what each of them had.

Note.

Scholer. I imagine that the first had 9 s̄.

Master. Consider euermore in your imaginations that you take a likely summe, as in this question take such a summe that hauing 2 added vnto it, may be diuided into 3 partes euen.

Scholer.

Scholer. Why? I remember you saide before, it forced not how fondly soeuer I gessed.

Master. As for the possibilitie of the solution it is truth, but for easines in worke, the aptest numbers are most conuenient.

Scholer. I thought no lesse, and therefore I tooke 9 as an apt number to be parted into three: but I perceiue I should haue considered the aptnesse of that partition after the addition of two vnto it, and then 7 had beene more meeter.

Master. That is truth, and then should the second man his summe be 5: for although he haue now but the third part of 9, that is 3, yet you must remember that he lent the first man 2, and so had he 5.

Scholer. Then to go forward: if the second man had three of the first man, then should he haue 8, and the first man but 4, so hath he double to the first man: yet he sayd in the question they should haue equall: wherefore it appeareth that he hath foure too much. Therefore I note that errour with his supposition, and gesse agayne that hee hath 10 s̄: whereunto I adde 2 shillings borrowed of the second man, and then hath he 12 shillinges, so the second man hath remayning but foure: whereunto I adde the 2 that hee lent to the first man, so had he but 6 s̄ at the beginning. Then take 3 shillings from the first man, and

giue

giue to the second, then hath the first man but 7, and the second hath 9, which are not equall, but there are two too many, wherefore I set downe both the positions with their errors, as here you see, and multiplie a crosse, so commeth there 40 and 14: and because the signes be like, I take 14 out of 40, and so resteth 26 to be the diuidend, then likewise I take 2 out of 4, and there resteth 2, by which I diuide 26, and the quotient will be 13, which is the summe that the first man had. And so appeareth that two being added thereto, the summe will be 15, so hath the second man now but 5, and before he had 7: then take three from the first, and put to his seuen, so haue each of them 10, and that is equall, as the question would.

7 10

4+ 2+

A question of Lambes. The fift example.

Master. For the fift example take this question. One man said to another: I thinke you had this yeare two thousand Lambes: so had I said the other: but what with paying the tithe of them, and then three seuerall losses, they are much abated: for at one time I lost halfe as many as I haue now left: and at an other time the third part of so many: and the third time $\frac{1}{4}$ so many. Now gesse you how many are left.

Scholer. Because here is mention made of certaine partes, I must take a number that may

may haue all those parts: that is to say, $\frac{1}{2}$, $\frac{1}{3}$ & $\frac{1}{4}$, which will be 24, howbeit 12 hath the same partes. Therefore first I take 12 to be the number that doth remaine, so hath he lost 6, 4, and 3, that is 13, and in the whole 25, but it should be 2000.

Master. Ye are deceiued yet still: you haue forgotten the 10 part, which must be defalked, that is 200, so there remaineth but 1800 and now go on againe.

Scholer. Then to find the errour, I take 25 out of 1800, and there remaineth 1775 too few, which I set for the first errour. Then for the second position I take 24, whose halfe is 12, the third parte 8, and the quarter 6, whereby riseth 50, which is too little by 1750, therefore I set downe both the positions with their errours thus.

12 24

1775 — 1750 +

And multiplie in crosse wayes 1775 by 24, whereof commeth 42600. Also I multiplie 1750 by 12, and there ariseth 21000. And because the signes are like, I do subtract the one from the other, and so remaineth the diuidend 21600: then doe I subtract 1750 out of 1775, and there resteth 25: by which I diuide 21600, and the quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added

added together will make 1800. And if you adde thereto the tenth which was abated before, then will the whole summe be 2000. And now doth there come a question to my memorie which was demanded of me, but I was not able to answere to it, and now me thinketh I could solue it.

864	
432	
288	
216	
1800	

Master. Propone your question.

A question of sheepe and tillage. The sixth example.

Scholer. There is supposed a law made, that (for furthering of tillage) euery man that doth keepe sheepe, shall for euery ten sheepe eare and sow one acre of ground: and for his allowance in sheepe pasture, there is appointed for euery foure sheepe one acre of pasture. Now is there a rich sheepemaster which hath 7000 acres of ground, and would gladly keepe as many sheepe as hee might by that Statute. I demaund how many sheepe shall he keepe.

Master. Answere to the question your selfe.

Scholer. First I suppose he may keepe 500 sheepe, and for them he shall haue in pasture after the rate of foure sheepe to an acre, 125 acres, and in arable ground 50 acres, that is 175 in all: but this errour is too litle by 6825. Therefore I gesse agayne, that he may keepe 1000 sheepe, that is in pasture 250 acres: and and in tillage 100 acres, which maketh 350: that is too little by 6650.

500 1000



6825— 6650—

These both errours with their poſitions I ſet downe as you ſee, and Multiply in croſſe 6825 by 1000, and it maketh 6825000. Then I multiplie 6650 by 500, and it doeth amount to 3325000, which ſumme I doe ſubtract out of the firſte, and there remaineth 3500000 as the diuidend. Alſo I doe ſubtract the leſſer errour out of the greater, and ſo remaineth 175, by which I diuide the ſayd diuidend, and the quotient will be 20000, ſo that I ſee that by this rate he that hath 7000 acres of ground, may keepe 20000 ſheepe: & thereby I coniecture, that many men may keepe ſo many ſheepe, for many men (as the common talke is) haue ſo many acres of ground.

Maſter. That talke is not likely, for ſo much ground is in compaſſe aboue 48 ¾ miles: Leaue this talke and returne to your queſtiõs, leaſt your pointing be ſcarce well taken.

Scholer. In deed I do remember, that the Egyptians did grudge ſo much againſt ſhepheardes, till at length they ſmarted for it, and yet they were but ſmall ſheepemaſters to ſome men that be now, and the ſheepe are waxen ſo fierce now and ſo mighty, that none can withſtand them but the Lion.

Maſter. I perceiue you talke as you heare

An other way of working, vpon the fixth example.

some other: but to the other worke of your question: both this last question, and the next before might be wrought without the second position, by the Rule of proportion, as this. When in this question ye found in the first errour, that for 500 sheepe, there must be 175 acres, then might you reduce it to the golden rule, thus.

175 — 500
7000 — 20000

If 175 Acres will admit in allowance 500 sheepe, then 7000 will haue 20000. And so by one position with the helpe of the Golden Rule may you answere that question. Likewise for the question of Lambes, when you had found that 12 came of 25, you might haue set the figure thus as you see & haue said:

25 — 12
1800 — 864

If 25 doe leaue but 12, what shall 1800 leaue? and it would appeare to be 864.

Scholer. Sir, I thanke you for this aide, for it doth much shorten the worke of this Rule.

An other way yet of working vpon the fixth question.

Master. Yet againe I will shew you an other way, to answere to this last question without this Rule of False position, and that by the Rule of Felowship, for it appeareth in the proponing of the question, that 10 sheepe must haue in pasture two acres and ½, and for them must there be eared but one acre: so it fol-

followeth, that for two acres eared, there must be 5 set to pasture. And if you put them both into one summe, they will make 7. Therefore looke what proportion 7 being this totall, doth beare to 5 and to 2, such proportions shall any totall in this question beare to the pasture ground, and the eared ground.

Scholer. This serueth wonderous aptly. Therefore to proue it, I demaund this by the former supposition: If a man haue 300 Acres, how much shall he leaue in pasture, and how much shall he turne to tillage? You say that as 7 is to 5, so shall 300 be to the Acres of pasture: and as 7 is to 2, so is 300 to the acres of tillage, whereof for both I haue set examples here following, whereby appeareth that of Pasture there shall be $214\frac{2}{7}$ Acres, and of Tillage $85\frac{5}{7}$, which both summes added together, do make 300.





Master. Now take an other example: A man hath three siluer cuppes with one couer, the couer wayeth 18 ounces, the second cuppe wayeth euen halfe the waight of the first and the third. Now if the couer be put to the first cuppe, they waigh iust as much as all the three cups doe waigh: and if the couer be ioyned with the second cup, they waigh as much as the second twise, and the third: and if the

Another question, of the proportiō of the waight of 3 cups, the 7 example.

couer be put to the third cup, they will make twise as much as the first and the second cup. Now trie you what was the iust waight of euery cup.

Scholer. I doe set the weight of the first cup to be 9 ounces: then in as much as these two (that is to say, the couer and the first cup) do waigh the waight of the three cuppes, I see that the three cups must weigh 27 ounces, for so much is 18 and 9. Also because the first and the third doe waigh double so much as the second, therefore is it the third parte of that waight, that is 9, and then would it follow, that the third cup also should waigh 9 ounces, but then the question saith, that the couer being ioyned to the second cup, they waigh as much as the second twise, and the third once, that should bee 27, and so it doth: then beyng ioyned with the third cup, they should waigh twise as much as the first and the second, that should be 36, and they waigh but 27, so is that errour 9 too little. Then begin I againe, and say, that the first cup doth waigh 12 ounces, which I ioyne with the couer, and they make 30 ounces: then seeing the second is $\frac{1}{3}$ of that waight, it must needes waigh 10 ounces, & the third must waigh 8 ounces, seeing the first and the third must waigh 20 ounces. Now put I the couer to the second cup, and they waigh 28 ounces, which should be euen so: then ioyne I the couer with the third cup, and so should

it

it wey twice the first, & the second, that is 44 ounces, and they doe wey but 26, that is 18 too litle: those errours with their positions I set downe, and multiplie in crosse wayes 9 by 12, whereof commeth 108. Also 9 by 18, and that yeeldeth 162: and in as much as the signes be like, I abate the lesser out of the greater, and there doth remaine 54. Then doe I also abate the lesser errour from the greater, and so remayneth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the first cup: which beyng ioyned with the couer must wey as much as the three cups, so do they wey but 24 ounces. Then seeing the second cup is the third parte of that weight, for the other two cuppes (you say) must wey double his weight, the weight of the second cuppe is 8 ounces, and so the weight of the third must be 10 ounces. Now put the couer to the second cuppe, and it will make 26 ounces: that must be the weight of the second twise, and the third ounce, that is twise 8, and once 10, and so it is. Againe, put the couer to the third cuppe of 10 ounces, and they must wey twice as much as the first and the second, that is 28: and so is all agreeable.

9 12
9— 18—

Master. Then answere to this question.

A question of water. The eight example.

There is a Cesterne with foure cockes,

contayning 72 barrels of water: and if the greatest cocke be opened, the water will auoid cleane in 6 houres: at the second cocke it will aske eight houres: at the third cocke it will auoide in no lesse than nine houres: and at the smallest it will require twelue houres: Now I demaund, in what space will it auoid, all the cockes being set open?

Scholer. First I imagine that it will auoid in two houres.

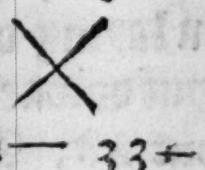
Master. Then must there auoide by the first cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the second cocke $\frac{1}{4}$ that is 18, and by the third cocke $\frac{2}{9}$ that is 16 barrels, and by the smallest cocke $\frac{1}{6}$, and this 12 barrels, all which summes put togither do make 70, as by their addition it doth appeare, but it should be 72, therefore the error is 2 too few.

Scholer. Then I begin againe by your fauour, because I thinke I vnderstand the worke, and put three houres for the due time: so shall there run out at the greatest cocke $\frac{1}{2}$, that is 36 barrels, and at the second hole $\frac{3}{8}$, that is 27, and at the third cocke $\frac{1}{3}$ that is 24, and at the smallest hole $\frac{1}{4}$, that is 18 barrels, which altogither doe make 105, and should be but 72, so is it too much by 33, therefore do I set the errors in order of the figure with their positions, and worke by multiplication,

24	
18	
16	
12	
70	

in

in crosse, saying: 2 times 3 is 6, and 2 times 33 maketh 66: and because the signes are vnlike, I must adde those two totals togither, which make 72: also I adde the two errors, and they make 35, by which I diuide 72, and the quotient riseth $2\frac{2}{5}$, whereby I see that all the cockes being set open, the water will auoid in two houres and $\frac{2}{5}$, of an houre.



Master. This exercise maketh you to grow expert in the rule. Therefore I will inure you somewhat more with a question or two.

A question of partners. The ninth example.

There were two men that had beene partners, and had in account betweene them 300 duckets: whereof the one should haue for his part 180, and the other 120: but in the parting of them they fell at variance, so that each of them catched as many as he could: yet afterward being reconciled, they agreed that hee which had gotten most part of them, should lay downe $\frac{1}{4}$ of them againe, and he that had gotten least, should lay downe $\frac{1}{3}$ of those which he had taken, and then parting them into two equall partes, each man to haue halfe thereof, and so had they their iust portions, as they ought: now I demaund of you what each of them had gotten by the skambling.

Scholer. I suppose he that had least, got

108 buckets, then the other had 192: wherefore in laying downe againe of the 192, there was put downe $\frac{3}{4}$ that is 144, and so had hee left but 48. Also of the 108: there was laid downe 36, that is $\frac{1}{3}$, and so he hath left 72. Then I put togither 144, and 36, and it maketh 180 which I part into two partes euen, and so commeth 90 to be giuen to ech of them: which sum put to 72, maketh 162, and ioyned to 48, it maketh 138: and now I doubt how I shall go forward.

Note. Master. You neede not to take but one of them which you list, the greater or the smaller, for all commeth to one purpose: and so may you compare it that you take to any of the other summes, remembring that you make comparison to the same in the second worke, as for example of the first parte, If you compare 138 with the lesser summe due, that is 120, so is it 18 too much: and if you compare it with the greater summe, then is it 42 too little. Againe, if you compare 162 to the greater summe, the error will be 18, as it was in the other: but it will haue a contrarie signe: and if you compare it with the lesser summe, it will be 42 too much: so that the error both waies is either 18 or 42: and as for the signes it litle forceth, for in them is nothing considered here, but likenesse and vnlikenesse, which in this case doth neyther further nor hinder. But now go on with the worke.

Scholer,

Scholer. If it be so, then am I out of my greatest doubt. Then I ioyne that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and so hath hee 138, which (I may say) is 18 too many, for the least should be but 120: that errour do I note, and then make a new position, supposing the one man to haue 204, and the other to haue 96, wherefore of the 204 there must be laid downe 153, and so remaineth with him 51. Also of the 96 there must bee layd downe $\frac{1}{3}$, that is 32, and so resteth with that man 64. Now of the 153 and 32 I make one summe as 185, which I must diuide into two equall partes, and so each man shall haue 92 $\frac{1}{2}$, whereunto if I adde their former portions reserued, then the one shall haue 156 $\frac{1}{2}$, and the other hath 143 $\frac{1}{2}$. Wherefore I take the lesser summe now againe, as I did before, that is 143 $\frac{1}{2}$, and finde that he hath too many by 23 $\frac{1}{2}$, for he should haue but 120, so haue I for my two positions two errors, which I set downe, as here may be seene, each errour vnder his position, and then by the Rule I doe multiply in crosse waies 108 by 23 $\frac{1}{2}$ and there riseth 2538 which I note, then againe I multiplie 96 by 18, and thereof amounteth 1728.

108	96
18+	23 $\frac{1}{2}$+

Now

Now because the signes are both like, that is both too many, I must worke by Subtraction, and so abating 1728 out of 2538, there will rest for the deuidend 810: then for the diuisor I subtract 18 out of 23 ½, and there remaineth 5 ½, by which I diuide 810, and the quotient will be 147 $\frac{3}{11}$, which is the iust portion of him that had the least summe. And if I do subtract it out of 300 being the totall sum, then will there remaine 152 $\frac{8}{11}$ as the portion that the other did get.

Master. For the proofe of this worke, you may choose whether you will examine those numbers according to the forme of the question, or else worke by other two positions for to finde the second number: and if those positions bring the same numbers that did amount by the first two positions, then doth each worke cõfirme other.

Scholer. By your patience, I will proue both wayes, not onely to see their agreement, but also to accustome my mind to those workes: for I perceiue it is exercise that must be the chiefe engrauer of these rules in my memorie.

Master. You consider it well: then go to.

Scholer. First I will by two other positions trie to finde the portion of him which had most.

Master. Although you may do it with any positions, yet to see the agreement of your worke

worke the better, take the same positions that you did before, comparing them now to the greater, as you did before vnto the lesser.

Scholer. Then I suppose, that he that had most, had 192, so had the other 108. Now if I take $\frac{3}{4}$ out of 192, that wil be 144, and there will rest to that man but 48. And from the second which had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof beyng 90, if I adde to each of those two mennes portions remayning with them, the one shall haue 138, and the other 162, of which two I take the greater (that is 162) and see it to be 18 too few, for it should be 180, that erroz I note vnder his position. Then for the second position I take (as I did before) 204 for the one, and so resteth 96 for the other: then take I $\frac{3}{4}$ of 204, and it will be 153, and there resteth to him 51. Also of the 96 I take $\frac{1}{3}$ that is 32, and there remaineth to him 64. Now put I that 32 to 153, and it yeeldeth 185: which being parted in equall values, maketh $92\frac{1}{2}$ to be added to each mans remainder, and so the one hath $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greatest summe, and it is $23\frac{1}{2}$ too little, that do I note also, and set both these errors vnder their positions, as in this example following doth appeare.

And then multiplying 192 by $23\frac{1}{2}$, there doth

doth arise 4512.

Againe, I multiply 204 by 18, and it maketh 3672, which I do subtract out of 4512,



because the signes be like, and there resteth 840 for the diuidend: then subtracting 18 out of 23 ½, there will remayne 5 ½, which I must take for the diuisor. And so diuiding 840 by 5 ½, the quotient will be 152 8/11, whereby I haue found an agreeable sum to that which I found by the former positions, for him that had most, which if I doe subtract out of 300, that is the totall, there will rest 147 3/11, which was the portion of him that had the least part.

Master. So by diuerse positions you see, that one doth confirme the worke of the other. Now examine those two nũbers by the forme of the question, and so shall you proue your worke good also.

Scholer. If that he which gate most, had 152 8/11, then must he lay downe 3/4 of his summe, that is 114 6/11, and so shall remaine with him but onely 38 2/11. The other which had least, that is 147 3/11 must put downe of his sum 1/3, that is 49 1/11, and so doth there remaine with him yet 98 2/11. Then do I adde togither 114 6/11 and 49 1/11, and it will make 163 7/11, which I must part into two equall partes, and that will be 81 9/11 to be giuen to each of them: so putting 81 9/11 vnto 38 2/11, there doth amount 120 iust, which

which is the true portion of him that should haue the lesser summe: and adding 81 $\frac{2}{11}$ to 98 $\frac{2}{11}$, the totall will be 180, the true portion of the other. And so is the worke by this proofe also tried to be good. And this I marke by the way, that in their scambling, he gate most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your studie be to learne truth and iust Arte of Proportion, and to distribute and part according thereunto, as often as occasion shal be ministred. And here would I make an end of this rule, saue that I remember one pleasant question which I can not ouerpasse, which I will declare somewhat largely, because you shall as well vnderstand some reason in the pleasant inuention, as apt proceeding in the wittie working thereof.

An exãple of mixture of gold and siluer. The tenth example.

Hiero king of the Syracusans in Sicilia, had caused to be made a crowne of Gold of a wonderfull waight, to be offered for his good successe in warres: in making whereof, the Goldsmith fraudulently tooke out a certaine portiõ of Gold, and put in Siluer for it, so that there was nothing abated of the full waight, although there was much of the value diminished. Which thing at length being vttered, (as no euill can alwaies lie hid) the king was sore mooued, and beyng desirous to trie the trueth without breaking of the Crowne, proponed the doubt to Archimedes, vnto whose wit

wit nothing seemed vnpossible, which although presently hee could not answere vnto, yet he had good hope to diuise some policie for that inuention. And so musing thereon, as he chanced to enter into a baine full of water to washe him, he obserued that as his body entred into the baine, the water did runne ouer the tubbe: whereby his ready wit of such small effectes coniecturing greater workes, conceiued by and by a reason of solution to the Kings question, and therfore reioycing exceedingly more than if he had gotten the Crowne it selfe, forgate that he was naked, and so ran home, crying as he ran, εὕρηκα, εὕρηκα, I haue found, I haue found. And thereupon caused two massie peeces, one of Gold, and an other of Siluer to be prepared of the same waight that the said Crowne was of: and considering that Gold is heauier of nature than Siluer, and therefore Gold of like waight with Siluer must needes occupie lesse rowme, by reason it is more compact and sound in substance, hee was assured, that putting the masse of Gold into a vessell brimme full of water, there would not so much water runne ouer, as when he should put in the siluer masse of the like waight. Wherefore he tried both, and noted not onely the quantities of the water at each time, but also the difference or excesse of the one aboue the other, whereby he learned what proportion in quantitie is betweene Gold and Siluer of equall

waight.

waight. And then putting the Crowne it selfe into the vessell of water brimme full (as before) marked how much water did runne out then, and comparing it with the water that ranne out when the Golde was put in, noted how much it did exceede that: and likewise comparing it to the water that ran out of the Siluer, marked how much it was lesse than that: and by those proportions found out the iust quantitie of Gold that was taken out of the crowne, and how much Siluer was put in stead of it. But seeing Vitruuius which writeth this Historie, doth not declare the particular worke of this triall, it shall be no inconuenience to suppose an example for declarations sake, wherein although the true and iust proportions be not expressed, yet the forme of triall shall be truly set foorth. And for an example, I suppose the waight of the Crowne to be 8 pound, and so of each the other two masses. And when the masse of gold was put into the water, I imagine that there ran out 2 pound of water: and when the masse of siluer was put in, I suppose there ran out three pound $\frac{1}{2}$. Againe when the crowne was put in, there ran out 2 pound $\frac{1}{4}$. Now to know what quantitie of siluer was in the Crowne, worke by the rule of false position, and imagine that there was 2 pound of Siluer: then must there be sixe pound of Gold, then say thus by the rule of proportion: If 8 pound of gold do expel 2 pound of water, what

shall

shall 6 pound expell ? and it will be 1 pound $\frac{1}{2}$. Againe for the siluer: If 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 2 pound of siluer put out? it will be $\frac{7}{8}$. Now adde those two waights of water togither, and they will make 2 pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{1}{4}$, so is it too much by $\frac{1}{8}$.

Scholer. Now do I vnderstand the worke as I thinke, therefore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position, and his errour, and take a new position, esteeming the siluer to be but one pound, so must there be in Gold 7 pound. Then say I: If 8 pound of Gold doe yeeld two pound of water, what shal 7 pound yeld? and it will be 1 pound $\frac{3}{4}$. Againe if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall one pound expell? and it will be $\frac{7}{16}$. Now must I adde those two summes together, and they make 2 pound $\frac{3}{16}$, and they should make 2 pound $\frac{1}{4}$, so is it too litle by $\frac{1}{16}$. Therefore I set the positions with their errours in order, as here followeth. And then I multiply in crosse wise two by $\frac{1}{16}$, and it maketh $\frac{1}{8}$: likewise one multiplied by $\frac{1}{8}$: maketh $\frac{1}{8}$. And because the signes be vnlike, I must adde those two summes which make $\frac{1}{4}$, and that is the diuidend. Againe I must adde $\frac{1}{8}$ to $\frac{1}{16}$, and it will be

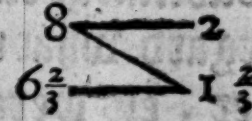


be $1\frac{3}{6}$ that is the Diuisor. Now I shall diuide $\frac{1}{4}$ by $1\frac{3}{6}$, and the quotient will be $\frac{1}{12}$, that is $1\frac{5}{3}$ wherby I know that there was put 1 ℔ and $\frac{1}{3}$ of siluer into the crowne, and so much gold taken out for it.

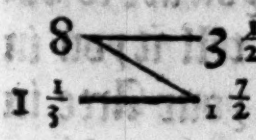
Master. Proue it now by examination according to the question.

Scholer. If there were 1 pound $\frac{1}{3}$ of siluer, then was there of Gold 6 pound $\frac{2}{3}$. Now say I by the Rule of proportion:

If eight pound of Gold expell two pound of water, what shall 6 pound $\frac{2}{3}$ expell?



It will be 1 pound $\frac{2}{3}$. Again If eight pound of siluer expell three pound $\frac{1}{2}$ of water, what shall $1\frac{1}{3}$ expell? It will be $1\frac{7}{12}$.



Now must I adde together 1 pound $\frac{2}{3}$, and $1\frac{7}{12}$ and they will make 2 pound $3\frac{9}{36}$, that is two pound $\frac{1}{4}$, according to the supposition of the question, whereby I perceiue the worke to be well done. And as I cannot but much reioyce of this excellent inuention, so my desire is kindled vehemently to bee perfectly instructed in euery part thereof, & namely in this point, whether the portion betwene water and gold be such, that for 8 pound of golde into a vessell full of water there shall runne out two pound of water: & for as much siluer, whether three pound $\frac{1}{2}$ of water would auoid?

Master. I perceyue your meaning, and

coniecture your imagination to be thus: that if you knewe the exact proportion betweene golde and siluer & water, both in their waighte and quantities, then could you easily finde out the mixtures of them, which thing I haue reserued for another worke that intreateth of such matters especially. And at this time you must consider, that you learne Arithmetike, which entreateth of the maner to solue doubtfull questions touching number, without regard what matter is signified by that number, els were it necessarie in Arithmetike to teach all Artes, seeing in it may bee mooued questions of all Arts. But seeing you are so desirous to know this thing, I will tell it you in such a sort, that you shall practise your Arte in finding it, & propound it in forme of a question. Golde beareth greater proportion to water than siluer doth, & their two proportions be in proportion togither as $\frac{48}{25}$. But to helpe you somewhat in this riddle, you shall note that the proportion of quicksiluer vnto water, is the iust middle nūber proportional in Progression Geometricall, betweene the proportions of Gold and siluer vnto water. And his proportion is $\frac{240}{21}$. Now if you will know the iust nūbers of these two proportions, then must you find out 3 numbers in Progression Geomitricall, whereof the middlemost must be $\frac{220}{21}$, and the first must be vnto the last, as 25 to 48. And thus I will leaue you to finde those numbers

Note,

1 question of the proportion of gold, siluer and quicksiluer vnto water.

when

when you be at leysure.

Scholer. Yet sir, I thanke you hartily for thus much, for now I see the possibilitie to find them out. Howbeit, because this question seemeth straunge, if it might please you to instruct mee somewhat in the order of working for it, I should the more easily finde the true working:

Master. You desire too much ease if you will studie for nothing: therefore to occasion you to studie the better, I will leaue this doubt wholy to your owne search. But as touching the generalitie of the rule, Archimedes needed not to take two masses of golde and siluer equall in weight with the crowne, for the proportion might as well bee found in any other waight, yea although the masse of golde were of one waight, and the masse of siluer of another. As for example: If the crowne were of 8 pound waight, as I did suppose, and I haue not so much other fine Golde, but onely one pound, and trying that by water, and finding that it doth expell but ¾ of an ounce of water, yet then by it may I inferre, that 8 pound of Golde would expell 6 ounces of water. And likewise of the siluer: whereof if I had but two pound, and finde that it doth expell three ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is one pound waight: and so is it as good as if the 3 masses were all of one waight. And thus

for this time I will make an end of this other part of Arithmetike.

Scholer. Although I cannot sufficiently thanke you for this, yet your promise made me to looke for the Art of extraction of rootes, wherof hitherto I haue learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within these three or foure monethes, if I perceyue this my paines to bee well taken in the meane season. And you shall not repent the tarrying for it: for it shall be increased by the tarrying. And in the meane time, you shall take this Addition, not for the second part of Arithmetike which I promised, but for an augmentation of the first part, vnto which I would haue annexed the extraction of rootes, square & cubike, namely for examples of the Statute of Assise of wood, but that in the second part I must write of diuers other roots, and thought it best to reserue those Rules also with their examples vnto the same second part.

Scholer. Sir, although I cannot recompence your goodnes, yet I shall alwayes doe mine endeuour to occasion you not to repent your benefite on me thus employed.

Master. That recompence is sufficient for your part.

FINIS.

The third part

or Addition to this Booke entreateth of briefe Rules, called Rules of Practise, of rare Pleasant and *commodious effect, abridged into* a briefer Method than hitherto hath bene published.

With diuerse other necessarie Rules, Tables, and Questions not onely profitable for Marchants, but also *for Gentlemen, and all other Occupiers* whatsoeuer, as by the Contents of this Booke may appeare

Set forth by *Iohn Mellis* Schoolemaster.

The first Chapter of this Addition entreateth of briefe Rules, called rules of PRACTISE, *with diuerse necessarie Questions, profitable, not onely for Marchants, but also for all other Occupiers whatsoeuer.*

1. Rule.



The working of Multiplicatiõ in Practise, is no other thing than a certain maner of multiplying of one kind by another: whereupon is brought foorth the product of the proponed number, which is accomplished by the meanes of Diuision in taking the halfe, the third, the fourth, the fift, or such other parts of the summe which is to be multiplied.

And for the better vnderstanding of such conuersions: you shall vnderstand that in the maner and vse of these rules of Practise, you ought first to know the euen or aliquot partes of a shilling, which in this Table following doth appeare.

Item d { 6, 4, 3, 2, 1 } in the { 1/2, 1/3, 1/4, 1/5, 1/2 } of a s.

Where-

Wherein as you see according to the order of these rules of Practise at 6 d the yard of any thing, you must take the ½ of your number which is to be multiplied, and the product that commeth thereof shalbe shillings, if any vnite doe remaine it is 6 d.

For 4 d take the ⅓ of the number that is to be multiplied, and the product also produceth shillings, if any vnites doe remaine, each one shall be worth in value 4 pence. The like is to be vnderstood of the other 3, &c.

Example I

At 6 pence the yard: what 379 yards?

189 s̄ — 6 d

II

At 4 d the yarde: what 104 yards?

34 s̄ — 8 d

III

At 3 pence the yard: what 5014 yards?

1253 s̄ — 6 d

IIII

At 2 d the yarde: what 532 yards?

88 s̄ — 8 d

V

At 1 penie the yard: what 409 yards?

34 s̄ — 1 d

Here you may see in the first example that 379 yards at 6 pence the yard, are worth 189 s̄ 6 d, in taking the ½ of 379. And in the second example the 104 yards at 4 pence the yard, are

worth 34 s̄ 8 d̄ in taking ỹ ⅓ of 104. Likewise in the third example 5014 yardes at 3 d̄ the yard bringeth forth 1253 s̄ 6 d̄ in taking the ¼ of 5014. Also in the fourth example at 2 d̄ the yard maketh 88 s̄ 8 d̄.

And lastly in the 5 example: 409 yardes at 1 d̄ the yard, amounteth to 34 s̄, and 1 d̄, in taking ỹ ½ of 409: And so is to be done of al other questions ỹ like, when the number of the pence is any of the euē or aliquot parts of 12 d̄

Item to bring the products of these shillings & all other the like into pounds is verie easie in diuiding of it into your minde by 20, for it is to be vnderstoode that as often as 20 is found in that product, so many pounds doth it containe, which with facilitie to performe, alwayes strike off the figure towarde your right hande, with a right downe dash of your pen for the 0 that appertaineth to 20. And then begin at the left hand, in taking the ½ of the rest. And if at the last any vnite doe remaine the same shall be ioyned with the figure that is cut off, which shall represent the odde shillinges contained in that worke.

As for example in your third question at 3 d̄ the yard which amounteth to 1253 s̄ 6 pence the product wherof maketh 62 l̄ 13 s̄ 6 d̄: as here you may see is easily performed by this example.

1
125|3
62—13—6

Item also for the working of 1 penny the

yard

yard, it is something harsh and hard to take the $\frac{1}{12}$ of some products: Therefore to ease that hard worke, you shall first bring your deliuered summe into groates, by taking the $\frac{1}{4}$ part of the product. And if any vnites remaine of that $\frac{1}{4}$ part, as sometimes there may, they are pence: and must bee signified with a line from the groats with their title of pence: And because that 60 groates maketh a pound or twentie shillings, strike off the first figure toward your right hand for the 0 that appertaineth to 60 (as you did euen now for the 0 that belongeth to 20:) then in taking the $\frac{1}{6}$ of that product, if there doe remaine any vnites, the same shall you ioyne with the figure that you cut off, esteeming thē as groats: which keepe in your minde. And by taking the $\frac{1}{5}$ part of them, you shall turne into shillings: and so haue you done: as for example by a question or two hereafter proponed shall more plainly by the worke appeare.

At 1 d the yard what 54368 yards.

	1359 | 2 groates.	
$\frac{1}{6}$ p	—226—	10 s — 8 d.

Here in taking the $\frac{1}{6}$ part of 1359: in comming to the last worke the $\frac{1}{6}$ part 39 being taken, the remainer is 3, which ioyned with the 2 that was cut off, maketh 32 groates: which conuerted into shillings by taking the $\frac{1}{5}$ part; maketh as appeareth 10 s 8 d.

8 d. Many other wayes there are, but none more apt for a young learner to vnderstande than this: wherefore this one way well empressed in memorie, is better than 20 wayes doubtfully vnderstood.

At 1 peny the yard, what 4533 yardes?

113	3 groates — 1 d

£ 18 — 17 — 9 d

At 1 peny the yard, what 64768 yards?

1619	2 groates.

£ — 269 — 17 — 4 d

2. Rule.

NOw followeth also to be vnderstood that if the number of pence bee not an aliquot part of 12, you must reduce them into some aliquot part of 12. And after the aforesayde manner, you shall make of them 2 or 3 products as neede shall require, and adde them togither into one summe. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

For pence	5. take	3 & 2	or	4 and 1
	7. take	4 & 3		6 and 1
	8. take	4 & 4		6 and 2
	9. take	6 & 3		4.4 & 1
	10. take	6 & 4		4.4 & 2
	11. take	6.4 & 1		4.4 & 3

Here in the first note of this Table at 5 d, you shall first take for 3 d the ¼ of the number

that

that is to be multiplied: and likewise for 2 d the $\frac{1}{6}$ of the same number, adding together both the products. But if you will worke by 4 and 1, you must for 4 d, first take the $\frac{1}{3}$ of the number that is to be multiplied. And for 1 d take the $\frac{1}{2}$ of the whole sum, or rather which is more better, for 1 d, you may take the $\frac{1}{4}$ of the product which did come of the 4 pence: because that 1 d is the $\frac{1}{4}$ of 4 d: The total sums of these two numbers shall bee the solution to the question. And in like maner is to be done of all others, as by these examples following shall appeare.

I

At 5 pence the yard what 758 yards?

3 d	189	6 d
2 d	126	4 d
	315 s	10 d

Otherwise,

At 5 pence the yard what 758 yards?

4 d	252	8 d
1 d	63	2 d
	315 s	10 d

II

At 7 pence the elle, what 563 elles?

4 d	187	8 d
3 d	140	9 d
	328 s	5 d

III

At 8 pence ŷ pound, what 112 pounds?

4 d	37	4 d
4 d	37	4 d
	74 s	8 d

Otherwise.

At 8 pence ỳ pound what 112 pounds?

6 d	56—0
2 d	18—8
	74 s̄—8 d̄

IIII

At 9 pence the elle what 356 elles?

6 d	178—0
3 d	89—0
	267 s̄—0 d̄

V

At 10 pence ỳ peece what 795 peeces?

6 d	297—6
4 d	265—0
	662 s̄—6 d̄

VI

At 11 d̄ the pound what 7579 pound?

6 d	3789—6
4 d	2526—4
1 d	631—7
	694 7 s̄—5 d̄
	℔ 347—7 s̄—5 d̄

Here in this first example where it is demaunded (at 5 d̄ the yarde) what will 758 cost: First for 3 d̄ I take the $\frac{1}{4}$ of 758: & thereof commeth 189 s̄—6 d̄: Then for 2 d̄ I take the $\frac{1}{6}$ of the same product which amounteth to 126 s̄ 4 d̄, these two summes added together, doe make 315 s̄, 10 d̄: And so much are the 758 yards worth at 5 d̄ the yard.

Item also for the same againe: First for 4 d̄ I take the $\frac{1}{3}$ of 758: and thereof commeth 252 s̄ 8 d̄: then for 1 penie I take the $\frac{1}{4}$ of the same product, that is to say, of 252 s̄—8 d̄, and it yeeldeth me 63 s̄ 2 d̄: which both added together

gether maketh 315 s̄—10 d̄, as before.

Item for 7 d̄ there is taken the $\frac{1}{3}$ and the $\frac{3}{4}$ of the whole summe: which is to bee multiplied, and adde them together, that is to say, first, for 4 d̄ there is taken the $\frac{1}{3}$ of 563: which comes to 187 s̄—8 d̄, as appeareth by the worke: and for 3 d̄ there is taken the $\frac{1}{4}$ of the whole summe which amounth to 140 s̄ 9 d̄, Both which products added together maketh 328 s̄—5 d̄: And so much comes 563 elles to at 7 pence the elle.

Item, for the first 8 d̄ there is taken for 4 d̄ the $\frac{1}{3}$ of the whole summe: and another $\frac{1}{3}$ for the other 4 d̄, which added togither as in the example doth evidently appeare, amounteth to 74 s̄—8 pence.

Againe, for the second worke of 112 l̄, there is taken first the $\frac{1}{2}$ of the whole summe for 6 d̄, which comes to 56 s̄: then for the 2 pence you haue to take $\frac{1}{6}$ of the whole summe, or if you will the $\frac{1}{3}$ of the product that came of 6 d̄, either, which maketh 18 s̄ 8 pence. These two summes being added togither doe make 74 s̄ 8 d̄, as in the third example appeareth.

Item, for 9 d̄ there is taken for 6 pence the $\frac{1}{2}$ of the whole summe: & the $\frac{1}{4}$ of the whole sum for 3 d̄, or otherwise for the 3 d̄ you may take the $\frac{1}{2}$ of the product that came of 6 d̄, because 3 pence is the $\frac{1}{2}$ of 6 pence: which added togither, as plainely appeareth in the fourth example, amounteth to 267 s̄—0 d̄.

Item,

Item, for 10 d, first there is taken for 6 pence the ½ of the whole summe, which amounteth to 397 s 6 pence. Thē for 4 d there is found 265 s: both which added togither maketh 662 s 6 pence, as appeareth in the 5 example. It may also be wrought, as appeareth by the secound note in the table by 4 pence twise taken, and the ½ of the product of 4 pence: or els by the ½ of the whole summe, &c.

Item, for 11 d, there is first taken the ½ for 6 pence: then the ⅓ of the whole summe for 4 d: lastly, the ¼ of ỹ last product for 1 d: All which 3 summes added together, maketh in s 6947, 5 d, ℔ in pounds 347—7 shillings—5 pence.

3. Rule. ITem likewise by the same reason, when you will multiplie (by shillings) any number that is vnder 20 shillings, you shall haue in the product pounds, if you know the euē or aliquot parts of 20, which are here in this little Table set downe to right.

Item s	10	is the	½
	5		¼
	4		⅕
	2		1/10
	1		1/20

of one ℔

So that for 10 s which is the ½ of a pound you may take the ½ of the number which is to be multiplied: and you shall haue in your pro-

duct

duct pounds: if a vnite doe remaine, it shall be worth 10 shillings.

Likewise for 5 shillings you must take the $\frac{1}{4}$ of the number which is to be multiplied: And if there doe remaine any vnites, they shall bee fourth parts of a pound, euerie vnite being in value 5 shillings.

For 4 shillings take the $\frac{1}{5}$ of the number which is to be multiplied: and if there doe remaine any vnites, they shall be fift parts of a pound, each vnite being worth 4 shillings.

For 2 shillings you must take the $\frac{1}{10}$ of the number to be multipled: wherefore to take the $\frac{1}{10}$ of any number you must cut of the last figure of the same nũber (which is nearest your right hand) from all the other figures with a small right downe line or dash with a pen, & so haue you done: for all the other figures which do remaine toward your left hand frõ the same figure that you do separate shall be the said $\frac{1}{10}$ of a pound: And that figure so separated towards your right hand, shalbe so many peeces of 2 shillings the peece: the which figure you must double to make thereof the true number, of shillings, as by the example shall appeare.

Finally, for 1 shilling needeth small worke, for it is so many shillings as bee proponed in the summe, which to bring into pounds hath beene alreadie taught in the first rule.

Example.

At 10 s the peece
What are —6543— peeces worth ?

½ £—3271—10 s

At 5 s the Elle
What comes —4373— Elles to ?

¼ £—1093—5 s

At 4 s the yard
What—7839 yards ?

⅕ £—1567—16 s

At 2 s the pound weight
What are—752|7—pounds weight worth ?

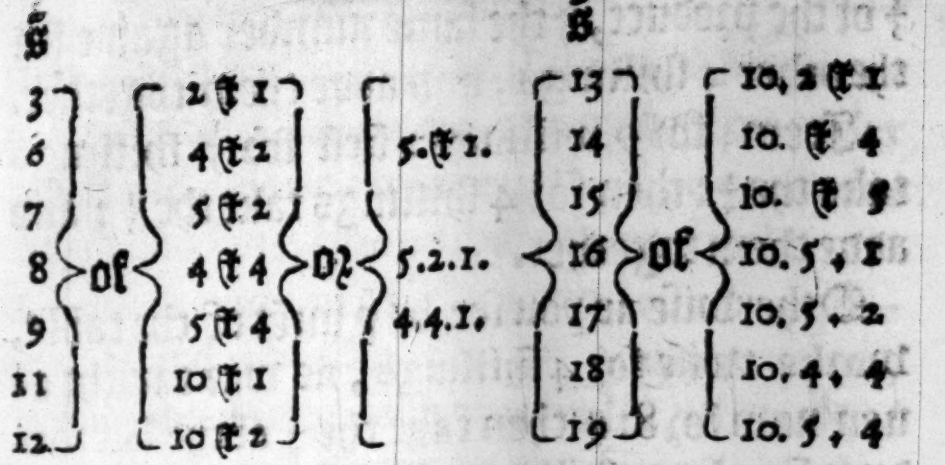
1/10 £—752—14 s

At 1 s the peece
What comes—757|3 peeces to ?

2/10 £—378—13 s

4. Rule. NExtly, now followeth in order to bee vnderstoode, that if the number of shillings be not some euen or aliquot part of 20, you must then conuert the same number of shillings into the aliquot parts of twentie, and thereof make two or three products, as neede shall require: which done, adde them together, and bring them into poundes. And here for thy furtherance I haue set downe a note of the order of their partes, as they are to be taken.

For



For 3 s̄ according to the tenor that you see is expressed in the Table, you must first take for 2 shillings the $\frac{1}{10}$ of the number that is to be multiplied: Then for 1 shilling you must take the $\frac{1}{2}$ of the product which did come of the same $\frac{1}{10}$ part, which two summes added together, produceth the effect desired.

Item, for 6 shillings according to the note set forth in the Table, first for 4 s̄ I take the $\frac{1}{5}$ of the number that is to be multiplied: Then for 2 s̄ the $\frac{1}{2}$ of the product that came of 4 s̄, & adde them together.

Or els, as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ part of the product that came of 5 shillings, and adde them together.

Item, for 7 s̄, first take for 5 s̄ the $\frac{1}{4}$ of the product, that is to be multiplied, then for 2 s̄, take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, &c.

Item for 8 s̄, according to reason, and the intent of the Table, for the first 4 s̄ take the

$\frac{1}{5}$ of the product, & the same number againe for the other 4 shillings, and adde them together.

Item, for 9 shillings: first for 5 shillings take the $\frac{1}{4}$: then for 4 shillings take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by ỹ intẽt of the table, worke twise for 4 shillings, as was taught euen now for 8: & then take the $\frac{1}{4}$ of the last product for the 1 shilling: But 5 & 4 is ỹ shorter.

Itẽ, for 11 s̃, first dispatch 10 s̃: for which you must take the $\frac{1}{2}$ of the product, then lastly for 1 shilling take the $\frac{1}{10}$ part of the summe produced of the $\frac{1}{2}$ of the product, & adde thẽ togither.

Item, for 12 s̃ where I will end with the first part of my Table: First take the $\frac{1}{2}$ for 10 shillings. And thẽ for 2 shillings take the $\frac{1}{5}$ of ỹ sum that came of 10 shillings, & adde them together: or els, if you please, for 2 shillings you may take the $\frac{1}{10}$ of the whole giuen number.

To write more of the maner of taking the true parts, I omit. The desirous practitioner will (no doubt) conceiue it. Also the Table is some aide to helpe the vnperfect, whereupõ by & by I will set down three or foure of these notes in examples, and the rest I will leaue to thine owne industrie and practise to labour vpon.

This is the order most commonly vsed in Practise whẽ the number of the shillings is not an aliquot part of a pound. But (louing Reader) after I haue touched the euen or aliquot partes of a ℔ that falleth out in

pence

pence and shillings, I will deliuer two new rules that shall drowne this common order quite and cleane: wherein shall be comprehended in one line and working both euen and odde parts of shillings vnder 20: without regard whether it be an aliquot or not an aliquot part: which two rules, when they come in place, I cõmit to thy friendly iudgment in working.

Now follow the examples vpon the notes before said.

At 6 shillings the yard

What	—3215—	yardes
4 shillings	643—	
2 shillings	321—10	

l—964—10 shillings.

Otherwise by Multiplication of 6.

	3215—
6 shillings	1929|0
	l—964.—10 shillings

At 7 shillings the Elle

What	—4563 Elles
5 shillings	1140.. 15
2 shillings	456.. 6

l—1597.. 1 shilling.

Dd ii Other-

Otherwise by multiplication of 7.

4563

7s 31941

1597.. 1s

At 8s the peece what 7563 peeces?

4s 1512..12
4s 1512..12

l—3025..4s

Otherwise by Multiplication.

7563

8s 60504

l—3025—4s

At 13s the peece what 401 peeces?

10s 200..10
2s 40.. 2
1s 20.. 1

l—260..13s

Otherwise by Multiplication.

401

13s 1203
~~401~~

5213

l::260—13s

These

These & such like Questions of compoũd numbers which I haue here in this fourth rule for orders sake set downe, for that it hath bin heretofore a common course of worke, I account but superfluous. For in the 8 and 9 Chapters of this my simple Additiõ shall appeare that the giuen price of any euen or odde number of shillings either vnder or aboue 20, shall be wrought at one or two workings at the most, how difficult soeuer the Question be.

ITem, there resteth yet a kinde of Practise, how to bring pence into pounds at the first working: whereupon you must vnderstand, that 240 pence maketh one pound, or 20 s. In consideration whereof I cut off the last figure or 0: and there remaineth but 24 (of which 24) 8 d is the $\frac{1}{3}$ part thereof: 6 d is the $\frac{1}{4}$ part, 4 d the $\frac{1}{6}$ part: and 2 pence is the $\frac{1}{12}$ part thereof. 5. Rule.

Whereupon if it were demaunded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in pricking or cutting off the first figure towards your right hand, for the 0 that appertaineth to 240: There is remaining of the said summe 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 l, and there resteth 1: which 1 I put to the 6, that I pricke or cut off, and it maketh 16 peeces of 8 pence, which I double to make into groates,

and they make 32, whereof the $\frac{1}{3}$ part maketh 10 s̄, and there remaineth $\frac{2}{3}$ s̄, which is 8 pence, whereby it followeth, that the 1486 yards at 8 pence the yard, maketh 49 l̄, 10 s̄, 8 d̄, as by the example shall appeare.

Item for 6 pence, take the $\frac{1}{4}$ part of the number from the prickt figure: And if any vnites do remaine, they are so many six pences, whereof taking the $\frac{1}{2}$ they are shillinges, if there doe remaine yet one, it is in value sixe pence.

Item, for 4 pence, take the $\frac{1}{6}$ part of the number from the prickt figure: If any vnites remayne, they are so many groates, which to conuert into shillings take the $\frac{1}{3}$ part: And if any thing yet remayne, they are thirds of shillings, each one in value beyng worth 4 pence.

Item for 3 pence, take the $\frac{1}{8}$ part from the prickt figure, if any vnites remaine, they are so many peeces of 3 pence whereof in taking the $\frac{1}{4}$ part, maketh shillings: If any thing yet remaine, they are fourth partes of shillings, each one being in value 3 pence.

Item, for 2 pence, as appeareth also by the table, take the $\frac{1}{12}$ part of the number from the prickt figure: If any thing remaine, they are so many peeces of 2 pence, which by taking the $\frac{1}{6}$ part, you shall turne into shillings: and if any vnites remaine, they are so many first part of shillings, or peeces of 2 d̄, whether you will.

If

If one cost 8 pence
What are 1486 worth?

maketh pounds. 49—12—0 pence.

If one cost 6 pence
What are 7865 worth?

maketh pounds 196 —12—6 pence.

At 4 d the yard
What are 8736 yards worth?

maketh pounds 145. —12—0 d.

If one cost 3 pence
What are 6874 worth?

maketh pounds 123. —8—6 d

At 2 d the Elle
What comes 7894 Elles to?

maketh poũds—65.—15—8 d

BUt if your number of pence be not an aliquot or euen part of 24: then must you bring them into the aliquot parts of 24, and make thereof diuers products, which must be added together, as by the questions hereafter following shall appeare. 6. Rule.

Item for 5 d. first take for 3 d, then for 2 d: and adde them together, according to the instruction of the second rule: Or else first take

for 4 pence, then for 1 peny.

Item for 7 d, first take for 4 pence: then for 3 pence and adde them togither.

Item for 9, first take for 6 pence: then for 3 pence, and adde them togither.

Item for 10 pence, first take for 6 pence: then for 4 pence, and adde them togither.

Item for 11 pence first take for 8 pence, thē for 3 pence, & adde them togither: as by these examples following doth appeare.

Examples.

If one yard cost 5 pence
what are 7596 worth?

4 pence	126.	—12
1 peny	31.	—13
maketh poūds	158—	5s̄

Otherwise.

	1—5—7596	
3 pence	94.	—19
2 pence	63	—6
maketh pounds	158	—5s̄

If one cost 7 pence
what are 987 worth?

4 pence	16.	—9—
3 pence	12	—6—9
maketh pounds	28—	15—9 pence.

Other-

Otherwise.

1—7—987

6 pence	24.	13	0
1 peny	4	2	3
maketh pounds	28	15	9

If one cost 9 pence
what are 987 worth?

6 pence	24.	13	6
4 pence	12	6	9
maketh pounds	37	0	3

Otherwise.

1—9—987

3 pence	12.	6	9
maketh pounds	37	0	3

If one cost 10 pence
what are 987 worth?

6 pence	24.	13	6
4 pence	16	9	0
maketh pounds	41	2	6

If one cost 11 pence
what are 987 worth?

8 pence	32.	18	
3 pence	12	6	9
maketh pounds	45	4	9

But if you haue any shillings and pence to be multiplied togither: Then are you to take for the shillings according to the instruction of the third rule: And for the pence according to the first rule before mentioned: vnlesse you can spie the aduantage thereof, and thereby helpe your selfe: as appeareth in this second example, where first I worke for 6 d: which is to be rebated out of the giuen number, and I haue 719 l 11 s my desire.

At 19 s 6 d the yard
what 738 yards

	738	
10 s	369—	
5 s	184—10	
4 s	147—12	
6 d	18— 9	
l	719—11 s	

Otherwise by Rebating.

738	
6 d	18—9
l	719—11 s

The like againe is doone by rebating as by these 2 examples appeareth:

At 18 s the Elle what 418 Elles.	
2 s	41—16
l	376—4 s

At 16 s the Elle what 517 Elles.	
4 s	103—8
l	413—12 s

And

And now I will touch a little the euen parts of a pound that falleth out in pence and shillinges, whereof for those parts you shall take such like parte of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which also for thy better ayde is here set downe.

1 s. 8 d		$\frac{1}{12}$
2. 6	is the	$\frac{1}{8}$
3. 4		$\frac{1}{6}$
6. 8		$\frac{1}{3}$

part of a pound.

Item first for 1 shilling 8 pence take the $\frac{1}{12}$ part of the giuen number, and if any thing do remaine, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.

Item for 2 shillings 6 pence, take the $\frac{1}{8}$ part of the number that is to be multiplied. And if any thing do remaine, they are eight parts of a pound, each one being in value 2 shillings 6 pence.

Item for 3 shillings 4 pence as appeareth by the Table, you must take the $\frac{1}{6}$ part of the giuen number. And if any thing do remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item for 6 shillings 8 pēce take the $\frac{1}{3}$ part of the number that is to be multiplied: And if any vnites do remaine, they are thirds of a pound,

a pound euery one being worth 6 shillings 8 pence.

Other infinite numbers there are, that may be reduced by abbreuiation into the proportionate parts of a pound: as 16 shillings 8 pence maketh $\frac{1}{6}$: which 16 shillings 8 pence is easily reduced into groates by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats: Then set 60 the groates of a pound vnder 50, cutting off the 2 ciphers as is here performed in the margent: And then haue you brought 16 shillinges 8 pence into the knowne partes of a pound which maketh $\frac{1}{6}$.

16 — 8
3
5|0
6|0

But yet gentle Reader, for thy further instruction, I haue hereunto annexed in a Table, how pence and shillings beareth proportion to a pound, which I commit to thy friendly beneuolence, it will be some aide vnto the vngrounded practitioner: but I count him the best workeman that can presently reduce his giuen price vnto the knowne & proportionate parts of a pound.

s	d	r
0	2	1/120
0	3	1/80
0	4	1/60
0	6	1/40
0	8	1/30
1	0	1/20
1	3	1/16
1	8	1/12
2	0	1/10
2	6	1/8
3	0	3/20
3	4	1/6
3	9	3/16
4	0	1/5
5		1/4
6		3/10
6	3	5/16
6	8	1/3
7		7/20
7	6	3/8
8		2/5

s	d	r
8	4	5/12
8	9	7/16
9		9/20
10		1/2
11		11/20
11	3	9/16
11	8	7/12
12		3/5
13		13/20
13	4	2/3
13	9	11/16
14		7/10
15		3/4
16		4/5
16	8	5/6
17		17/20
17	6	7/8
18		9/10
18	4	11/12
18	9	15/16
19		19/20

6/20
120
3
10/20
0

Here follow foure Examples vpon the 4. notes deliuered.

At

At 1 s̄ 8 d̄ the yard
what are 3884 yards worth?

maketh pounds—323—13—4 pence

At 2 s̄ 6 pence the yard
what are 4563 yards worth?

maketh pounds—570—7—6 pence

At 6 s̄ 8 d̄ the Elle
what comes 7562 Elles to?

maketh pounds—2520—13—4 pence

Now by custome you are able to worke by all sortes of summes being deliuered in shillings and pence, as 1 shilling one peny, 2 shillings two pence, 3 shillings 3 pence, and so of all other: wishing you to haue some considerations of your questions, when they are set downe, for there are many subtill abbreuiations, and great aduantages to be gotten, and easily to be perceiued.

As 3 s̄ 8 d̄ of 2 s̄ and 1 s̄ 8 d̄.

4 s̄ 2 d̄, of 3 s̄ 4 d̄, and 10 d̄, which 10 is ¼ of 3 s̄ 4 pence.

5 s̄ 8 d̄ of 4 s̄, and 1 s̄ 8 pence.

5 s̄ 10 d̄, of 5 s̄ and 10 pence, which 10 pēce is ⅙ of 5 shillings.

And by this meane when you haue taken one product, you may oftentimes vpon the same

same take an other more briefly than vpon the summe which is to be multiplied, &c.

NOw (gentle Reader) that you haue seene the vertue of the euen or aliquot parts of a pound in shillings alone, and also in the aliquot parts of shillings and pence: according to my promise, hereafter followeth a briefer and easier method for anie euen number of shillings eyther vnder or aboue 20, than euer yet hath beene published: Notwithstanding Master Humfrey Baker, whose trauel is worthy commendation, and whom for knowledge sake I reuerence, hath in some part touched this first part, though not in this method. The worke of the rule is both pleasant, ready and briefe, as by the varietie of the examples deliuered thereupon shall appeare. And first I will set forth a question, thereby the better to expresse or teach you the order therof: which is this.

If one yard cost 6 s what 8574 ?

1	6	8574
maketh pounds	—2572.	—4

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawen vnder it, you shall presently set a pricke vnder your first figure 4, towards your right hand, drawing

drawing from the pricke as heretofore hath beene practised, a little short line, thereto set downe the shillings anon, which done, multiplie the first figure 4 by 6 the value of your price, (which here you see standeth in sight aboue the line) it maketh 24: which is 1 pound 4 shillings. The 1 pound keepe to carie to the next place, and the foure shillings set downe at the end of the prescribed line towardes your right hand. Thus haue you done now with fire aboue the line, and also with 4 in the first place (for the pricke vnder foure doth represent that 4 hath done his office.) Then secondarily for a general rule take but the ½ of the giuen price which here is 3, which 3 is the number that shall now continue the rest of the multiplication and end the worke, whereupon I multiplie three into seuen standing in the second place it maketh 21, and with the one I I kept in minde 22, set downe 2 and keepe 2 in mind working according to the rule of Multiplication, deliuering the tens in mind in their due place, which done, the product from the pricke to your left hand representeth the pounds, and the other at the end of the line the shillings, as appeareth by the examples.

Note.
A general rule.

If one yard cost 2 s̄ what 7536 ?

1	2	7536
maketh pounds—		753.—12 s̄

If

If one yard cost 4 s̄ wh[illegible] 8792 ℔:

1	4	8792
maketh pounds—		1758 · — 8 s̄

If one peece cost 6 s̄ what 9537 ℔:

1	6	9537
maketh pounds—		2861 · — 2 s̄

If one cost 8 s̄ what 7509 ℔:

1	8	7509
maketh pounds—		3003 · — 12 s̄

If one cost 12 s̄ what 5794 ℔:

1	12	5794
maketh pounds—		3476 · — 8 s̄

If one cost 14 s̄ what 3705 ℔:

1	14	3705
maketh pounds—		2593 · — 10 s̄

If one cost 18 s̄ what 5703 ℔:

1	18	5703
maketh pounds—		5132 · — 14 s̄

If one cost 22 s̄ what 953

1	22	953
maketh pounds—		1048 · 6 s̄

Let these suffice (gentle Reader) for an entrance into euen numbers: And now I will shew the like rule for any odde or vneuen parts of a pound.

9. Rule. TO helpe you to the vnderstanding of these other questions that hereafter follow: wherein my first example the giuen number is 6487. At 3 shillings the yard. I multiplie 3 aboue the line into 7, it maketh 21. The one shilling I set downe, and the 1 ℔ I keepe. Now am I to take the ½ of three, which because it is an odde number I cannot. Therefore I shall keepe & continue my multiplication by three still, and worke by the ½ of the rest of the giuē figures or numbers, to wit 648. And first the ½ of 8 which is 4 multiplied into three maketh 12, thereto ioyne the 1 ℔ in minde, it maketh 13: set downe 3, keepe one. Then againe multiplie by ~~two~~ 3 the ½ of foure, it maketh fiue, and with one in minde it maketh 7. Then lastly take the ½ of fiue which is three, saying 3 times three is nine, which nine set downe, & so is the question answered, as appeareth by the practise, and the examples following.

At 3 s̄ the yard, what 6487 ?

1	3	6487
maketh pounds		—973 —1 s̄

If one yard cost 5 s̄ what 4269li:

1	5	4269

maketh pounds— 1067li— 5 s̄

At 7 s̄ the Elle what 6489li:

1	7	6489

maketh pounds— 2271li — 3 s̄

If one Elle cost 9 s̄ what 2807li:

1	9	2807

maketh pounds— 1263li— 3 s̄

At 11 s̄ the Pistolet what 8263li:

1	11	8263

maketh pounds— 4544li— 13 s̄

If one peece cost 13 s̄ what 4629li:

1	13	4629

maketh pounds— 3008li— 17 s̄

But now note (gentle Reader) when the giuen price falleth vpon any odde number. As 3.5.7.9.11.13. &c. Then it is to be presuppoſed, that the giuen ſumme to be multiplied muſt be a ſumme made of euen numbers, as 2.4.6. 8.10. &c. els cannot that queſtion be wrought at one line or working.

Prouiding alwayes that it may beare an odde figure in the firſt place towardes your right hand: as appeareth in theſe 6 examples which laſt were wrought, and ſuch like. &c:

which may beare an odde number for the price, and be done at one line or working very well.

But if the giuen price be an odde number, and the summe to be multiplied odde numbers also. Then can it not be done at one working, but requireth the aide of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall rule: First worke for the euen number, contained in that question, or giuen price, according as you haue learned, and then afterwards for the one odde shilling, take the ½ of the summe giuen to be multiplied, omitting the first prickt place, as was taught for the working of one shilling in my first rule of Practise, and adde those two together, and you shall haue your desire.

A generall Rule.

Example.

At 3 s the yard
what are 7539 yards worth?

2 s	753 · —	18
1 s	376 · —	19
maketh pounds —	1130 —	17 s

At 7 s the Elle what 7539?

1	7	7539
6 s		2261 · — 14
1 s		376 · — 19
maketh pounds —		2638 — 13

At

At 13 s the yard, what 7534 ?

1	13	7534	
12 s		4520·	— 8
1 s		376·	—14
maketh pounds		4897	—2

And thus haue I abridged into these two rules how to bring any number of shillinges: whatsoeuer they be into pounds, with a briefer method than euer yet hath beene published, which I commend vnto thy friendly censure and iudgement in the vse of practise thereof.

If one cost 6 s 5 d, what 1231 ?

1	6	5	1231	
6 s			309·	—6
4 d			20·	—10—4
1 d			5·	—2 —7
maketh pounds			394	—18—11

At 14 s 2 d, what 2825 ?

1	14	2	2825	
14 s			1977·	—10—
2 d			23·	—10—10
maketh pounds			2001	—00—10 d

At 16 s 4 d, what 2531 ?

1	16	4	2531	
16 s			2024·	—16
4 d			42·	—3 —8
maketh pounds			2066	—19—8

At 3 s the Pistolet, what 8325 ?

1	3	8325

maketh pounds 1248 — 15 s

At 7 s the crowne, what 6529

1	7	6529

maketh pounds 2285 — 3 s

At 9 s the peece, what 6767 ?

1	9	6567

maketh pounds 2955 — 3 s

These three last questions may seeme something hard, yet are they easie inough if you marke them well, if I should explane them, then are they too easie: therefore I leaue them to whet the minds of the desirous.

10. Rule. ITem, when any one of the summes which is to be multiplied, is composed of many denominations, and the giuen number but of one figure alone: then shall you multiplie all the denominations of the other summe by the same one figure, beginning first with that summe which is least in value towardes your right hand, & bring the product of those d into shillings, and the product of the shillings into pounds, as by this example doth appeare.

At 3 l 7 s 4 d what are 9 worth ?

maketh pounds	30	6	0 d

But

BAt if in any of the summes that are to be multiplied there be a broke̅ number: First worke for the whole according to the instructions that you haue learned: and then take such part of the giuen price, as that broken number beareth in proportion to the price, as in the example. After you haue wrought for 3 s̄ and for 6 d̄, then are you to take the ½ of 3 s̄ 6 d̄ for the ½ yard, and adde that to the summe: So adding all 3 products togither, which maketh 43 l̄ 2 s̄ 9 d̄ the iust price of 246 ½ Elles: and thus must you do of all other:

At 3 s̄ 6 d̄ the Elle, what 246 ½?

1	3	6	246 ½
3 s̄			36 — 18
6 d̄			6 — 3
½			1 — 9
maketh			43 — 2 — 9

At 16 s̄ 4 d̄ the peece, what 14 ¼?

1	16	4	14 ¾
16 s̄			11 — 4 —
4 d̄			0 — 4 — 8
¾			0 — 12 — 3
maketh pounds			12 — 00 — 11 d̄

If one peece cost 4 l̄ 3 s̄ 6 ½ d̄ what are 12 peeces worth?

4 l̄	48 —
3 s̄	1 — 16
6 d̄	0 — 6
½	0 — 0 — 6
maketh pounds	50 — 2 — 6 d̄

The Proofe.

If 12 peeces cost 50 l. 2 s. 6 d what one peece?

50 2 6

maketh pounds 4 — 3 — 6 ½

12. Rule. ITem touching the manner how to vnderstand the order of this question, and others the like: first seeke how many times 12 is contained in 50, which is 4 times, and so resteth 2 l, which 2 l conuerted into shillinges, and ioyned with the other 2 shillings, maketh 42 shillinges, wherein is found 12 three times: resteth 6 shillings, which turned into pence, putting thereto the 6 d in the first place, it maketh 78: wherein 12 is found 6 times, resteth 6 d, which containeth 12 but ½ a time, put that ½ to the 6 d: And then the solution is 4 l, 3 s 6 ½, as appeareth by the practise thereof.

13. Rule. ITem, the like is to be done of any thing that is bought or sold after 5 score to the hundreth, or the Quintall: As for example.

If

If 100 coſt 27 ℔ 13 ſ 4 d
What one pound?

	27 — 13 — 4
	20
ſ	5 | 53
	| 12
	1 | 10
	5 | 3
d	6 | 4 | 0
	10 | 0 $\frac{3}{5}$

maketh 5 ſ — 6 $\frac{2}{5}$ d

I haue wrought this at length for the ayde of the young learner, because he ſhould vnderſtand how all the multiplication is ſet downe.

	27 — 13 — 4
	20
ſ	5 | 53
	| 12
d	6 | 40
	| 100 $\frac{2}{5}$

makes 5 ſ 6 $\frac{2}{5}$ d

But to worke it more neately, it is by a litle vnderſtanding ended thus.

Item to the vnderſtanding of this and ſuch like queſtions, the right downe line is all the guide, which is pulled downe cloſe by 20, as you ſee in the example, where 27 pound 13 ſhillings is reduced all into ſhillings, and maketh 553 ſhillings.

The 5 towardes your left hand being ſepa-

rated

rated with the hanging or right downe line, is the iust number of shillings that answereth to the question: Nextly, 53 shillings is multiplied by 12 to reduce them to pence, putting to the 4 d, it yeeldeth for the multiplication of the first figure two 110: the one beyond the line towards the left hand, is 1 peny towards the rest of the pryce: then 53 also multiplied by 1 yeeldeth 53: but the 5 behinde the line towards the left hand is also 5 pence more, towards the pryce, which 1 and 5 I adde together vnder the line: it maketh 6 d. So is there found now as appeareth by the Titles of shillings and pence 5 shillings 6 pence.

Finally, I come now on this side the line towards the right hand, and vnder 12 I finde first 10, and then 3, which added together maketh 40, vnder which 40 you must put the 100, and it maketh $\frac{40}{100}$, which abbreuiated commeth to $\frac{2}{5}$. So the iust pryce of one pound after 5 score to the hundreth, maketh 5 s 6$\frac{2}{5}$ d.

One example more, and so wil I leaue this rule.

a brace is not so much as an ell of Antwerpe by halfe a quarter of an ell a nayle 5 yt $\frac{1}{5}$ of a nayle If An ell of Antwerpe is not so much as a yard of Lond by a q of a yard a nayle 6 yt $\frac{1}{3}$ of a nayle

If 100 cost $10\frac{3}{4}$ d

What 9874

100 .. $10\frac{3}{4}$ .. 9874

6 d	246.	17	0
4 d	164.	11	4
$\frac{2}{4}$ d	20.	11	5
$\frac{1}{4}$	10.	5	$8\frac{1}{2}$
l 4	42 ..	5.	$5\frac{1}{2}$
s 8	20		
	45		
maketh	12		
d 5	$45\frac{1}{2}$		
	100	$\frac{91}{200}$ parts of a d.	

Also the like may be done of our vsuall weights here in England (which is 112 l for euerie hundreth weight) in case you know the aliquot parts of a hundreth weight, which are these, 56 l, 28 l, 14 l, and 7 l: For 56 l is the $\frac{1}{2}$ of 112 pound, 28 l is the $\frac{1}{4}$ of 112 l, 14 l is the $\frac{1}{8}$, and 7 l is the $\frac{1}{16}$ part.

Therefore for 56 l take the $\frac{1}{2}$ of the summe of money that 112 l weight is worth.

For 28 l take the $\frac{1}{4}$ of the summe of money that 112 l weight is worth.

For 14 l, take the $\frac{1}{8}$ of the summe that 112 l is worth.

And for 7 l, the $\frac{1}{16}$ of the summe of money that C is worth.

As for example: At 17 l 19 s the hundreth pounds

pounds weight, that is to say 112 ℔, what shall 3 quarterns and 7 ℔ cost?

1 · ·	17 ·	19 · ·	3—7
2 quarterns	8 ·	19· 6·	
1 quarterne	4 ·	9· ·9	
7 pounds	1	2· ·5¼	
maketh pounds	14 ·	11· ·8¼	

The second Chapter entreateth of the Reduction of diuers measures to others value by Rules of Practise.

14. Rule. NOw will I shew a fewe examples of Practise in reducing of measures: as Elles, Yards, Braces, Pawnes of Ieanes, &c. Much more would I haue touched but that I feare the booke will rise to too great a volume.

In 864 Elles of Antwerpe, how many yardes of London?

864	864
432	216
216	648

maketh 648 —— yardes of London.

Item, in these and such like questions of Flemmish measure to be brought into yardes English: first take the ½ of the giuen number, as

as appeareth in the first example towardes your left hand. Then take the ½ of that product, or the ¼ of the giuen nũmber, and adde those 2 productes together they shalbe yardes English, as by the example you may perceyue.

The second example towardes your right hand is yet briefer than the first, whose worke is this: take the ¼ of the deliuered number, and that product subtract out of the giuen number, and the reste sheweth your desire. Of these two wayes vse which you thinke best.

The Proofe.

In 648 yards of London,
how many Elles of Antwerpe?

648
216

maketh 864 Elles of Antwerpe.

Item for the vnderstanding of this worke: first take the ⅓ parte of the yardes of London, which found, adde that ⅓ part and the yardes together, as appeareth by the practise: and the product sheweth the Elles of Antwerpe. 15. Rule.

In

In 320 yards of London, how many Elles of Antwerpe, maketh $426\frac{2}{3}$ Elles.

320 yards	Proofe.
$\frac{1}{2}$ $106\frac{2}{3}$	$426\frac{2}{3}$ Elles
$426\frac{2}{3}$ Elles	$\frac{1}{2}$ $106\frac{2}{3}$
	320 yards

Other Reductions.

16. Rule. Item you shall vnderstand, that for as much as sixe Braces of Millane make 5 Elles of Antwerpe, whereupon, according to the rules of Practise, you may reduce the one into the other by the like reasons aforesaid, in taking the $\frac{1}{6}$ part, and then subtract the same to make Elles of Antwerpe: And againe by the contrary in taking the $\frac{1}{5}$ parte with adding the giuen number, to turne the Elles to Braces, as for example.

In 876 Braces how many elles of Antwerpe?

876 Braces	The contrary.
146	730 Elles Flem:
Elles 730 Antwerpe	146
	876 Braces.

Elles 730 Antwerpe.

182 $\frac{1}{2}$

Yards $547\frac{1}{2}$ English.

Thus

Thus appeareth, that 876 Braces by practise, make 730 Elles Flemmish: which Elles Flemmish reduced into English yards by the rules aforesaid, make 547 ½ yards.

So againe vpon the same first question of Braces, I would know how many yards English they make. 17. Rule.

After the rate that 100 Braces are worth 62 ½ yards.

876 Braces

438

109 ½

I answere 547 ½ Yards

Item, to the vnderstanding of this worke and such like, first take the ½ of the giuen Braces, and after take the ¼ of that halfe, or the ⅛ of the giuen number, and adde them together, and the product are also yards English.

Item three Elles of Rochell make 5 Elles at Lisbone: So likewise three Elles at Lions make 5 Elles at Antwerpe. 18. Rule.

To worke these and such like, double the Elles of Lyons, and the Elles of Rochell: and from their productes subtract the ⅙: And the rest shall be Elles of Antwerpe, or Elles of Lisbone.

Ex-

Example.

In 63 elles of Lions how many Elles of Antwerpe?	In 100 Elles of Rochell, how many Elles of Lisbone.
63	100
63	
126	200
$\frac{1}{6}$) 21	$\frac{1}{6}$) $33\frac{3}{6}$
Ans. 105 Elles Ant.	Ans. $166\frac{1}{3}$ els of Lisb.

Touching the proofe or returne of these & such like questions, for a generall rule, you shall first take the $\frac{1}{5}$ of the giuen number: and adde that $\frac{1}{5}$ & the giuen number together, and the $\frac{1}{2}$ of that product shall be your desire.

Example.

In 105 Elles of Antwerpe, how many Elles of Lions?	In $166\frac{2}{3}$ elles of Lisbone, howe many Elles of Rochell?
105	$\frac{1}{5}$) $166\frac{1}{3}$
$\frac{1}{5}$) 21	$33\frac{1}{2}$
$\frac{1}{2}$) 126	$\frac{1}{2}$) 200
Ans. 63 els of Lyons.	Ans. 100 els of Roch.

The

The third chapter treateth of the order & worke of the Rule of three in broken Numbers, after the trade of Marchants, digressing something from M. Recordes. which is cõprehended in 3 Rules.



NOw that I haue somewhat intreated of the Rules of Practise, I will giue a few instructions, after my simple order, for ỹ working of the Rule of three in broken numbers, wherein I shall need to say the lesse, because I hope the studious Learner, that hath trauelled any thing in the Ground of Artes, is not vnfurnished of knowledge capable to vnderstand me. But before I deliuer any instructions for broken numbers, I will propone a question, which shall be wrought three sundry wayes, thereby to shewe as it were three degrees of comparison: how far the Rule of three in broken, for more speede of worke, differeth from whole, which I rather set downe for a view, that the studious herein may be more desirous to attaine brokẽ: leauing any more to discourse in Dialogue forme, but onely to giue instructions where need is: and in the rest to put forth the questions with their answeres.

My first question is this.

If one yard cost 6 s — 8 d
What are —— 789 worth at that rate?

1 — 6 — 8 — 789

12 80

80 63120 d.

Here the product of the summe are pence according to the nature of the middle number.

ℓ ℓ

ℓ 37 | ℓ
63 ℓ 20 | 5260 (263
ℓ 2222 | 2220

ℓ ℓ ℓ

I answere 263 ℓ

1 — 6⅔ — 789

3 20 20

15780

Here the product of the summe is s, according to the nature of the middle number.

ℓ

15750 | 5260 | 263 pounds.
3333 | 2220 |

1 — ⅓ — 789

3 1 1

789

Here

Here the product is pounds according to the title of the second number.

£
789 (263
333
I answere 263 £

Now that you haue seene the 3 former vertues of the Rule of three, whose products haue first brought forth d, next s, & lastly pounds: I will deliuer three notes in order following, & with them a dosen questions, that shall shewe the worke of the Rule of 3 in broken numbers or Fractions.

1 The first foure shall be sundry questions of a Fraction comming in the second place.

2 The second foure shall be of two Fractions comming in the second or third place.

3 The third foure of Fractions in all three places.

My first question is this.

If one yard cost me 3 s — 4 d what are 756 worth at that price?

In setting downe the questiõ to performe the worke I turne 4 d into the part of a s, which is $\frac{1}{3}$, and then the question standeth thus. 1. Rule.

1 — 3 $\frac{1}{3}$ — 756

TO the ready working of this queſtiō, and all ſuch other like, my firſt note is this: which take for a generall rule, that when any one Fraction ſhall come either in the ſecond or third place, that the Denominator of that Fraction or Fractions, muſt alwayes be brought vnto the number or Numerator of the firſt place: and thereby multiplied the one into the other.

And this benefite is alwayes gotten by the vertue of bringing the denominator of the ſecond numbers fraction vnto the firſt place. For the Fraction in the middle number is now releaſed: & the product that commeth of the multiplication, is of the nature and like Denomination of the whole number in the ſecond place which here are ſhillings.

Whereupon now to worke the queſtion I bring three the denominator of the Fractiō in the ſecond place, vnto my firſt number one with a line ſet vnder $\frac{1}{}$ thus: and the three vnder it $\frac{1}{3}$ thus: ſaying, once 3 is 3 my Diuiſor: That done, reduce three $\frac{1}{3}$ ſaying, three times 3 is nine, and the 1 ouer 3 make 10: my ſecond number in the rule of three: by which 10 I do multiplie my laſt number 756 as appeareth by the worke thereof, and it yeldeth 7560 ſhillings my diuidend.

Thē diuiding 7560 by 3 my diuiſor, it yeldeth in quotiēt 2520 ſhillings which maketh 126 pounds, as appeareth here moſt plainely, both

both by the example and the worke.

At 3 s 4 d the yard what 756 yardes?

1 —— 3 :: 1 : 756

3 10

10 7560 s

£	£	
7560	2520	126 I answere 126 £.
3333	2220	

Yet otherwise vpon the same question, altering the price now into the proportion it beareth to a pound, for the 3 s 4 d is $\frac{1}{6}$ parte of a pound: which example first standeth thus, as appeareth on the left hande, and afterwardes wrought as appeareth on the right hand.

1 —— $\frac{1}{6}$ —— 756

$\frac{1}{6}$ —— 756

1 1

1 —— 1

756 pounds

As soone as I haue caried 6 the denominator of my middle number vnto my first place, as afore hath beene taught, I pull downe one, the numerator of 6, with a line vnder 6 thus: $\frac{1}{6}$: And that one of custome I pull downe in sight beyng the figure that I shall multiplie my third or last number by, according to the tenor of the rule of three: And because one can

neither multiplie nor yet diuide (though here it is set downe in forme of Multiplication the rather for your vnderstanding) the product of the multiplication according to the declaration of this my first rule or note is conuerted into the title of my second number, which here are pounds: Now followeth the diuision performed by my diuisor 6, to make an end of the question.

```
  33
 756 (126: which maketh 126 l, as before.
 666
```

And thus much for the varietie in working that question.

And now followeth another.

If one yard of cotten cost 8 ¼, what 859 ?

```
1 ——— 8 ¼ ——— 859
                1 ——— ¼ ——— 1
4      33          33
                  2577
                  2577
                 28347

2(3          11 l   s   d
28347 | 7088(6  590 | 29 — 10 — 6 — ¼ d.
4444  | 12     220 |
```

This

This question was also wrought like the first, and bringeth forth 29 l — 10 s — 6¾ d, the price of 859 yards.

If 7 l of any thing cost 3 l — 10 s, what comes 987 pound to?

7 — 3½ — 987

2 / 7 / 7

14 / 6909

(7
6909 | 493 7/14 | ½ I answere 493 l 10 s.
14

Notes vpon my second Rule, for two Fractions comming in the second and third place.

My first question is this.

If one Elle cost 1 3 s 4 d: what halfe a quarter 8½ of an Elle?

Answere. First bring 1 3 s 4 d into the parts of a l, which is ⅔ and then will the question stand thus.

1 — ⅔ — ⅛

Item for the performance of this worke, do as before was taught in the first Rule, first

bring 3 the denominator of the second fraction vnto your first number 1: setting a line vnder 1 thus: Saying once 3 is 3: that done, bring 8 the denominator of the third fraction setting it vnder 3, and multiplie them together, saying 3 times 8 makes 24: which 24 is your Diuisor: (Now haue you done with the denominator 3, and also with the denominator 8.) Therefore you shall put a line vnder 3 thus. And the like line also vnder 8, setting or pulling downe vnder them their owne numerators, that is 2 vnder 3, and also 1 vnder 8, as appeareth in the example, which numerators for a generall rule are euermore to be pulled downe of custome in sight, to multiplie the one by the other, according to the tenor of Rule of 3. Then I multiplie the one by the other, saying: once 2 is 2 which signifieth 2 ℔, beyng of the nature and like denomination of the middle number, which 2 ℔ is to be reduced into shillings, other wise it cannot be diuided by my first number 24. Then diuiding 40 by 24, the quotient bringeth forth $1\frac{2}{3}$ s̄, So much is $\frac{1}{8}$ of an elle worth after that rate. Otherwise although 2 pound could not be diuided by 24: yet it might haue beene abbreuiated to $\frac{1}{12}$ of a pound: which is worth 1 s̄ 8 d̄, as before.

1 — ⅔ — ⅛

3 2 1 16

8 20 40 (1 ⅔ ⅝

24 24

24 40

Second queſtion.

If one pound of any weight coſt 13 ſhillings 4 pence, what are ⅞ of the pound worth after that rate? Anſwere. Reduce the 13 ſhillinges 4 pence into the partes of a pound: which is ⅔ of it, and then will the queſtion ſtand thus.

1 — ⅔ — ⅞

Item for the vnderſtanding of this, if you marke well the laſt example, this and the reſt lieth open, and neede ſmall inſtruction. For as you did laſt, ſo now againe bring the denominator of the ſecond and third fraction vnto the firſt figure 1, multiplying the one into the other, which maketh alſo 24 your diviſor.

Then making a line vnder 3 thus, and a line vnder 8 thus, and pulling downe their Numerators vnder each figure, that is, 2 vnder 3, and 7 vnder 8, which as I ſaid before for a generall rule I pull downe of cuſtome in ſight: to be the two numbers that of duetie ought

ought to be multiplied togither, which done I bring 2 being the lesser figure vnder 7: multiplying them togither, it maketh 14: which are of the nature of the middle number. That is to wit poundes: which 14 cannot aptly be diuided among 24, therefore are reduced into s, as is plainely to be seene in the example. Then 280 s parted among 24, yeldeth for his quotient 11 s 8 d your desire, and the iust price of $\frac{7}{8}$ of an elle. Otherwise, 14, though it could not be diuided by 24, might by mediation or diuision in broken numbers haue beene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his knowne parts, maketh 11 s 8 d as before. But my good will and meaning is to aide young beginners: Therefore haue I reduced the 14 l into s, which is the easier way.

Now followeth the example.

$\frac{1}{\frac{3}{8}}$. $\frac{2}{3}$. $\frac{7}{8}$

24

14

20

280 s

(1

4

46

280 | 11 $\frac{2}{3}$

144

The third example.

If

If one yard coſt me 2 s̄ 6 d̄, what 345 ¼ yards?

Anſwere. Firſt put 6 d̄ into the parts of a s̄: and then the queſtion ſtandeth thus.

1 —— 2 ½ —— 345 ¼:

Item, to the readie vnderſtanding of this and all ſuch like, according as before hath bin declared, bring the denominators of the ſecond and third fractions vnto the firſt place, multiplying them the one into the other, all which make 8 your Diuiſor common. Then next reduce your ſecond number, ſaying 2 times 2 is 4: and 1, is 5: as was taught in the example aforeſaid. Laſtly reduce your third number 345 ¼ all into fourths, and they make 1381: which 1381 is to be multiplied by 5, according to the tenor of the rule of 3: which done, maketh 6905 s̄: and diuided by 8 your diuiſor, yeeldeth in quotient 863 ⅛ s̄, which maketh in pounds 43 l̄ — 3 s̄ — 1 d̄ ½: And ſo much are the 345 and ¼ yards worth at that price.

The ſame queſtion wrought againe by 2 s̄ 6 d̄, is now conuerted into the parts of a l̄, and ſtandeth thus:

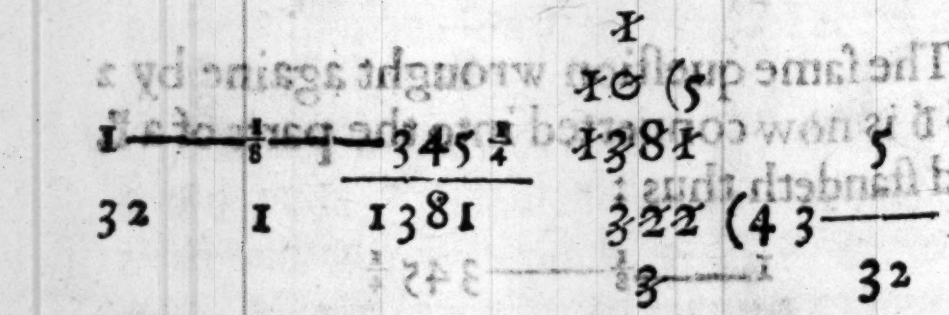
1 —— ⅛ —— 345 ¼

Item, after I haue brought here my ſecond & third fraction vnto my firſt place, and found 32 to be my diuiſor: hauing thus furniſhed

shed my first place with all thinges vnto him belonging (which is meant of bringing and multiplying the Denominators of the second & third Fractions into him) I then go in hand to see what is to do in my second place, where presently of custome I pull downe my Numerator 1 vnder 8: beyng the figure in sight that shall multiplie my third number.

Then lastly I reduce $345\frac{1}{4}$ al into fourths, as afore was practised, which maketh 1381, the which 1381 I am to multiplie by one my second number: they are nothing increased, but by the Metamorph. of my worke, they are now 1381 pound: being of the nature of the middle number, as I haue often shewed you, which diuided by 32 my diuisor, yeeldeth 43 pound and $\frac{5}{32}$: which $\frac{5}{32}$ of a pound reduced into knowne numbers, make 3 shillings $1\text{d}\frac{1}{2}$ as before.

Example.



3. Rule. Now follow 4 other questions, which are in all three places, broken numbers, or whole and broken together.

Item

Item first for the finding out of your Diuis. for: you shall take this for a most certaine and generall rule. That you must multiplie the numerator of the first number in the question by the denominator of the second: And also all that againe by the denominator of the third: & the totall thereof shall be your diuisor.

Secondarily, for a generall rule to find out your diuidend, multiplie the denominator of the first number by the numerator of the secõd, and the whole thereof by the denominator of the third. And the totall thereof shall euermore be your diuidend.

Now for an example I propone this question thereby to make my meaning the more plaine, and to shew you as I haue done in the rest, the maner and order of the worke.

If $\frac{2}{3}$ of any waight or measure cost $\frac{1}{6}$ of a £ or 20 s, what are $\frac{3}{8}$ of the like waight or measure worth after that rate?

Example.

$\frac{2}{3}$ —— $\frac{1}{6}$ —— $\frac{3}{8}$

Item for the more playner vnderstanding hereof, and all other the like, in broken numbers: First you shall pull downe two the Numerator of the first number or fraction, with a line vnder $\frac{1}{2}$ thus: that done, according as you

haue

haue learned before, bring 6 the denominator of the second fraction, and set it vnder 2 multiplying the one into the other which maketh 12. Then lastly bring 8 the denominator of the third fraction, and set it vnder 12, multiplying that 12 by 8, which amounteth to 96: (or els for more briefe multiple 6 by 8: saying 6 times 8 makes 48, which 48 set vnder two, and multiplie the one into the other, maketh 96 as before.) And this 96 is the first number in the rule of three. That shall alwaies for a most generall rule be your diuisor.

Secondly to worke for your diuidend, you shall, as hath bene sufficiently declared afore, pull downe 5 the Numerator of your second fraction. And set it vnder 6 with a line vnder $\frac{6}{}$ thus. That done, as you know, you are to pull downe 3 the numerator of the third fraction, and set it vnder 8 with a line vnder $\frac{8}{}$ thus, multiplying the one into the other, according to the tenour of the rule of 3, which maketh 15. Then according to my note, forget not to bring the Denominator of the first Fraction which is 3 vnder 15, and multiplie them together, which maketh 45, which 45 is your diuidend. Which 45 are of the nature or denomination of the middle number, as I haue oft taught you before. And therefore are 45 ℔, which aptly cannot be diuided by 96. Therefore you shall reduce that 45 ℔ into s. as you see is performed in the example, which

amoun-

amounteth to 900 s̄, which diuided by 96 your diuisor, it yeeldeth 9 s̄, and $\frac{36}{96}$ of a s̄, which in lesser termes is $\frac{3}{8}$, which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$ d̄: And so much will the aforesaid $\frac{3}{8}$ cost, as by the worke following shall appeare.

Example.

$\frac{2}{3}$ — $\frac{5}{6}$ — $\frac{3}{8}$

2 5 3

48 15 | 36

96 3 | 900 | $\frac{36}{96}$ | $\frac{3}{8}$

45 96

20

900

Otherwise though 45 could not be deuided by 96: yet by Diuision in broken numbers it might haue bene abbreuiated to $\frac{1}{2}\frac{5}{2}$ of a £, which reduced into knowen partes will make 9 s̄ $4\frac{1}{2}$ d̄ as before.

Now my second example shall be the proofe of this question.

If $\frac{3}{8}$ yards cost $\frac{1}{2}\frac{5}{2}$ of a poūd or 20 shillings, what shall $\frac{2}{3}$ cost?

Answere. Worke as was taught you before, and you shall haue your desire.

$\frac{3}{8}$	$\frac{15}{32}$	$\frac{2}{6}$
3	15	2
32	2	
96	30	
3	8	
888	240	

Here, as appeareth by the worke, the multiplications being ended, 240 is to be diuided by 288, which to some perchance may seeme hard, yet notwithstanding is the worke good: Therefore abbreuiate 240 by 288, as you see here is practised: and the end of your abbreuiation shall come to $\frac{5}{6}$ your desire $\frac{240}{288}|\frac{120}{144}|\frac{60}{72}|\frac{30}{36}|\frac{5}{6}$

Otherwise. $\frac{240}{288}|\frac{120}{144}|\frac{60}{72}|\frac{30}{36}|\frac{5}{6}$

Otherwise. $\frac{240}{288}|\frac{40}{48}|\frac{5}{6}$

The third question.

If $\frac{3}{4}$ elles cost 13 s 4 d, what $156\frac{1}{2}$ elles?

Answere. To worke this question the shortest way: reduce 13 s 4 d into the parts of a £, which is $\frac{2}{3}$.

Then as you did afore, after you haue set downe the question, the numerator of the first fraction 3, is pulled downe vnder 4, and the De-

Denominators of the other 2 fractions multiplied into him which maketh 18 your diuisor.

Then the numerator of the second fraction two is pulled downe vnder three of custome now in sight readie to multiplie my third nũber by: which is performed as soone as the last number 156 ½ is reduced into halfes.

Then lastly I multiplie that product by 4. the denominator of my first fraction: it yelderh 2504: which I diuide by 18, and my quotient is 139 ℔ ℈ 1/9 of a ℔ remaining, which is worth 2 ß 2 2/3 ₰: And so much will 156 ½ elles cost, as by the worke following doth appeare.

3/4	2/3	156 ½	℔
3	2	313	47
6		2	76 (2
18		626	2504 | 139 1/9.
		4	1888
		2504	℔

The fourth Example.

If two ½ Elles cost 1 2/3 ℔, what commeth 29 1/4 elles to?

Item to the workemanship of this question: first reduce your first number to one direct Numerator: in saying two times 2 is 4, & 1 is 5: Then bring the multiplication of the Denominators of the second and third Fracti-

ons, which maketh 12: and multiplie that 12 by fiue your first numerator, it maketh 60, which is your diuisor.

Then the reduction of the second number, which is 5 multiplied 117 the product of the last numbers reduction, make 585, which 585 yet resteth to be multiplied by two, the denominator of the Fraction in the first place yeldeth 1170: which diuided by your diuisor 60: yeldeth 19 pounds—10 s̄, as appeareth by the worke thereof.

Thus hauing now touched the 12 questiõs wherof I first pretended, which with diligence and oft practise I trust are sufficient to ayde the desirous, vnto the working of any broken numbers, I will nowe intreate of diuers necessarie Rules incident vnto traffike, as hereafter followeth.

The fourth Chapter treateth of Losse and Gaine, in the trade of Marchandise.

IF one yard cost 6 s̄—8 pence: and the same is sold againe for 8 s̄—6 pence: the question is, what is gayned in 100 pounds laying out on such commoditie.

Answere. The rule of three direct, applieth two maner of wayes to doe the same, the one is, to say: If $6\frac{2}{3}$ giue $8\frac{1}{2}$, what giueth 100?

Mul-

Multiplie and diuide, & looke what your quotient bringeth forth aboue your laying out, is the neate gaynes, and the solution to your question: If you follow the worke, your quotient will bring forth 127 ℔ --10 shillings, which is 27 ℔ —10 s̃ more then your principall, & so much is gained in the 100 pounds laying out.

Item, to worke it the other way, which I take the neerest, seeke the difference betwixt the iust price, and the ouer price, which is one shilling 10 pence: Then say by the rule of 3: If $6\frac{2}{3}$ s̃ gaine $1\frac{1}{2}$ s̃, what shall 100 pounds gaine? Multiplie and diuide, and you shall finde 27 pounds--10 shillings, and so much is gained in 100 pounds laying out.

Use which of these two wayes you thinke good.

The proofe.

If a yard of cloth be deliuered for 8 s̃ —6 d̃, whereupon was gained after the rate of 27 ℔ 10 s̃ in 100 pounds laying out. The question is what the yard cost at the first hand?

Answere. Put your gaines to 100 ℔, all maketh 127 pounds—10 shillings: thē say, if 127 ℔ —10 s̃ giue but 100 ℔, what giueth $8\frac{1}{2}$ s̃. Worke, and you shall find 6 s̃ —8 d̃, the true solution to your question.

Yet another Branch or proofe vpon the same first question

If one yard cost 6 s̄—8 d̄, the question is what price the same is to be sold againe for, to gaine 27 l̄—10 s̄ in 100 l̄ laying out.

Answere. Say by the rule of three, if 100 l̄ giue 127 l̄—10 s̄, what giueth 6 $\frac{2}{3}$ s̄? Multiplie and diuide, and you shall finde 8 s̄ sixe d̄, your true solution.

If one elle cost 7 s̄—8 pence, & sold againe for 8 s̄—6 d̄, Quest̄. What is gained in 20 l̄, laying out in such commodities.

Answere. Seeke the difference betwixt the iust price & the ouer price, which is 10 pence, and then applie the Rule of three, as before is taught, saying, If 7 $\frac{2}{3}$ s̄ giue $\frac{1}{0}$ shillings, what giueth 20 l̄? Multiplie and diuide, and you shall finde 2 l̄ 3 $\frac{1}{2}\frac{1}{3}$ s̄, and so much is gained in 20 pound laying out.

The proofe also by an example of losse.

A Marchant hath bought Holland cloth at 8 s̄—6 d̄ the Elle, which proueth not to his expectation, whereupon he is content to loose 2 l̄—3 s̄—$\frac{1}{2}\frac{1}{3}$ s̄ in 20 pounds laying out. The question is what price ought to be made of the elle abating this losse.

Answere. Doe as before in gaines hath bene taught, putting 2 l̄—3 $\frac{1}{2}\frac{1}{3}$ s̄ to your 20 l̄, all together maketh 22—3 $\frac{1}{2}\frac{1}{3}$ s̄. Then say by the rule of three: if 22—3 $\frac{1}{2}\frac{1}{3}$ s̄ giue but 20 l̄, what

what shall come of 8 ½ s: worke and you shall find 7 s—8 d the iust price that the elle ought to be sold for after the rate of this losse.

Thus it appeareth euidently as in companie the rule is appliable as well to gaines and losse.

If 20 ¼ yards cost 26 l—10 s, how shall I sell the same againe to gaine ⅓ of the principall, or to make of 3, 4: which is all one.

Answere. By the rule of 3, if 3 do giue 4, what will 36 ½ giue? Multiplie and diuide, and you shall finde 48 l—⅔. Then say againe, if 20 ¼ yardes doe giue 48 ⅔ pounds, as well principall as gaine, what will one yard bee worth at that price? Multiplie and diuide, and you shall finde 2 l—8 $2\frac{16}{43}$ s.

If one Elle of cloth cost me 8 s—8 d, and afterwardes I sell 10 ½ elles thereof for 5 l 13 s—4 d, I would know whether I winne or lose: and how much vpon the 100 l of money.

Answere. See first, at 8 s—8 d the Elle, what 10 ½ elles comes to, and you shal find 4 l 11 s, and I sold the same for 5 l—13 s—4 d so that I did gaine vpon the 10 ½ yardes 1—2—4 d. Then if you would know how much is gained in the 100 l, say by the rule of three, if 4 l—11 s did gaine 22 ⅓ s, what wil 100 l

gaine: multiply & diuide, & you shal find — 24 pound — 10 s̄ — $\frac{10}{91}$ d̄. And so much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ yards cost me 11 pound — 5 s̄, and I sell the yard againe for 16 s̄, the question is, whether I do winne or loose, and how much in or vpon the pound of money.

Answere. Looke what the 12 $\frac{1}{2}$ yards come to at 16 s̄ the yard, and you shall find 10 pound. But they cost 11 pound — 5 s̄. So there is lost vpon the whole 1 ponnd — 5 s̄. Then to know how much is lost in the pound, say by the rule of 3, if 11 $\frac{1}{4}$ pound do loose 1 $\frac{1}{4}$ pound, what wil 1 pound loose. Multiplie and diuide, & you shal find 2 s̄ — 1 d̄ $\frac{2}{3}$, & so much is lost in the pound of money.

If I sell the C weight of any commoditie for 4 l̄, whereupon I do loose after 10 pound in the 100 pound, I demaund how much I shal loose or gaine in the 100 pound, if in case I had sold the same for 4 pound — 10 s̄.

Answere. Say if 90 pound yeeld 100, how much will 4 giue? Multiplie & diuide, and you shall find 4 $\frac{4}{9}$. Then say againe, if 4 $\frac{4}{9}$ giue me 4 $\frac{1}{2}$, what will 100 come to? Multiplie and diuide, and you shall finde 101 pound $\frac{1}{4}$, which is more than 100 pound by 1 pound — 5 s̄. And so much is gained in the 100 pound.

A Marchant hath solde Currans for the summe of 430 pound, & he hath gained therein after

after 10 pound in the 100 pound. The question is to know how much he gained in all.

Answere. Say by the rule of three, If 100 pound doe gaine 10 pound, what will 430 ℔ gaine? Multiply and diuide, and you shall find 43, and so much hath he gained in all.

If one yard be worth 28 ½ s̃, for how much shall 10 yards be solde, to gaine after 8 ℔ 6 s̃ 8 d̃ in the 100 pound.

Answere. First adde 8 ℔ 6 s̃ 8 d̃ to 100: Then say if 100 ℔ do giue 108 ⅓ for principal and gaine, what will 28 ½ s̃ principall yeelde? Multiplie and diuide, and you shall finde 30 ⅞ s̃. Then say againe by the rule of three, If 1 yarde doe giue 30 ⅞ s̃ (which is as well the principall as the gaine, what shall 10 yards giue? Multiplie and diuide, and you shall find 15 ℔ 8 s̃ 9 d̃. And for the same price shall the 10 yardes be solde, for to gaine after the rate of 8 ℔ 6 s̃ 8 d̃ vpon the 100 ℔.

A Branch or Proofe out of this Question.

A Marchant hath sold Clothes for 15 ℔ 8 s̃ 9 d̃, and hee hath gayned in the whole, the summe of 1 ℔ 3 s̃ 9 d̃. The question is to know how much hee hath gained in the 100 pound.

Answere. To knowe this, first rebate the gaines from the price, and there will remaine

14 l---5 s--0 d. Then say by the rule of 3 direct, if 14 l ¼ giue me 1 l---3 s---¾ d, what will 100 l giue? Multiplie and diuide, & you shall finde 8 l---6 s---8 d the effect desired: the proofe is apparant in the question before.

Yet an other Branch or Proofe of the first question.

If 10 yards bee deliuered for 15 l 8—9, whereupon was gained after the rate of 8 l 6 s—8 d vpon the 100 l, the question is, what the yard did cost at the first hand.

Answere. First say by the rule of three, if 10 with principall and gaine yeeld 15 l—8¼, what shall 1 yeeld? Multiplie and diuide, and you shall finde 30 ⅞ s. Then say againe by the rule of three, if 108 ⅓ principal and gaine giue but 100, what shall 30 ⅞ s of principall and gaine yeeld? worke and you shall finde 28 ½ s, And so much did the yard cost at the first penie.

If one yard cost 36 s, how much shall 12 yards be sold for to gaine after the rate of 10 l in the 100 l.

Answere. First say, if 100 giue 110 l principall and gaine, what will 36 s giue? Multiplie and diuide, and you shall finde 39 ⅗ s. Then say againe by the rule of three: If 1 yard of principall and gaine yeelde 39 ⅗ s, what

what shall 12 yardes gaine? Multiplie and diuide, and you shal find 23 l—15 $\frac{1}{5}$ s, which $\frac{1}{5}$ s in knowne number is 2 $\frac{2}{5}$ d. And for the same price shall the 12 yards be sold, to gaine after the rate of 10 in the 100.

The proofe

If 12 yards be sold for 23 l—15 s-2 $\frac{2}{5}$ d, whereupon is gained after 10 l in the 100 l. The question is, what the yard cost at the first penie.

Answere. First say, if 12 giue 23 l—15 $\frac{1}{5}$ s, what 1 yarde? Multiplie and diuide, and you shall finde 39 $\frac{3}{5}$ s. Then say againe by the rule of 3, if 110 l giue but 100, what shall 39 $\frac{3}{5}$ giue? worke and you shall finde 36 s, the iust price of the yard at the first hand.

Item, when one Marchant selleth wares to another, & hee giueth to the buier 1 l—6 s 8 d vpon the score, or 20 l. The question is, how much shall the buyer gaine vpon the 100 pound after that rate.

Answere. First adde 1 l—6 s—8 d vnto 20 pound, and they are 21 $\frac{1}{3}$. Then say, if 20 l giue 21 $\frac{1}{3}$, what shall 100 giue? Multiplie and diuide, and you shall find 106 $\frac{2}{3}$. So the buyer getteth

getteth after the rate of $6\frac{2}{3}$ ℔ vpon the 100 ℔.

Gentle Reader, other necessarie questions appertaining to Losse and Gaine, you shall haue in the eight Chapter of this Treatise.

The fifth Chapter entreateth of Losse and Gaine vpon Time, wrought by the double Rule of three: or by the Rule of three cõposed, which is contained in foure special selected branches or questions of diuerse formes, each one of them springing from the first question, and each one of them also being a proofe to other, &c.

IF one yarde cost mee 2 s̃ 8 d̃ readie money, and after I sel the same againe for 2 s̃ 10 d̃, to be paid for it at the ende of the 3 moneths. The question is, what I gaine vpon the 100 ℔ in 12 monthes.

Answere. First say, if $2\frac{2}{3}$ gaine $\frac{1}{6}$, what shall 100 ℔ gaine? Multiplie and diuide, and you shall find $6\frac{1}{4}$ ℔. Then say againe by the rule of three, if three moneths gain $6\frac{1}{4}$ ℔, what shall 12 monethes gaine? Worke and you shall finde 25 ℔, and so much shall I gaine in 12 moneths after that rate.

Item, you may also worke it at one working by ỹ first part of the rule of 3 composed, saying

saying, if $2\frac{2}{3}$ s̄ in three moneths do gaine $\frac{1}{6}$ of a s̄, (which is 2 d̄) what wil 100 l̄ gaine in 12 moneths: which for thy further encouragement, the worke of this one example I haue here put downe, to verifie that I affirmed in the first part of this Ground of Arts, that this rule, and so all others more reioyceth in Broken then in Whole.

s̄	moneths	s̄	l̄	moneths
$2\frac{2}{3}$	: 3 ,	$\frac{1}{6}$	· 100 ·	12
8				100
3		2		1200
24		7		3
6		162		3600
144		3600 | 1444	25.	
		14		

Where the Multiplication and the Diuision being ended maketh 25 l̄ your desire.

If a yard be deliuered for 2 s̄ — 10 d̄, to be paid at 3 moneths, whereupon was gained after the rate of 25 l̄ in the 100 for 12 moneths. the question is now, what the yarde cost at the first hand.

Answere. First say, if 12 moneths gaine 25 l̄, what shall 3 moneths gaine: Worke and you shall finde $6\frac{1}{4}$ l̄, Then say againe the

second

second time, if 106 ¼ ℔ giue but 100, what shal 2 ⅓ s̄ giue? Worke and you shall find 2 s̄ 8 d̄, which is the iust price that the yard cost at the first hand.

If one yard of Cloth cost me 2 s̄ — 8 d̄ readie money, for what terme shall I sell the same againe for 2 s̄ — 10 d̄, so that I might gaine after the rate of 25 pound vpon the 100 pound in 12 moneths.

Answere. First say, if 2 ⅔ gaine ⅙, what shall 100 pound gaine? Multiplie and diuide, and you shall finde 6 ¼ pound. Then say againe for the second worke, if 25 pound become of 12 moneths, what shall come of 6 ¼? Worke, and you shall finde 3 moneths, the iust terme of time that the cloth ought to be deliuered at 2 s̄ 10 d̄ to gaine 25 pound vpon the 100 pound in 12 moneths.

If one yard cost me 2 s̄ 8 d̄ readie money, for what price shall I sell the same againe to be paid at the end of three moneths, so that I may gaine after the rate of 25 pound in the 100 pound for 12 moneths.

Answere. First say, if 12 gaine 25 ℔, what shall 3 moneths gaine? Multiplie and diuide, & you shall find 6 ¼ ℔. Then say for the second worke, if 100 pound giue 106 ¼, what giueth 2 ⅓ s̄? Worke and you shall find 2 s̄ — 10 d̄, & for that price must the yard bee solde to gaine after 25 pound in ℔ 100 pound for 12 moneths.

Many

Many other of these questions I might here haue deliuered, but for feare the book would rise to too thicke a volume, and so to make the price so much the dearer, wherby it might not be so partable to my Countrimẽ as I wish it. But these foure, I haue of purpose framed in this order, hauing relation one to another, assuring you, that what questions soeuer may bee proponed within the compasse of this Rule, you shall find by one of these foure to make a solution. And moreouer, diuers other are yet to bee deliuered: where the Creditor giueth diuers dayes of payment, which can neuer be well wrought, nor yet vnderstood, vnlesse you can first find by Art, the iust time that all those payments, how different soeuer they be, ought to bee paid at once: whereupon first I think good here to giue some instructions into such a Rule, for it is the onely ayde for the finishing of such questions as hereafter shall follow.

The sixt Chapter intreateth of Rules of Payment, which is a right necessarie Rule, and one of the chiefest handmaydes that attendeth vpon buyiug and selling, &c.

Example.

A Merchant doth owe a summe of money, whereof the $\frac{1}{3}$ is to be paid at 6 moneths, the $\frac{1}{2}$ at 8 moneths, and the rest at a yeare. If he would pay at one payment, the question is, what time ought to be giuen him?

Answere. I haue omitted the quantitie of the summe, for you shall vnderstand, the rule is appliable, and yeeldeth a true solution to what summe soeuer shall be proponed. But now for orders sake in teaching, I doe imagine the summe to be 60 pounds, whereupon the maner of this worke is to multiply ỹ proportionate part of the money by the time, as in companie. Then 20 being the first paiment and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers by 6, his time of payment maketh $\frac{6}{3}$, which in whole numbers as appeareth by the example in the margent, maketh 2 moneths: next 30, which is ỹ $\frac{1}{2}$ multiplied by his terme 8, yeldeth 4 moneths, then the rest

$\frac{1}{3}$ by $\frac{6}{1}$	$\frac{6}{3}$	2 moneths
$\frac{1}{2}$ by $\frac{8}{1}$	$\frac{8}{2}$	4 moneths
$\frac{1}{6}$ by $\frac{12}{1}$	$\frac{12}{6}$	2 moneths
		8 moneths

which is 10 l, must needes be abreuiated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 moneths, produceth $1\frac{2}{6}$, maketh 2 moneths. All which added togither, as appeareth in the margent, maketh 8 moneths, which is the iust time that all those payments ought to be paid at once.

A Marchant hath 800 l to pay, the $\frac{1}{6}$ thereof ready money, the $\frac{1}{4}$ at two monethes, the $\frac{1}{2}$ at foure moneths, and the rest at a yeare. The question is, if hee would pay all at one payment, what time ought to be giuen him.

Answere. The readie money is neuer multiplied: then $\frac{1}{4}$ multiplied by 2 moneths as you did before, maketh $\frac{1}{2}$, then $\frac{1}{2}$ by 4 produceth 2 moneths, as appeareth here in the margent. But now for the rest of the money you can not multiplie it vntill you haue sought what proportion it beareth to 800 pounds. Therefore you must subtract the readie money, the $\frac{1}{4}$ & the $\frac{1}{2}$ out of the principall. The rest will be 66 $\frac{2}{3}$ l, which you must look what part it beareth to ỹ principal, which you shall find to be $\frac{1}{12}$, ỹ same you must also multiplie by his time 12 monethes, and it yeldeth 1 moneth, so all make three $\frac{1}{2}$, moneths as appeareth in the margent.

$\frac{1}{4}$ $\frac{2}{1}$ $\frac{2}{4}$ $\frac{1}{2}$
$\frac{1}{2}$ $\frac{4}{1}$ $\frac{4}{2}$ 2
$1\frac{1}{2}$ $\frac{12}{1}$ $1\frac{2}{12}$ 1
$3\frac{1}{2}$

A Marchãt is to pay 1200 ℔, in three termes, That is to wit 400 ℔ at two weekes, & 600 ℔ at foure monethes, lastly 200 ℔ at 5 moneths. The question is in what time, they ought to be paid at once.

Answere. Proportionate the partes, and you shall find that 400 is $\frac{1}{3}$ part, and for 600 you shall finde $\frac{1}{2}$, & likewise 200 is the $\frac{1}{6}$ part, which multiplie by their times as before, and you shall haue $\frac{2}{3}$ weekes. more 8 weekes, and lastly three $\frac{1}{2}$ weekes, which together maketh 12 weekes, or three moneths your desire.

A Marchãt is to pay 600 ℔ in three termes, whereof 100 ℔ is paid present, more 300 ℔ at 20 dayes, and the rest at 5 monethes, accounting 30 dayes to a moneth. The question is, what time ought these payments to bee payd at once.

Answere. Worke and you shall finde two moneths.

The

The seuenth chapter intreateth of buying and selling in the trade of Merchandise, wherin is takē part ready money, and diuerse dayes of payment giuen for the rest, & what is won or lost in the 100 ℔ forbearance for 12 moneths more or lesse according to the quantitie of money, or proportion of time, &c.

A Merchant hath bought Sattens which cost 8 shillings the yard ready money. And he selleth the same againe to another man for 10 shillinges the yarde, but he giueth two dayes for the payment, that is to say, three monethes for the one halfe, and fiue moneths for the other halfe. The question is to know how much the seller doth gaine vpon the 100 ℔ in 12 moneths after that rate.

Answere. Seeke first by the rules of payment, at what time those two paymentes ought to be payd at once, and you shall finde foure monethes, at which time the second Merchant ought to haue payd the whole entire payment. And therefore say by the first part of ỹ Rule of three composed. If 8 s̄ in 4 monethes do gaine 2 s̄, what wil 100 ℔ gain in 12 moneths

8—4—2—100—12

4 × 12 = 32

12 × 100 = 1200

2 × 1200 = 2400

2400 | 75

232

Multiplie and diuide, and you shall finde 75 pounds, as appeareth in the example, and so much doth the first Merchant gaine vpon 100 pounds in 12 moneths.

A Merchant hath sold fiftie clothes at 9 ½ l the peece, to be paid the one ½ at foure moneths, the ⅓ at fiue moneths, and the ⅙ at 7 monethes, and the sellers mynde is to take no more but after 8 poundes in the 100 for 12 monethes. The question is now what the first Merchant gayneth in the sale of these clothes after that rate.

Answere. First looke what the 50 clothes come to at that price, and you shall finde 475 pounds. Then secondly, according to your direction in the Rules of payment, seeke at what time all the payments are to be performed at once. And you shall finde 4 ⅚ moneths. Then thirdly say by the first part of the rule of three compoled. If 100 l in 12 monethes gaine 8 l, what wil 475 l gaine in 4 ⅚ moneths? Worke, & you shal find 15 pounds, & ⅘ of a pound, which is the neate gaines that the first Merchãt hath after the rate aforesaid.

A Merchant hath bought Holland at 7 s̃ 3 d̃ the Elle ready money, and he selleth the same again for 8 s̃ 4 pẽce the Elle, to be paid ¼ part in ready money, more ⅓ part at two monethes, and the rest at 4 monethes. The que-

stion

tiō is now to know how much the first Merchant doth gaine vpon the 100 poundes in 12 moneths after the same rate.

Answere. According to the direction deliuered you in the rule of payment, the ready money is not to be multiplied. Then working for the other two payments, to finde out the true proportion at what time they ought to be payd at once, you shall finde for $\frac{1}{3}$ at 2 moneths $\frac{2}{3}$ of a moneth. And the rest of ỹ money which is $\frac{5}{12}$ multiplied by his terme 4 moneths yeldeth $1\frac{2}{3}$ moneths, both which added together make 2 & $\frac{1}{3}$ moneths, the iust time that both the paymēts ought to be performed at once. And therefore say by the first part of the rule of 3 composed, if $7\frac{1}{4}$ in $2\frac{1}{3}$ moneths doe gaine $\frac{13}{240}$ of a ℔, what shall 100 pounds gaine in 12 monethes after that rate, worke and you shall find 76 pounds 16 shillings 11 pence $\frac{7}{20}\frac{1}{3}$ pounds. And so much doth he gaine vpon 100 pound in 12 moneths.

A Merchant hath bought 30 clothes at sixe pounds the peece for ready money. Afterward he selleth 10 of them for 7 pounds the peece, for three moneths terme. And the other 20 he selleth for 8 pounds the peece for foure moneths terme. The question is now, what he gaineth vpon 100 pounds in 12 moneths.

Answere. First finde the value of the 30 clothes, which amoūt to 180 pounds. Secōdarily, seeke what ỹ 10 peeces come to at 7 poūds, and what the 20 peeces come to at 8 poundes

the one comes to 70, and the other to 160: both which together make 230, which is 50 ℔ more than they coste. Thirdly, as I haue taught you in the rule of payment, proportionate the first and second prices, vnto the proportion they beare vnto 230 the product of their two prices, and you shall finde $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then fourthly, multiplie those parts by their times, and you shall haue $\frac{21}{23}$ and $\frac{23}{46}$: both which together maketh 3 whole monethes, and $\frac{15}{23}$ of a moneth, which is the iust time that both these payments are to be paid at once.

Then say by the first part of the Rule of 3 composed: If 180 ℔ in $3\frac{16}{23}$ moneths do gaine 50 ℔, what shall 100 gaine in 12 monethes? Multiplie and diuide, and you shall finde $90\frac{10}{51}$ pounds. And so much doth he gaine vpon 100 pounds in 12 moneths.

A Merchant hath bought Synamon which cost him 9 s̄ the ℔ ready money. The question is now, at what price he ought to sell the 100 waight. To wit 112 pounds, to be paid the $\frac{1}{4}$ at two moneths, and the residue at the ende of 3 moneths, so that he may gaine after the rate of 10 pounds vpon a 100 ℔ for a 12 moneths.

Answere. Seeke first by the Rules of payment what terme both the paymentes ought to be payd at once, where the $\frac{1}{4}$ multiplied by his terme two moneths, maketh $\frac{1}{2}$ monethes.

Like-

Likewise the next payment which is $\frac{1}{4}$ multiplied by his terme 3 monethes, maketh $2\frac{1}{4}$ moneths, both which added togither, maketh $2\frac{3}{4}$ moneths, which is the time, that both the payments ought to be paid at once. Then say by the Rule of 3, if 12 moneths doe giue me 10 pound, what will $2\frac{3}{4}$ moneths giue? Multiply and diuide, and you shal finde $2\frac{7}{24}$ pound. Thē say againe by the rule of 3: If one pound cost me 9 s̄, what will 112 pound cost? Multiplie and diuide, and you shall finde 50 pound —8 s̄. Then say once againe, If 100 pound do giue $102\frac{7}{24}$, what wil $50\frac{2}{5}$ pound giue? Multiply & diuide, & you shal find 51 pound —11 s̄ —$1\frac{1}{5}$ d̄. And for that price ought I to sell 112 poūd of Synamon to be paid at two seueral paiments aforesaid, to gaine thereby after the rate of 10 pound vpon the 100 pound in 12 moneths.

Briefe Rules for our hundreth *waight here at London, which is* after 11 pound for the C.

Item who that multiplieth the pence that one pound weight is worth by 7, and diuideth the product by 15, shall find how many pounds in money the 112 pound waight is worth.

And contrariwise, he that multiplieth the pounds that 112 pound waight is worth by 15, and diuideth the product by 7, shall finde how

many pence in money the 112 pound weight is worth.

Example.

At 10 d the pound weight, what is 112 ℔ weight worth?

Anſwere. Multiplie 10 by 7, and thereof commeth 70, the which diuide by 15, and you ſhall finde 4 2/3 ℔. And thus the 112 ℔ is worth 4 ℔—13—4 d after the rate of 10 d the poũd aforeſaid.

At 6 ℔ the 112 ℔ weight, what is one pound worth?

Anſwere. Multiplie 6 ℔ by 15, and thereof commeth 90: the which diuide by 7, and you ſhall find 12 d 6/7. So much is one pound worth when the 112 ℔ did coſt 6 pounds.

The eight Chapter entreateth of *Tares and allowances of merchandiſe* ſold by weight, and of loſſes and gaines therein, &c.

AT 16 ℔ the 100 ſuttle, what ſhall 895 ℔ ſuttle be worth in giuing 4 ℔ weight vpõ euerie 100 for treate.

Anſwere. Adde 4 vnto 100, and you ſhall haue 104. Then ſay by the rule of three, If

104

104 be worth 16 l, what are 895 l worth? Multiplie and diuide, and you shall find 237 l 13—10 2/13 d. And so much shall the 895 poũd weight be worth.

Item at 3 s̃ 4 d̃ the pound weight, what shal 754 1/2 be worth, in giuing 4 l weight vpon euery 100 for treate.

Answere. See first by the rule of three, what the 100 pound is worth, saying: If 1 cost 3 1/5 s̃, what 100? Multiplie and diuide, and you shall find 16 l 2/5. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three, if 104 be solde for 16 2/5, for how much shall 754 1/2 be sold for? Multiplie and diuide, and you shall finde 120 l'—18 s̃ 3 3/13 d̃. And for so much shall the 754 1/2 be sold for at 3 s̃—4 d̃ the pounde in giuing 4 vpon the 100.

Other necessarie briefe rules there are for the finding of treats, or casting vp of chestes of sugar, &c. which for that it is a misterie, I omit. If any lacke instruction that way, they shall finde me ready to pleasure them.

Item if 100 l be worth 36 s̃ 8 d̃, what shal 800 l be worth in rebating 4 pound vpon euery 100 for tare and cloffe.

Answere. Multiplie 860 by 4, and thereof commeth 3440, the which diuide by 100, and you shal haue 34 2/5 l, abate 34 2/5 from 860, and

there will remaine 825 $\frac{3}{5}$. Then say by the rule of three, If 100 ℔ cost 36 $\frac{2}{3}$ s̃, what will 825 $\frac{3}{5}$ cost after the rate? Multiply and diuide, and you shal finde 15—2—8 $\frac{16}{25}$. And so much shall the 860 cost in rebating 4 pound vpon euery 100: for tare and cloffe.

Item whether doth he loose more that giueth 4 ℔ vpon the 100: or he that rebateth 4 ℔ vpon the 100.

Answere. First note that he that giueth 4 ℔ vpō 100, giueth 104 for 100. And he which rebateth 4 pound vpō the 100 giueth the 100 for 96. Therefore say by the rule of three, if 104 be deliuered for 100, for how much shall the 100 be deliuered? Multiplie and diuide, and you shall finde 96 $\frac{2}{13}$, and he which rebateth 4 in the 100 maketh but 96 of 100, so that he looseth 4 in the 100, & the other which giueth 4 vpon the 100, looseth but 3 $\frac{11}{13}$ vpon the 100. Thus you may see that he which rebateth 4 in the 100, looseth more by $\frac{11}{13}$ in the 100 ℔, than the other which gaue 4 vpon the 100, for tare and cloffe.

If 100 ℔ of any thing cost me 23 s̃ 4 d̃, the question is how I shall sell the ℔ to gaine after the rate of 10 ℔ vpon the 100 ℔.

Answere. Say by the rule of three, if 100 ℔ giue 110 ℔, what shall 23 $\frac{1}{3}$ s̃ giue? Multiplie and diuide, and you shall finde 1 ℔ $\frac{17}{60}$. Then say

againe, if 100 pound be woꝛth 1 $\frac{17}{20}$ poũd, what is one pound woꝛth? Mulꝛiply and diuide, and you shall find 3 d $\frac{3}{25}$. And so much is the pound woꝛth in gaining 10 pound vpon the 100.

Item a Gꝛocer hath bought C weight of a commoditie foꝛ 6 pound 10 s̃. The questiõ is now to know how many poũds thereof he shall sell foꝛ 32 s̃ 4 d̃ to gaine 20 s̃ in the C weight.

Anſwere. Adde 20 s̃ vnto 6 pound 10 s̃, and they make 7 pound—10 s̃. Then say if 7 $\frac{1}{2}$ yeeld me 112 pound, what shal 1 $\frac{2}{3}$ poũd yeeld? Mulꝛiply and diuide, & you shal find 24 pound $\frac{8}{9}$. And so many pound ought he to sell to gaine 20 shillings in his C. weight.

If one pound weight cost 3 s̃—4 d̃, and I sell the same againe foꝛ 4 s̃, what is gained in 100 pound of money laid out in that commoditie.

Anſwere. You may say if 3 $\frac{1}{3}$ s̃ giue 4 s̃, what will 100 pound giue? But then whẽ you haue found, you must subtract 100 poũd out of the pꝛoduct, ẏ rest is your neat gaine, oꝛ else to pꝛoduce the neate gaine in your woꝛke at the fiꝛst. Then subtract the iust pꝛice out of the ouerpꝛice, as I taught befoꝛe in the fiꝛst beginning of losse and gayne, and your conclusion shall bee all one. Mulꝛiplie and diuide by which of the two wayes you thinke good, and you shall finde that hee gayneth 20 pound

fi

in the 100 pound.

Item if the pound weight which cost 4 s̄, be sold againe for 3 s̄—4 d̄, I demaund what is lost in the 100 ℔ of money?

Answere. Say, if 4 s̄ loose $\frac{2}{3}$ s̄, what shall 100 ℔ loose? Multiplie and diuide, and you shall find 16 ℔—13 s̄—4 d̄, and so much is lost vpon the 100 ℔ of money.

Item if a C weight of any commoditie cost 45 ℔, and the buyer repenting would loose 5 ℔ in the 100 ℔ of money, I demaund how the pound may be sold: his losse to be neither more nor lesse than after the rate aforesaid of 5 by the hundreth?

Answere. By the rule of 3, if 100 ℔ loose 5 ℔, what shall 45 ℔ loose? Worke and you shal finde 2 $\frac{1}{4}$ pound, which rebated from the principall 45, resteth 42 ℔—15 s̄. Lastly say if 112 ℔ yeeldeth but 42—15 s̄, what 1 pound? Multiplie and diuide, and you shall finde 7 s̄ 7 d̄ $\frac{1}{2}\frac{7}{8}$. And so much is the pound worth after that losse.

A Grocer hath bought 3 peeces of raisins, weying 175 $\frac{1}{2}$ pound, 182 $\frac{1}{4}$ ℔, 191 ℔, tare for each fraile 2 $\frac{1}{4}$ ℔, at 25 $\frac{1}{2}$ s̄ the C weight. The question is what they amount to in money?

I answere 6 ℔ 3 s̄—4 d̄ $\frac{25}{28}$.

A

A Grocer hath bought 3 sacks of Almonds waying 267 ½ l' tare 2 pound: 257 ½ poũd tare 2 ½ pound, 252 pound tare 3 pound at 2 s̃ 10 ½ the pound, what amount they to in money?

I answere 110 pound — 12 — 3 ¾ s̃.

The ninth Chapter intreateth of *Lengthes and Breadthes of Arras, and* other clothes, with other questions incident vnto Length and Breadth.

IF a peece of Arras be 7 elles and ¾ long, and 5 elles, and ⅔ broad: how many elles square doth the same peece containe?

Answere. Multiplie the length by the breadth, that is to say 7 ¾ by 5 ⅔. And thereof will come 43 ½ elles: so many elles square doth the same peece containe.

Item more a peece of Arras doth containe 22 elles square, & if the same were in length 3 ¼ elles, I demand how many elles in breadth the same peece doth containe?

Answere. Diuide 22 elles by 3 ¼, & thereof commeth 6 ⅔. So many elles doth the same containe in breadth.

Item more a Merchant hath 3 ¼ Elles of Arras,

Arras, at 1 $\frac{2}{3}$ elles broad, which he will change with an other man for a peece of Arras, that is $\frac{7}{8}$ elles square. The question is how many elles of that squarenesse ought the first Merchant to haue.

Answere. Multiplie the first Merchants peece his length by the breadth, and you shall finde it containeth 5 $\frac{1}{12}$ elles, which $\frac{1}{12}$ Elles you shall diuide by $\frac{7}{8}$, and you shall finde 6 $\frac{4}{21}$ elles, & so many elles of that squarenesse ought the latter Merchant to giue the first.

Item a student hath bought 3 $\frac{1}{2}$ yardes of broad cloth, at 7 quarters broad, to make a Gowne, and should line the same throughout with Lambe, at a foote square each skin: the question is now how many skinnes he ought to haue?

Answere. Seeke first the number of yards square that his cloth containeth, which to do, multiplie 3 $\frac{1}{2}$ his length, by 1 $\frac{3}{4}$ his breadth, and you shall find 6 $\frac{1}{8}$ yardes square, then say by the rule of three, if 1 yarde square giue 9 foote, what shall 6 $\frac{1}{8}$. Worke, and you shal find 55 $\frac{1}{8}$ skinnes.

Item more, a Lawier hath a rich peece of seeling come home, which is 24 foote and three Inches long, and 7 foote and 2 $\frac{1}{2}$ Inches high: the Ioiner is to be payd by the yard square: The question is, how many yardes this peece con-

containeth?

Answere. Multiplie his length by his breadth, that is to wit $24\frac{1}{4}$ foote by $7\frac{5}{24}$ foot, and you shall finde $174\frac{77}{96}$ foote square, which 174 you shall diuide by 9 (for so many foote make a yard square, and you shal find 19 yards, three foote, and $\frac{42}{77}$ of a foote, and so many yards doth this peece hold.

Item, bought a peece of Holland cloth conteining 36 Elles $\frac{1}{3}$ Flemmish. The question is, how many Elles English it makes?

Answere. You must note, that fiue Elles Flemmish do make but three Elles English. Therefore say by the Rule of three, if 5 Elles Flemish make but three Elles English, how many Elles English will $36\frac{1}{3}$ Elles Flemmish make? Multiplie and diuide, and you shall finde $21\frac{4}{5}$, and so many Elles English doth $36\frac{1}{3}$ Elles Flemmish containe. The like is to be done of all others.

Item more, I haue bought 342 Elles Flemish, of Arras worke at two Elles broad, Flemmish, and I would line the same with Elle broad Canuas of English measure. The question is, how many Elles English will serue my turne?

Answere. For as much as three Elles English are worth fiue Elles Flemmish, therefore put 3 Elles English into his square, and multiplying

plying 3 by himselfe, which maketh 9. Likewise multiplie the English elle which is fiue quarters euery way into himselfe squarely, & you shall find 25. Then multiply 342 which is the length of the peece, by two which is the breadth, and thereof commeth 684, then say by the rule of three, as before: if 25 Elles square of Flemmish measure be worth 9 elles square of English measure, what are 684 of Flemmish measure? Multiply and diuide, and you shall find 246 $\frac{6}{25}$ Elles English.

The same is also wrought by the backer rule of three, in seeking the squares contained in the Flemmish elle of two elles broad (which are 18): and also in seeking the squares conteined in the English elle (which are 25) then say by the rule of three backeward, if 18 quarters require 342 Elles, what shall 25 quarters giue? Multiplie and diuide by the rule of three. Reuerse, and you shall finde as before 246 $\frac{6}{25}$ Elles English.

Item more, at two shillings 4 pence the Flēmish Elle, what is the English elle worth after the rate?

Answere. Say if three quarters giue 3 $\frac{1}{3}$ s̄, what giueth fiue quarters? Multiply and diuide, and you shall finde 5 shillings — 6 $\frac{4}{9}$ d.

Item more, at 8 s̄ 4 d̄ the Flemmish Elle square, what is the English Elle worth after

ter that rate.

Answere. According to the reason of the last question, consider that a Flemmish Elle square is equall to 9 quarters of a yard English, & an English Elle square is equall to 25 quarters of a yard. Therefore say by the rule of 3, if 9 quarters giue 8 $\frac{1}{3}$ s. what 25 quarters? Worke & finde 23 s — 1 $\frac{7}{9}$ pence. And so much is the English Elle worth.

Item more, at 6 s — 8 d the Elle square, what shall a peece of cloth cost that is 7 $\frac{1}{2}$ elles long, and three $\frac{1}{4}$ elles broad?

Answere. Multiplie the breadth by the length and you shall finde 24 $\frac{3}{8}$ Elles square. Then say by the rule of three, if 1 elle square cost 6 $\frac{2}{3}$ s, what 24 $\frac{3}{8}$? Multiply & diuide, & you shall finde 8 pounds — two s — 6 pence, and so much the same peece of cloth shall cost.

Item more, a Mercer sold 3 peeces of silke. To wit 24 $\frac{1}{4}$ 23 $\frac{1}{3}$, & 25 yardes at 9 $\frac{3}{4}$ shillings the yard, & was glad to receiue in part of payment againe, a cloth containing 34 $\frac{1}{2}$ yards at 7 $\frac{2}{3}$ shillings the yard. The question is now, what the Debitor is in the Creditors debte. Worke, and you shall finde he oweth the Mercer two and twentie pounds — 3 s — two $\frac{1}{4}$ d.

The

The tenth chapter intreateth of the reducing of the Pawnes of Geanes into Englisſh yardes.

NOte, that 100 Pawnes doe make 26 yards, whereupon 3 Pawnes $\frac{11}{13}$ do make one yard, and one Pawne after that rate and proportion is $\frac{13}{50}$ of a yard.

In 4563 Pawnes of Geanes, how many yards Englisſh?

Anſwere. Say by the rule of three, if 100 pawnes doe make 26 yards, what will 4563 pawnes make? Multiplie and diuide, and you ſhall finde 1186 yards $\frac{12}{50}$. So many yardes do 4563 pawnes make.

Otherwiſe, take ſome other number at your pleaſure, as 10 pawnes, which is the $\frac{1}{10}$ part of 100, then to finde his proportion, take the $\frac{1}{10}$ part of 26, which is two $\frac{3}{5}$, and then ſay alſo by the rule of three, if 10 pawnes giue $2\frac{3}{5}$ yards, what will 4563 pawnes giue? Worke, and you ſhall finde 1186 $\frac{12}{50}$ yards, as before.

Woze, at two ſhillings—6 d the pawne of Geanes, what will the Englisſh yard be woꝛth after the rate?

Anſwere. Say by the rule of three, if $\frac{13}{50}$ of a yard coſt two $\frac{1}{2}$ s, what one yard? Multiply.

and

and diuide, and you ſhall find 9 s̄—7 $\frac{1}{13}$ d̄.

Note, if 346 $\frac{1}{2}$ Pawnes coſt 30 poūds—13 ſhillings 4 pence ſterling, what is that the Engliſh yard after the rate?

Anſwere. Say by the rule of three, if 346 $\frac{1}{2}$ Pawnes coſt 30 $\frac{2}{3}$ pounds, what are three $\frac{1}{13}$ Pawnes woꝛth (foꝛ ſo many Pawnes make a yarde:) Multiplie and diuide, and you ſhall finde $\frac{27}{27}\frac{00}{027}$ parts of a pound, which in knowen numbers is woꝛth 6 ſhillings—9 pence $\frac{6}{9}\frac{2}{009}\frac{7}{9}$.

The eleuenth chapter intreateth of Rules of Loane and Intereſt, with certaine neceſſarie Queſtions and Proofes incident thereunto, &c.

ITem lent my friend 326 ℔ foꝛ 5 $\frac{1}{2}$ moneths ſimplie without any Intereſt, vpon condition, to haue the like courteſie againe when I neede. But when I came to boꝛow, he could ſpare me but 149 pounds—8 ſhillings—4 d̄, the queſtion is now how long time I ought to haue ỹ vſe thereof, to counteruaile my friendſhip befoꝛe time ſhewed him.

Anſwere. Say by the backer rule of three if 326 giue 5 $\frac{1}{2}$ moneths, what time will 149 pounds $\frac{1}{2}$ giue? Multiplie and diuide, and you ſhall find 12 moneths, ỹ ſo long time ought I

to vse his money.

The Proofe.

Item, lent my friend 149 pounds—8 s—4 d, for 12 moneths. The question is now, how much money he ought to lend me againe for fiue ½ monethes to recompence my friendship shewed him.

Answere. Say by the backer or reuerse rule of 3: if 12 moneths giue 149 5/12, what shall 5 ½ moneths giue? Worke, and you shall finde 326 pounds, and so much ought he to lend me to requite my gentlenesse or good turne.

Two other Branches yet more for Proofe out of the same questions.

Item lent my friend 149 pounds—8 s—4 d for 12 moneths, to haue the like friendship againe when I neede. And comming to borow of him, he very courteously tooke me 326 l (for that he could well then spare the same.) The question is now, how long I ought to occupie it, not vsurping friendship, but in his due time to restore it againe.

Answere. Say by the Rule of three Reuerse, if 149 5/12 giue 12 moneths, what shall 326 pounds giue? Multiply and diuide, & you shall finde, that at 5 ½ monethes terme, I ought to restore it againe.

Proofe,

Proofe.

Item lent my friend 326 ℔ for 5 ½ moneths: The queſtion is now, how many pounds hee ought to lend me for 12 monethes to recompence the pleaſure againe.

Anſwere. Worke by the rule of three Reuerſe, as you haue done before, and you ſhall finde 149 pounds—8 s̄—4 pence.

Againe, foure other ſelected queſtions of Loane and Intereſt, all out of one braunch, and each one alſo a neceſſarie Queſtion, and a particular proofe to other.

ITem, lent my friend 430 pounds at Intereſt for 3 moneths to receiue after the rate of 8 pounds in the 100 for 12 moneths. The queſtion is, what the Intereſt commeth to?

You may if you pleaſe worke it at two workings by the rule of three direct, in ſaying: If 12 monethes giue 8 pound, what giueth three monethes? Multiplie and diuide, and it giueth two pound.

Thē for the ſecond worke ſay, If 100 poūds yeeld 2 pounds, what yeldeth 430 ℔? Multiply and diuide, and you ſhall finde 8 ℔—12 s̄, and ſo much comes the loane of 430 ℔ to for 3 moneths,

nethes, after the rate of 8 pounds in the 100 pounds of 12 moneths.

Otherwise wrought thus by the Rule of 3 at twise also.

If 100 ℔ giue 8 ℔, what giueth 430 ℔: Multiply & diuide, and you shall find 34 pounds $\frac{2}{5}$.

Then againe for the secõd worke say. If 12 moneths giue 34 pounds $\frac{2}{5}$, what giueth 3 moneths: Worke & find 8 poũds—12 s̃ as before.

Otherwise yet at one working. By the first part of the rule of 5 numbers forward, in saying, if 100 pounds in 12 moneths gaine 8 ℔, what shal 430 poũds gaine in three moneths: Multiplie the first by the second for your diuisor, and the other three the one into the other for the deuidend, and you shall find 8 pounds—12 shillings, as aforesaid.

Proofe.

Item a friend of mine receiued of me eight pounds 12 shillings for the Interest and vse of 430 pounds for three moneths terme: the question is now, what he tooke in the 100 pounds for the twelue moneths after that rate.

Answere. For most brief, say by ꝑ first part, or rule of 5 numbers forward: If 430 pounds in three moneths did pay 8 ℔—12 s̃, what doth 100 pounds in 12 moneths take after ꝑ rate: Worke, & you shall find 8 pounds, and so much he tooke vpon the 100 pounds for 12 moneths.

A third queſtion and Proofe alſo wrought by the backer rule of 5 Numbers.

Item, lent my friend 430 pound to receiue for the Intereſt thereof, after the rate of 8 ℔ in the 100 ℔ for 12 moneths. The queſtion is now, how long time my friend ought to giue the vſe thereof, that it may be returned with 8 ℔—12 s̄ gaines.

You may worke it if you pleaſe, by the rule of 3 direct at twice, in ſaying: If 100 ℔ yeeld 8 ℔, what yeeldth 430 ℔: Multiplie and diuide, and find 34 ℔ and $\frac{2}{5}$.

Then againe for the ſecond worke ſay, If 34 $\frac{2}{5}$ giue 12 moneths, what giueth 8 $\frac{3}{5}$ pound: Multiplie and diuide, and you ſhall finde 3 moneths, and ſo long time ought my friend to vſe it, to returne with 8 ℔—12 s̄ gaine.

Otherwiſe at one working by the backer rule of 5 numbers, in ſaying: If 100 ℔ in 12 moneths doo gaine 8 ℔, how long time ſhall 430 ℔ be a gaining of 8 pound—12 s̄: Multiplie the firſt and the ſecond into the laſt for your diuidend, and the third and fourth multiplie together for your diuiſor, and then diuide, and you ſhall finde 3 moneths the iuſt time that my friend ought to vſe it, to returne it with 8 pound—12 s̄ gaine.

A fourth deriued question out *of this Braunch which is a Proofe* of this last, and also of the other two going before.

ITem how much money ought a merchant to deliuer after 8 ℔ in the 100 for 12 monethes that in 3 moneths he may gaine 8 ℔—12 s̄.

Answere. You may also if you please worke it by the Golden Rule of three at twice, first saying, if 3 monethes gaine 8 ℔, what 12 monethes gaine? you shall finde 34 ℔. Then say againe, if 8 ℔—be come of 100 ℔, what shall come of 34 ℔—8 s̄? Worke and you shal find the answere to the question, which is 430 ℔, and so much ought the merchant to deliuer.

But most briefely it is answered by the Backer Rule of 5 numbers, where I argue thus, saying: If 100 ℔ be 12 moneths a gaining of 8 ℔, then but for 3 monethes terme, onely to take 8 ℔—12 s̄ must needs be a good round summe: to worke it, set your number thus, 100—12—8—3—$8\frac{3}{5}$: multiplying the first into the second, and also by 43 the product of the fift, for your diuidend, and the third & fourth together with 5 the Denominator of your fraction, for your Diuisor: then diuide, and you shall finde as before 430 ℔. The true solution to your question.

The

The twelfth Chapter treateth of *the making of Factors which is* taken in two sortes.

THe first is, when the estimatiō of the factor, is taken vpon the sending of the Merchant, as if the estimation of his person be $\frac{1}{4}$, it is vnderstood, that he shal haue $\frac{1}{4}$ of the gaine, and the Merchant the other $\frac{3}{4}$.

The other sorte is when the estimation of his making is out of the sending of the Merchant, as if the order and agreement betweene them were such, that the Merchant shall put in 800 ℔, and the Factor for his making shall haue $\frac{1}{4}$, neuertheless he shall haue but $\frac{1}{5}$ of the gaine or profite, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 pound make 1000 ℔, whereof the 200 ℔ is $\frac{1}{5}$.

A Merchant doth put in 800 ℔ into the hands of his Factor: vnder such condition, that the said Factor shall haue $\frac{1}{4}$. And after certaine time, they finde in profite 124 ℔—6 ß 8 ð, I demaund how much the Merchant shal haue hereof, and how much ought the Factor to haue?

Answere. When the estimation of the factor is out of the sending of the Merchant, it maketh

l	s	d	
99	9	4	for the Merchant.
24	17	4	for the Factor.

But if that his estimation be at the sending of the Merchant, then it maketh but

l	s	d	
93	5	0	for the Merchant.
31	1	8	for the Factor.

For the Merchant is then to haue $\frac{3}{4}$, and the Factor $\frac{1}{4}$.

A Merchant doth put into the handes of his Factor 800 l, and the Factor 400 l to haue the $\frac{1}{5}$ of the profite, I demaund now for how much his person is esteemed, when the same is counted vpon the sending of the Merchant.

Answere. According to the tenor and order before prescribed in the first rule, that is, if his estimate be $\frac{1}{4}$, he shall haue the $\frac{1}{4}$ of the gayne. Therefore say by the rule of 3 direct, If $\frac{1}{4}$ taken put in 400 l, what is the estimate, or putting in of $\frac{1}{5}$ taking? Multiplie and diuide, and you shall find 320 l, and so much is the person of the Factor estimated.

Otherwise.

To finde the estimation of the person of the

the Factor, you shal consider, that seeing it was agreed betweene them, that the Factor should take the $\frac{1}{5}$, then the Merchant shall haue the residue, which are $\frac{4}{5}$: wherefore the gaine of the Merchant vnto that of the factor is in such proportion as 5 vnto 4. Then if you wil know the estimation of the person of the Factor. Say if 5 giue 4, what will 400 giue? Multiplie and diuide, and you shall finde 320 l. And so much is the person of the Factor esteemed to be worth.

Other conditions than these aforesaid, may also be betweene Merchants & Factors without respect, either of sending or not sending of the Merchant, where most commonly the estimation of the body of the Factor is in such proportion of the stocke which the Merchant layeth in, as the gaine of the said Factor is vnto the gaine of the Merchant. As thus, If a merchant do deliuer into the hands of his factor 400 pound, and he to haue halfe the profite. The person of the said factor shal be esteemed to be worth 400 pound, and if the Factor do take but $\frac{1}{5}$ of the gaine, hee should haue but $\frac{1}{2}$ so much of the gaine as the Merchant taketh which must haue $\frac{2}{5}$, wherefore the person of the Factor is esteemed but the $\frac{1}{2}$ of that which the Merchant layeth in. That is to say 200 pound.

And if the factor did take the $\frac{1}{3}$ of the gaine,

then

then the merchant shall take the residue which are $\frac{2}{5}$, wherefore the gayne of the Merchant vnto the Factor is then in such proportion as 3 vnto 2: whereupon if you will then know the estimation of the person of the Factor. Say if 3 giue 2, what shall 400 giue: Worke and you shal finde 266 $\frac{2}{3}$ pounds. And so much is the person of the Factor esteemed to be worth.

And if the Merchant should deliuer vnto his Factor 400 pound and the factor would lay in 80, and his person, to the end he might haue the $\frac{1}{2}$ of the gaine, I demaund how much shall his person be esteemed.

Answere. Abate 80 from 400, and there will remaine 320. And at so much shal his person be esteemed.

A Merchant hath deliuered vnto his factor 900 ℔ to gouerne in the trade of merchandise, vpon condition that he shall haue the $\frac{1}{3}$ of the gaine, if any thing be gained, and also to beare the $\frac{1}{3}$ of the losse, if any thing be lost. Now I demaund how much his person was esteemed at.

Answere. Seeing that the factor taketh the $\frac{1}{3}$ of the gaine, his person ought to be esteemed as much as $\frac{1}{2}$ of the stock, which the Merchant layeth in. That is to say the $\frac{1}{2}$ of 900 pound which is 450. The reason is, because $\frac{1}{3}$ of the gaine that the Factor taketh, is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gaine that the Merchant taketh, and so

the

the factor his person is esteemed to be worth 450 pound.

A Merchant hath deliuered vnto his Factor 600 pound. And the Factor layeth in 250 pound and his person. Now because he layeth in 250 pound and his person, it is agreed betweene them, that he shall take the $\frac{2}{5}$ of the gaine, I demaund for how much his person was esteemed?

Answere. For as much as the Factor taketh $\frac{2}{5}$ of the gaine, hee taketh $\frac{2}{3}$ of that which the Merchant taketh: for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factors laying in, ought to be 400 pound, which is $\frac{2}{3}$ of 600 pound that the Merchant laid in. Then subtract 250, which the Factor did lay in, from 400 pound which should haue beene his whole stocke, and there remaineth 150 pound for the estimation of his person.

More, a Merchant hath deliuered vnto his Factor 800 pound, vpon condition that the factor shal haue the gaines of 160 l, as though he laid in so much readie money, I demaund what portion of the gaine the said Factor shal take?

Answere. See what part the 160 l (which the Factor layd in) is of 960 which is the whole stocke of their companie, and you shall finde $\frac{1}{6}$. And such parte of the gaine shall the Factor take.

But in case, that in making their couenants,

nants, it were so agreed betweene them that the factor should haue the gaine of 160 pound of the whole stocke which the merchant layeth in. That is to say of the 800 pound, thē should the Factor take $\frac{1}{5}$ of the gaine: for 160 is $\frac{1}{5}$ of 800 pound.

The thirteenth Chapter entreateth of Rules of Barter, & exchanging Merchandise, which is distinct into 7 Rules, with diuers others necessary Questions incident thereunto.

The first Rule.

TWo Merchants willing to change their merchandise the one with the other. The one hath 24 broad clothes at 10 pound—10 s̄ the peece. The other hath Mace, at 12 s̄ the pound. The question is how many pound of Mace he ought to giue him for his clothes, to saue himselfe harmelesse and be no looser.

Answere. Seeke first by the Rule of three, what the 24 clothes coste at 10 pound—10 s̄ the peece, & you shall finde 252 l. Then to find the quantitie of Mace, say againe by the Rule of 3, if 12 s̄ buy one pound, what shall 252 pound buy me? Worke, and you shal find 420 pound of Mace. And so many pound ought he

to

to giue for his clothes.

The Proofe.

Two barter, the one hath 420 pound of Mace at 12 s the pound, to barter or chãge for broad clothes at 10 poundes—10 s the peece. The question is how many broad clothes hee ought to giue for all his Mace.

Answere. First say if one cost 12 shillings, what 420? You shall find 5040 shillings. Thẽ say againe if 10 ½ poundes giue 1 cloth, what shall 5040 shillings giue? Worke, and you shall finde 24 clothes your desire.

The second Rule.

Two change merchandise for merchandise. The one hath Pepper at two s—4 pence the pound, to sell for ready money. But in barter he wil haue no lesse than 3 shillings the pound. And the other hath Holland at 5 shillings—6 pence the Elle ready money. The question is now at what price he ought to deliuer the Elle in barter to saue himselfe harmeless.

Answere. Say by the rule of three direct: if two ⅓ s ready money giue three s in barter, what shall 5 ½ shillinges giue in barter? you shall finde 7 ¼ shillings, & at that price ought the second Merchant to sell his Holland in barter.

The

The Proofe.

Two barter. The one hath Holland at 5 s̄ — 6 pence the Elle to sell for ready money. And in barter he will haue 7 $\frac{1}{14}$ s̄. The other hath Pepper at two s̄ — 4 pence the pound to sell for ready money. The question is now how he ought to sell in barter.

Answere. Say by the rule of three direct. If 5 $\frac{1}{2}$ ready money giue 7 $\frac{1}{14}$ s̄ in barter, what ought two $\frac{1}{3}$ s̄ to take in barter? Multiply and diuide, and you shall find 3 s̄ your desire.

The third Rule.

Two barter. The one hath cloth of Arras at 30 s̄ the Elle ready money, but in barter he will haue 35 $\frac{1}{2}$ shillings. And the other hath white wines, which he deliuered in barter for 16 l̄ for a Tunne. The question is now what his wines cost the Tunne in ready money.

Answere. Say by the Rule of three direct. If 35 $\frac{1}{2}$ s̄ in barter giue but 30 readie money, what did 16 pounds in barter cost? Worke & you shall finde 13 pounds — 10 s̄ $\frac{30}{71}$. And so much cost his wines for a Tun ready money.

The Proofe.

Two barter merchandise for merchandise. The one hath white wines at 13 l̄ — 10 s̄ $\frac{30}{71}$ the

the Tunne to sell for ready money. But in barter he deliuered it for 16 pounds. The other to make his match good and saue himselfe harmelesse, deliuereth Arras at 35 ½ s the elle. The question is now, what an Elle of his Arras cost in ready money.

Answere. Say by the rule of three direct. If 16 l in barter giue but 13—10 30/71 s in ready money, what shall 35 ½ shillings yeeld in barter? Worke and you shall find 30 s your desire.

The fourth Rule.

Two barter. The owne hath Kerseys at 14 pounds the peece ready money. But in barter he will haue 18 pounds. And yet he will haue the ⅓ parte of his ouerprice in readie money. And the other hath Ginger at eight groats the pound to sell for ready money. The question is how he ought to deliuer the Ginger by the pound in barter to saue himselfe harmelesse, & make the barter equall.

Answere. Item for the working of this question, and such other the like, you must vnderstand, if the partie ouerselling his wares, require to haue also some portion in ready money, as ½ : ⅔ : ¼ &c. Then shall you firste rebate the same demaunded part whatsoeuer it be from the ouer price, and also from the iust price. And those two numbers that shall remaine after the subtraction is made, shall be

the

the two first numbers in the rule of three. And the iust price of the same merchandise shall be the third number, which by the operation of the rule of three direct shal yeeld you a true solutiō how and at what price you shall ouersell that your merchandise, to saue your selfe harmelesse, and make the barter equall.

Example.

Take the $\frac{1}{3}$ (of eighteene) which is the ouer price of his cloth, which $\frac{1}{3}$ of eighteene is sixe, which as appeareth here in the margent, you must subtract from 18, there resteth 12. And also abate it from 14, which is the iust price of the cloth, and there remaineth 8, which eight and 12 are the two first nnmbers in the rule of three. Then take eight groates or two $\frac{1}{3}$ shillings for the third nūber. Then say by the rule of 3 direct, If 8 pounds giue 12 pounds, what shall $2\frac{2}{3}$ s̄ giue? Multiplie and diuide, and you shall find 4 s̄. And for so much shal the second Merchant sell his Ginger, or his commoditie in batter, to ballance the same equall.

14	18
6	6
8	12

The Proofe.

Two barter. The one hath fine Kerleyes at 14 pounds the peece ready money. But in barter

ter he will haue 18 ponndes, and yet he will haue ℔ $\frac{1}{3}$ part of his ouerprice in readie money. And the other hath Ginger, which he hauing cunning inough to make the barter equall, deliuered in barter for 4 shillinges the pound. The questiō is now what his Ginger cost him in ready money.

Answere. After you haue made the subtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 & 14, as before was taught you, then will there remaine 8 and 12 for your two first numbers in the rule of three. Then say, if 12 giue but 8, what shal come of 4 the ouerprice of the pound of Ginger? Multiply and diuide, and you shall find two shillings—8 pence your desire.

Two Merchants barter merchandise for merchādise. The one hath Denshire whites at 7 poūds—13 s̄—4 d̄ the peece ready money, but in barter he doth thē away for 8 ℔—3 s̄—4 d̄. And yet he will haue the $\frac{1}{3}$ part of his ouerprice in ready money. And the other hath Cottens at 3 ℔ the peece ready money. The question is now at what price he ought to sell or exchange his Cottens in barter to saue him selfe harmelesse, & make the barter equall.

7—13—4	8—3—4
2—14—5 $\frac{1}{3}$	2—14—5 $\frac{1}{3}$
4—18—10 $\frac{2}{3}$	5—8—10 $\frac{2}{3}$

Answere. First seeke $\frac{1}{3}$ part of 8 ℔—3 s̄—4 d̄, which is 2 ℔—14—5 $\frac{1}{3}$ d̄, which rebated from 8—3—4 d̄, there resteth as appeareth by the example aboue said 5—8—10 $\frac{2}{3}$ p̄ce, which $\frac{1}{3}$ of 8—3—4 d̄ also rebated from 7—13—4 d̄, there resteth 4—18—10 $\frac{2}{3}$ the two first numbers in the rule of 3. And the 3 pouds which is the neate price of the peece of Cotten is the third number. Then say by the rule of three direct, as was taught before: If 4—18—10 $\frac{2}{3}$ d̄ giue 5 ℔—8 s̄—10 $\frac{2}{3}$ d̄, what shall 3 ℔ giue? Multiply & diuide, and you shall finde 3—6—0 $\frac{7}{8}$ $\frac{2}{9}$ pence the iuste price that he ought to deliuer his Cottons in barter.

The fifth Rule.

Two Merchants will change merchandise for merchandise. The one hath Kerseyes at 40 s̄ the peece to sell for ready money. And in barter he will sell them for 56 s̄—8 d̄, and he will gaine after 10 ℔ vpon the 100 ℔. And yet he will haue the $\frac{1}{2}$ of his ouerprice in ready money. The other hath Flaxe at 3 d̄ the pound ready money. The question is now how he shal sell the pound of his Flaxe in barter.

Answere. See first at 10 ℔ vpon the 100 ℔ what the 56 $\frac{2}{3}$ s̄ commeth to, in saying by the rule of three direct. If 100 ℔ giue 110 ℔, what 56 $\frac{2}{3}$? Multiply and diuide, and you shall finde 3 ℔—2 s̄—4 d̄ of which the $\frac{1}{2}$ that he demaun-

deth

deth in ready money, is 1 ℔ — 11 s̄ — 2 pēce, the same 31 s̄ — 2 pence abated frō 40 s̄, and also from 56 s̄ — 8 d̄: there will remaine 8 s̄ — 10 pence, and 25 s̄ — 6 d̄, for the two first numbers in the rule of three, and 3 pence the price of the pound of flaxe for the third number. Thē multiplie and diuide, and you shall finde 8 $\frac{3}{5}$ pence. And for so much shall he sell the pound of flaxe in barter.

The sixth Rule.

Two are willing to exchāge Merchandise: The one hath Norwich Grograynes at 25 s̄ the peece ready money. And in barter he will haue 30 s̄, and he will haue the $\frac{1}{4}$ part of his ouerprice in ready money. The other hath Norwich stockings at 40 s̄ the dozen to sel for ready money. But in as much as the first Merchaunts Grograynes are no better, hee would deliuer them so to ballance the barter, that he may gaine after 10 pounds in the 100 pounds. The question is now how he shall sell his hose the dozen in barter, according to his request.

Answere. Say, if one 100 giue 110, what shall 40 s̄ giue, which is the iust price of the dozen of stockings: Multiplie and diuide, and you shall finde 44 s̄. Then take the $\frac{1}{4}$ of 30 s̄, which is 7 s̄ — 6 d̄. And subtract it from 25 s̄, & also from 30 shillings, & there will remaine

17 s̄—6 d̄—£ 22 s̄—6 pēce, for the two first numbers in the rule of three, and 44 s̄ which is the iust price (with his gaine in the dozen of stockings) for the third number. Then multiply and diuide, & you shall finde 56 shillings $6\frac{5}{7}$: and so much he is to sell his dozen of stockings in barter.

The seuenth Rule.

Two Merchants will change their merchandise one with the other. The one hath 720 Elles of Cambricke at 5 s̄ the Elle to sell for ready money, but in barter he requireth 6 s̄—8 d̄. And yet notwithstanding he looseth by it after 10 pounds vpō the 100 pounds, wherupon he requireth the $\frac{1}{2}$ of his ouerprice in ready money, and the other Merchant hauing skil inough to make the barter equall, deliuereth English Saffrons at 30 s̄ the pound. The question is now what his Saffron cost the pound in ready money.

Answere. You must first seeke what is lost vpon the 100 pounds, which to do you may say if you please, If 100 l̄ loose 10, what shall $6\frac{2}{3}$ loose? Worke & you shall find $\frac{2}{3}$ s̄ (or 8 d̄) which must be rebated frō 6 s̄—8 d̄. So resteth 6 s̄ still: or you may say if 100 poūds giue me but 90 pounds, what shall 6 s̄—8 d̄ giue? Worke this way eyther, and you shall finde also as before, directly in your quotient 6 s̄ your desire.

sire. Then are you next to cast vp what ỹ 720 Elles of Cambricke commeth to at 6 s̄ 8 d̄ the Elle, and you shall finde 240 pound: the ½ whereof the Cambricke Merchant will haue in readie money (which is 120 pound) nexti ly you must cast what the Cambricke commeth to after his losse in the 100 pound, which as you found is but 6 s̄ an elle, and you shall finde 216 pound. Now must you subtract his readie money (which is 120 ℔) out of 240 ℔, and also out of 216 ℔, and there will remaine 120 pound and 96 pound for your two first numbers in the rule of three and 30 s̄ the ouerprice of your Saffron for the third number. Then multiply and diuide, and you shall finde 24 shillings. And so much did his Saffron cost in readie money.

Two Merchants barter. The one hath 50 Clothes to put away for readie money at 11 pound the Cloth, and in barter putteth them away for 12 pound, taking Holland Cloth at 20 d̄ the Flemmish Elle, which was worth no more but 18 d̄. The question is now, what Holland payeth for the cloth, and what he winneth or looseth by the bargaine?

Answere. 50 Clothes at 11 ℔ the cloth cõmeth to 550 ℔, and put away at 12 ℔ the peece, maketh 600 ℔: Then to finde what Holland payeth for the Cloth, say by the rule of 3 direct, If 20 d̄ buy 1 Elle, what 600 ℔? Worke,

and you shall find 7200 Elles. Now to finde the estate of his gaine or losse, you must seeke what his 7200 Elles commeth to at 18 d the Elle. Worke by the rule of Proportion direct and you shall finde 540 l, which is not so much as his Clothes were worth in readie money by 10 pound, and so much lost the first Marchant by his exchange.

A Venetion hath in London 100 peeces of silke, to put away for readie mony at 3 l the peece. But in barter he deliuereth them for 4 l the peece, taking wools of a Felmonger at 7 l — 10 s the C weight, which was worth no more but 6 l the C in readie money. The question is now, what wools payeth for the silkes, and which of them winneth or looseth by the barter.

Answere. 100 peeces of silke at 3 l, is 300 l, and at 4 l is 400 l. Then to find what woolles payeth for the silke, say by the rule of three direct. If 7 ½ l buy me 1 C weight, what 400 l? Worke, and finde 53 ⅓ C weight of wooll. Now to finde the estate of their gaine or losse, cast vp his wooll at 6 l the C. (for so much they were worth readie money) and you shall finde 320 l, which is 20 l more than the silkes were to be sold for in readie money, whereupon the Venetian gained 20 l by the barter.

A

A Merchant hath 53 ⅓ C. weight of wooll at 6 ℔ the C. to sell for readie money, but in barter he will haue 7 ℔—10 s̄: and an other doth barter with him for Silkes which are worth 3 pounds a peece ready money. The question is now, how he ought to deliuer his silkes the peece in barter: and how many payeth for the woolles.

Answere. Say by the rule of Proportion, or ẏ rule of three direct, If 6 ℔ for a C. weight ready money yeeld me 7 ℔—10 s̄, what will 3 ℔ yeelde, which is the iust price of a peece of silke in barter, to make the Trucke equall: Worke, and finde 3 ℔—15 s̄ the price of a peece of silke in barter: then say, if 3 ℔—15 s̄ require 1 peece of silke, how many peeces of silke are bought with 400 ℔, which is the value of the 53 ⅓ C. weight of woll at 7 ℔—10 s̄. Worke by the rule of three direct, and you shal find 106 peeces of silke, and ⅔ of a peece, and so many peeces of silke paieth for the woolles, and neither partie hath aduantage of other.

Two men will change merchandise the one with the other. The one of them hath Beere at 6 s̄—8 d̄ the barrel to sel for readie money. But in barter he will sell the barrell for 8 s̄, and yet he will gaine moreouer after 10 ℔ vpon the 100 ℔. And the other hath white Spanish wooll at 20 s̄ the Roue to sell for readie money. The question is now, how he shall

deliuer the Roue of wooll in barter to saue himselfe harmelesse.

Answere. Say if 6 $\frac{2}{3}$ s̄ which is the iust price of the barrell of Beere, be sold in barter for 8 shillings, for how much shal 20 s̄ (which is the iust price of the Roue of wooll) be sold in barter? Worke by the rule of three direct, and you shall find 24 s̄. Then for because the first merchant will gayne after 10 l̄ vpon the 100 l̄, he maketh of his 100 l̄ — 110 l̄. And therefore say by the rule of three, if the second merchant of 110 pound doe make but 100 pound, how much shal he make of 24 s̄? Multiply & diuide, and you shall find 21 s̄ — 9 d̄ $\frac{2}{11}$ of a peny. And for so much shall he sell the Roue of wooll to be deliuered in barter, to the end the first merchant may gaine 10 in the 100.

Two merchants will change their commodities the one with the other. The one of them hath white paper at 4 s̄ the reame to sell for ready money. And in barter he will do it away for 5 s̄, and yet he will gaine moreouer, after the rate of 10 l̄ vpon the 100 l̄, and the other hath Mace at 14 s̄ — 6 d̄ the pound weight, to sell in barter. Now I demaund what the poūd did cost in readie money?

Answere. Say, if 5 s̄ which is the ouer price of the paper in barter be come of 4 s̄ the iust price, of how much shall come 14 s̄ $\frac{1}{2}$, which is the surprice of the pound of Mace in bar-

barter: Multiplie and diuide, and you shall finde 11 s̄ $\frac{3}{5}$. Then for because the first Merchant of Paper will gaine after 10 vpon the 100. Say, if 100 do giue 110, what shal 11 $\frac{3}{5}$ s̄ giue: Worke, and you shall finde 12 s̄—9 d̄ $\frac{13}{25}$, and so much did the pound of Mace cost in ready money.

The fourteenth Chapter entreateth of exchanging of money from one place to another.

EXchange is no other thing than to take or receiue money in one Citie, to render or pay the value thereof in an other Citie, or else to giue money in one place, and receiue the value thereof in an other, at terme of certaine dayes, moneths, or fayres, according to the diuersitie of the place.

But this practise chiefly consisteth in the knowledge of the Money or Coynes in diuers places, of which, for thy benefite, (after a fewe examples giuen to the Introduction of this worke) I will set downe certaine notes of the diuersitie of the common and vsuall Coynes in most places of Christendome for traffike.

And first I will begin at Antwerpe, where they vse to make their accounts by Deniers de gros:

gros: that is to say by pence Flemmish, whereof 12 do make 1 s̄ Flemmish, and 20 s̄ do make 1 l̄ de gros.

Item a Merchant deliuered at Antwerpe 400 pounds Flemmish, to receiue in London 20 s̄ sterling for euerie 23 s̄ — 4 d̄ Flemmish. The question is now, how much sterling money is to be receiued at London for the said 400 pounds Flemmish?

Answere. Say by the rule of three, if 23 1/3 Flemmish giue 20 s̄ sterling, what 400 pounds Flemmish? Worke, and you shall finde 342 l̄ — 17 s̄ — 1 5/7 d̄, and so much sterling shall I receiue in London for the 400 l̄ Flemmish.

Otherwise also wrought by Rules of Practise in taking the 1/7 of the Flemmish money deliuered, and abating the same from the principall, the rest is English money sterling, as before.

400 ·· 0 ·· 0

57 ·· 2 ·· 10 · 2/7

342 ·· 17 ·· 1 · 5/7 sterling.

A Merchant at London deliuereth 200 poūd sterling for Antwerpe at 23 s̄ 5 d̄ Flemmish the pound sterling. The question is, how much he must receiue at Antwerpe.

Answere.

Answere. Say by the Rule of three, if 1 ℔ sterling giue 23 s̄—5 d̄ Flemmish, what 200 pound sterling ? Worke, and you shall finde 234 ℔—3 s̄—4 d̄. So many pounds Flēmish shall he receiue at Antwerpe for the said 200 pound sterling.

Otherwise also by Practise.

1 •• 23 •• 5 •• 200			
3 s̄ 4 d̄	33 ••	6 •• 8	
1 d̄	••	16 •• 8	
maketh	234 ℔ ••	3 •• 4	sterling.

In London 200 ℔ sterling is deliuered by exchange, for Antwerpe at 23 s̄—9 d̄ Flemmish the ℔ sterling. The question is, what rate the Flemmish money ought to be returned to gaine 4 ℔ vpon the 100 ℔ sterling at London.

Answere. First say by the Rule of three direct. If 1 pound sterling giue 23 ¾ Flēmish, what 200 ℔ sterling? Multiplie and diuide, and you shall finde 237 pound—10 s̄. The which to returne to gaine 8 pound sterling in London, say by ỹ backer Rule, If 200 pound sterling require the exchange 23 s̄—9 d̄ Flemmish, what the exchange to make 208 pound sterling ? Worke by the Rule, and finde 22 s̄—10 d̄ 1/26 d̄ Flemmish the effect in the question required.

If

If I take vp money at Antwerpe after 19 s̄—4 d̄ Flemmish, to pay for the same at London 20 s̄ sterling, and when the day of payment is come, I am forced to returne the same money againe in London to pay my Bil of exchange, so that for 20 s̄ which I take vp here at London, I must pay 19 s̄—6 d̄ at Antwerpe, I demaund whether I do win or loose, and how much in or vpon the 100 pound of money.

Answere. Say by the rule of three: If 19 ½ giue 19 ⅓, what will 100 l̄ giue? Multiplie and diuide, and you shall finde 99 l̄—2 s̄ $\frac{106}{117}$ which being abated from 100 l̄, there will remaine 17 s̄ $\frac{11}{117}$, and so much do I loose vpon the 100 l̄ of money.

If I take vp at London 20 s̄ sterling to pay at Antwerpe 22 s̄—4 d̄, and when the day of payment is come, my Factor is cōstrained to take vp money againe at Antwerpe, wherewith to pay the aforesaid summe, and there he doth receiue 23 s̄—4 d̄ Flemmish, for the which I must pay 20 s̄ at London, the question is now whether I do winne or loose, and how much vpon the 100 pound of money after that rate.

Answere. Say by the Rule of Proportion, If 22 ⅓ s̄ giue 23 ⅓ s̄, what will 100 l̄ giue? Mul-

Multiply & diuide, & you shall find 104 poũds $9\frac{111}{201}$ s̃, frõ the which abate 100 pounds, & there will remaine 4 pounds $9\frac{111}{201}$ s̃, and so much is there gained vpon the 100 pounds of money.

In Antwerpe is deliuered 200 poũds Flẽmish by exchange for London, at 20 s̃ sterling for euery 23 s̃—4 d̃ Flemmish. The question is at what rate the same is to be returned to gaine 10 pounds vpon the 100 pounds Flemmish in Antwerpe.

Answere. First say by the rule of three, if $23\frac{1}{3}$ Flẽmish giue 20 s̃, what shall 200 poũds giue? Worke, & you shall find 171 pounds—8 s̃ $6\frac{4}{7}$ d̃. Thẽ say againe by the rule of 3 direct, if 171 l̃ 8 s̃ $6\frac{4}{7}$ sterling giue me 210 l̃ Flẽmish, what shall 20 s̃ sterling giue? worke, and you shall find 24 s̃—6 d̃ Flẽmish. And at the same rate ought ỹ same to be returned at Antwerpe to gaine 10 pounds vpon the 100 Flemmish.

A Merchant of Antwerpe deliuereth 234 l̃—3 s̃—4 d̃ Flemmish, to receiue at London 200 poũds sterling. The question is now, how the Exchange goeth after this rate.

Answere. Say by the Rule of 3 direct, If 200 giue 20, what giueth $234\frac{1}{2}$? Multiply & diuide, and you shall find 23 s̃—5 d̃. And for so much goeth the Exchange.

Item the Exchange from London into France,

France, is not like as it is in Flaunders, but is deliuered by the French Crowne, which is worth 50 soulx Tournois the peece.

Whereupon also you must note, that in France they make their accounts by Francks, Soulx and Deniers Tournois, whereof 12 Deniers maketh one soulx Tournois, and 20 soulx maketh 1 ℔ Tournois, which they call a Liure or Francke. But the Merchants to make their accounts do vse French crownes, which is currant among them for 51 Soulx Tournois. But by exchange it is otherwise, for it is deliuered but for 50 Soulx Tournois the Crowne, or as the taker vp of the money can agree with the deliuerer. And note that this v Caracter representeth the Crowne by exchange, and is euer 50 Soulx Tournois or French money.

A Merchant deliuereth in London 240 pounds sterling after 5 s̄ — 6 d̄ the crowne, to receiue at Paris 50 soulx Tournois for euery crowne, I demaund how much Tournois or French money payeth the bils for the said 240 pounds sterling.

Answere. Say by the rule of three, if 5 ½ s̄ sterling giue me 50 s̄ Tournois, what shall 240 ℔ sterling giue? Reduce the pounds into shillings, then multiplie and diuide, and you shall find 2181 Liures — 16 Soulx, — 4 Deniers

niers, and $\frac{4}{1}$, Tournois, & so much payeth the Bils at Paris for the said 240 pound sterling.

A Merchant deliuereth in Roan, or elsewhere in France 1430 pounds, or Francks, the which Francke or pound is 20 soulx, or a pound Tournois, to receiue in London 6 s̄—4 d̄ sterling for euery △ of 50 soulx Tournois. The question is how much sterling money I ought to receiue at Lódon for my 1430 poūd Tournois.

Answere. Say, if two ½ poūds giue me 6 ¼ s̄, what will 1430 giue me? Worke, & you shall finde 3622 s̄ ½ sterling, which maketh 181 ℔ 2 s̄—8 pence, and so much money is to be receiued at London for the said 1430 Liures Tournois, after 6 s̄—4 pence euery Crowne of 50 soulx.

In London is deliuered 200 ℔ sterling by exchange for Paris, at 5 s̄—9 d̄ the ▽ of 50 soulx Tournois. The questiō is at what price the said crowne is to be returned to gayne 6 ℔ vpon the 100 pounds sterling at London.

Answere. First say by the rule of three direct, if 5 ¾ s̄ sterling giue 50 soulx Tournois, what shall 200 poūds sterling giue? Worke, & you shall finde 1739 Franckes or Liuers, two soulx $\frac{14}{23}$. Then, the which to returne & gaine 6 pounds vpon the 100 ℔ in London, say by the rule of 3 direct, if 1739 Franckes two soulx $\frac{14}{23}$

yeeld

yeld 212 pounds, what the Δ of 50 Soulr: woꝛke, and finde 6 s̄—1 d̄ $\frac{7}{50}$. the effect requi-red in the queſtion.

A Marchant deliuered in London 160 l. ſterling, to receiue in Biskay foꝛ euery 5 s̄ 6 d̄, 1 Duckate of 374 Marueides. The queſtion is, how many Marueides I ought to receiue at Biskay.

Anſwere. Say, if 5 $\frac{1}{2}$ s̄ ſterling giue 374 Marueides, what ſhall 160 pounds ſterling giue: Multiply and diuide, and you ſhall finde 217600 Marueides, and ſo many I ought to receiue at Biskay foꝛ my 160 pounds ſterling.

A Marchant deliuereth in Bayon 20000 Marueides to receiue in London 5 s̄—6 pēce ſterling, foꝛ euery Duckate of 374 Mar-ueides. The queſtion is now how much ſter-ling money payeth the Bils of Exchange foꝛ the ſaid 20000 Marueides.

Anſwere. Say, if 374 Marueides make 1 Duckate, what 20000 Marueides: Multiply and diuide, and find 106 Duckates $\frac{178}{187}$.

Then ſay againe, if 1 Duckate giue 5 $\frac{2}{3}$ s̄, what giueth 106 $\frac{178}{187}$ Duckates: Woꝛke, and find 30 poūds—6 s̄ and $\frac{34}{50}$. s̄ which is woꝛth $\frac{408}{561}$ parts of a peny.

Otherwiſe it is wꝛought moꝛe bꝛiefer at one woꝛking, as in the laſt queſtion befoꝛe, in-con-

considering that 5 s̄ —8 d̄ containeth one Duckate, or 374 Marueides. Therefore say by the rule of three, if 374 Marueides giue 5 $\frac{2}{3}$ s̄, what 40000 Marueides ? Worke, and you shall also finde in your quotients 30 pounds—6 s̄ $\frac{34}{561}$. And so many pounds sterling is to be receiued for the 40000 Duckates.

In London 200 l̄ deliuered by Exchāge for Vigo 374 Marueides the Duckate, of 5 s̄ 10 d̄ sterling, maketh 256457 $\frac{1}{7}$ Marueides, ỹ which to returne and gaine 10 pounds vpon the 100 pounds in London, say by the rule of three direct, if 220 pounds require 256457 $\frac{1}{7}$ Marueides, what 5 s̄—10 d̄ ? worke, and finde 340 Marueides, prices of euery Duckate in return: which is the effect in the question required.

These may seeme sufficient for instructions.

NOtwithstanding for thy further aide and benefite hereafter follow 6 speciall & most briefe rules of Practise for English, French and Flemmish money.

1	teacheth	how to turne Flemish to English sterling.
2		how to turne Engl. sterling to Flemish.
3		how to turne Flemish to French.
4		how to turne French into Flemish.
5		how to turne sterling into Frēch.
6		how to turne Frēch into sterling.

The fifteenth Chapter intreateth of the said sixe Rules of Breuitie, and of valuation of Englisch, Flemmisch, and French money, and how each of them may easily be brought to others value.

How briefly to reduce ℔, s̄, and d̄ Flemmisch, into ℔, s̄, and d̄ Englisch Sterling.

IT is to be noted, that 7 ℔ Flemmisch maketh but 6 pounds sterling, 7 s̄ Flemmish maketh 6 s̄ sterling, & 7 d̄ Flēmish 6 d̄ sterling, so that 7 yeldeth but 6. Wherein is euident, that there is lost $\frac{1}{7}$, (if it may be so called) when it is reduced into Englisch money. Wherefore to know how much 233 pounds, 13 s̄, 4 d̄, Flemmish maketh Englisch, you must subtract from it $\frac{1}{7}$, beginning with the pounds, &c. and that which resteth after this subtraction is the summe required: so that 233 pounds, 13 s̄, 4 d̄ Flemmish maketh 200 pounds, 5 s̄, $8\frac{4}{7}$ peuce sterling.

Example.			Another Example.		
℔	s̄	d̄	℔	s̄	d̄
233	13	4	311	0	0
$\frac{1}{7}$ 33	7	$7\frac{1}{7}$	$\frac{1}{7}$ 44	8	$6\frac{6}{7}$
200	5	$8\frac{4}{7}$ ster.	264	11	$5\frac{1}{7}$

To

To reduce ℔, s̄, and d̄ Sterling, into ℔, s̄, and d̄, Flemmish.

Note that a pound sterling, maketh 1 poũd, 3 s̄ 4 d̄ Flemmish, that is 1 poũd $\frac{1}{6}$: 1 s̄ sterling maketh 1 s̄ $\frac{1}{6}$ Flem. & 1 sterl. maketh 1 $\frac{1}{6}$ Flem. So that there is gained (if it may be so called) $\frac{1}{6}$ of the summe beyng thus reduced to Flem. For of $\frac{6}{6}$ is made $\frac{7}{6}$, which is one whole, and $\frac{1}{6}$. Then to knowe how much 237 pound, 7 s̄ 6 d̄ sterl. maketh flem: subtract from your sterl. the $\frac{1}{6}$ of the whole summe, and adde it to the same summe, and it maketh 276 pounds 18 s̄ 4 pence, which is the summe required.

Example.				Another Example.			
℔	s̄	d̄		℔	s̄	d̄	
237	7	6		337			
$\frac{1}{6}$ 39	11	3		$\frac{1}{6}$ 56	3	4	
276	18	9	ster.	393	3	4	

Yee shall note, that the equalitie of Flemmish and French money is this, that is to say the pound Flemmish maketh 7 pound $\frac{1}{5}$ Frẽch or Turnois, 1 shilling Flemmish maketh 7 s̄ $\frac{11}{5}$ French, and a groat Flemmish, maketh 7 d̄ $\frac{1}{5}$ French.

Wherefore to know how much 143 pounds 4 s̄, 9 d̄ Flemmish maketh French, yee must multiply the whole number twise by 6, begin-

ning at 8 d, & so forward, & the product of your second multiplication diuide by 5, so that worke is finished. Or multiply the said sum by 7, and take out of it $\frac{1}{5}$, adding it to the product of your multiplication by 7, & that is your number required. So that as well by the one as by the other, 143 pōds 4 s̄ 9 d̄ Flemmish maketh 1031 pound 6 s̄ 2 d̄ $\frac{2}{5}$ French or Turnois.

Example.				The same otherwise.			
℔	s̄	d̄		℔	s̄	d̄	
143	4	9	flem.	143	4	9	
		6				7	
859	8	6		1002	13	3	
		6		$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$	
5156	11	0		1031	6	2 $\frac{2}{5}$ frē.	
$\frac{1}{5}$ 1031	6	2 $\frac{2}{5}$ frē.					

Another Example.				Or thus.			
143	℔	Flem.		143			
6				7			
858				1001			
6				$\frac{1}{5}$	28	12	
5148				1029	℔	12	s̄ frēch.
$\frac{1}{5}$ 1029	℔	or 12	s̄ Frēch.				

A briefe Reduction of ℔. s̄ and d̄ French, into ℔. s̄ and d̄ Flemmish.

Multiplie 233 ℔, 8 s̄, 4 d̄ French by 5, and diuide the product twice by 6, that is the said number by 6, and the product againe by 6, and the quotient of this second Diuision is the thing required. So that 233 ℔, 8 s̄, 4 d̄ French, maketh 32 ℔, 8 s̄, 4 d̄, $\frac{1}{9}$ flemmish.

Example.				Another.			
℔	s̄	d̄		℔	s̄	d̄	
233	8	4	fre.	753			fren.
		5		5			
1167	1	8		2756			
$\frac{1}{6}$ 194	10	$3\frac{2}{6}$		$\frac{1}{6}$ 627	10		
$\frac{1}{6}$ 32	8	$4\frac{1}{9}$	fle.	$\frac{1}{6}$ 104	11	8	flem.

To reduce ℔. s̄ and d̄ Sterling, into ℔. s̄ and d̄ French or Turnois.

The ℔ sterling maketh 8 ℔, 8 s̄ french, that is to say 8 ℔ $\frac{2}{5}$: the s̄ maketh 8 s̄ $\frac{2}{5}$, and the peny 8 d̄ $\frac{2}{5}$ French. Wherefore to know what 231 ℔, 13 s̄, 4 d̄ sterling maketh French, yee must multiply your whole summe by 42, that is by 7, and the product of it by 6, and diuide this

second product by 5, and that is the summe required.

Otherwise multiplie the summe sterling by 8, and adde twice to the product $\frac{2}{5}$, and it shall produce the summe required. So that both waies 231 l, 13 s, 4 d sterling, maketh 1946 l French, as here vnder followeth.

Example.				The same otherwise.			
l	s	d		l	s	d	
231	13	4	ster.	231	13	4	ster.
		6				8	
1390	0	0		1853	6	8	
		7		46	6	8	
$\frac{1}{5}$ 9730	0	0		46	6	8	
$\frac{1}{5}$ 1946	0	0	fren.	1946	0	0	fren.

An other Example.				The same.			
		753	ster.			753	ster.
		6				8	
	4518				6024		
	7			$\frac{1}{5}$	150	12	
				$\frac{1}{5}$	150	12	
	31626						
$\frac{1}{5}$	6325	4	fre.		6325	4	fren.

To

To reduce ℔, s̄, and d̄ Fren, into ℔, s̄, d̄ Ster.

To know how much 1256 ℔ 12 s̄ 6 d̄ fren. maketh in sterling money, multiply the summe by 5, and diuide the product by 7 and 6 at twice, and the last quotient shall be the thing required, that is to say, 1256 ℔ 12 s̄ 6 d̄, maketh 149 ℔ 11 s̄ 11 d̄ $\frac{4}{7}$ sterling.

Example.

℔	s̄	d̄	
1256	12	6	fren.
		5	
6283	2	6	
$\frac{1}{6}$ 1047	3	9	
$\frac{3}{7}$ 149	11	11 $\frac{4}{7}$	ster.

An other Example.

℔	s̄	d̄	
2531			fren.
5			
12755			
$\frac{1}{6}$ 2109	3	4	
$\frac{1}{7}$ 301	6	2 $\frac{2}{7}$	ster.

Note that when any money is giuen by exchange at London for Roan at 71 d̄ $\frac{1}{2}$, or rather 71 $\frac{1}{7}$ for the crowne of 50 s̄ french, there is neyther gaine nor losse, for it is one money for another, accounting 8 ℔ 8 s̄ French for 1 ℔ sterling. So the giuer loseth the time of payment which is about 15 dayes, and hee that taketh it, hath gaine of the same.

They of Roan that put forth or take money by exchange for London, ought to haue like consideration.

Item when any man giueth at London 64 d $\frac{1}{3}$, or rather 64 d $\frac{2}{7}$, to haue at one of the Fayres of Lions a crowne de Marc, he that so giueth the money looseth the time, and he that taketh it, gaineth the same: for 62 d $\frac{2}{7}$ is equall in value to 45 s̄ French. He that putteth or taketh money at Lions for London, ought to consider the same.

Item, when any deliuer in Antwerpe 75 d, to receyue at Lions a crowne of Marke, hee that putteth it foorth, looseth the time, and hee that taketh it, gayneth the same. For 75 groats Flemmish is equall in value to 45 s̄ French.

Thus for this time I make an ende of the practise of Exchange and the instructions therunto belonging, and according to my promise, yet further to gratifie such as are desirous to know the common Coynes vsed for traffike among Merchantes in these Cities following, here followeth a briefe declaration of their moneys and the reckonings and accounts of them.

The

The sixteenth chapter containeth a declaration of the valuation and diuersitie of Coines of most places of Christendome for traffike. And the maner of exchange in those places from one Citie or Towne to an other, which knowne is right necessary for Merchants, by meanes whereof they do find the gaine or losse vpon the exchange.

ITem for as much as the greatest diuersitie of mouey of exchange is at Lions. Therefore I will begin duly of the money of that place.

At Lyons they vse Franckes, Soulx and Deniers Tournois: A Francke maketh 20 Soulx, and one Soulx 12 Deniers. But the Merchants to keepe their bookes of accounts doe vse French Crownes of the marke at 45 Soulx the peece, and do diuide it into 20 s̄, 1 s̄, and 12 d̄.

Item a Marke of Golde maketh 65 A of the Marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graynes: And so the summe or whole by imagination or gesse.

Also at Lyons there are 4 faires in a yeere, at the which they doe commonly exchange, which are from three moneths to 3 moneths.

At

At Geanes they vse the soulx: one Ducket maketh ℔ 3.

At Naples they vse Duckets, Tarie, and Graines, The Ducket maketh 5 Taris, and one Tarie 20 graines: but they take 6 Duckets (which maketh thirtie taries) for the ounce.

A Ducket maketh 10 Carlins, and a Carlin 10 graines, so that 2 Carlins make a Tarie, and 100 graines make a ducket.

At Rome they vse the duckets of the chamber: one ducket is worth 12 Guylis, & a Guylis 10 soulx.

At Venice they vse Duckets Currants at 124 soulx a peece or 24 deniers, and 1 denier maketh 32 picolis.

At Palerme and Mesine they write, after ounce, tarie, and graines, and 1 ounce is worth 6 duckets or 30 taries, & 1 tarie is 20 graines, and 1 graine 6 picolis, 1 ducket is also woorth 24 Carlins.

At Millan they vse ℔, s̃, d̃, of duckets Imperials, and ∆ of exchange is worth 4 ℔.

At Lucques, Florence and Ancone, they vse the ∆ of golde: in golde the French crowne is worth ℔ 7, but at Buloigne ℔ 3, 10 s̃:

At Barselone they vse the soulx, the ducket of exchange is worth 22 soulx.

At Valence and Saragosse they vse the Liuer, Soulx and Denier, the French crowne of exchange is worth 20 Soulx, and 1 Soulx is

12 deniers.

At the Fayres of Castil they vse the Marueides, the ducket is worth 375 Marueides.

At Lisbone they vse the Raies, one ducket of exchange is worth 400 rayes.

At Noremburge, Franckford, and August, in Germanie, they vse the Krentzers, whereof 60 make a Florin.

At Antwerpe they vse s̄, and d̄ de Gros, and they exchange into the denier de Gros, to wit our English penie.

At London they vse the 1 l̄ sterling, and 1 d̄ sterling, and they exchange in 1 d̄ sterling.

The exchange of Lyons at sundrie places.

Item at Lyons there is exchange in three sorts, at the cities and townes following.

First they deliuer at Lyons one Marke to haue or receiue at Naples almost 41 ½ duckets, at Venice 70 duckets currant, at Rome 63 duckets of the Chamber, Luques and Florence 65 Δ of gold, at Millan 82 Δ.

And contrariwise at the said Cities aforesaid they do giue so much of money to haue a marke at Lyons.

Secondly they giue at Lisbone one Δ of Marke of 45 Soulz Turnois a peece, to haue at Gennes almost 68 Soulz. At Palerme and Mesine almost 24 Carlins. At Barselone 22 Soulz, at Valence or Saragosse 20 soulz. At the faire at Castil 350 Marueides, at Lisbone

bone 360 Rais, in Antwerpe 57 deniers de Gros, and at London 70 d sterling.

And contrariwise they do giue in the said Cities almost as much of their money to haue a French crowne, of the marke at Lions.

Thirdly they doo giue at Lions a ∆ of the summe to haue almost 93 Krentzers at Franckford, Ausburg, Norēberge, or other Cities in Almaine.

Also at Lyons onely they do pay the change the $\frac{2}{3}$ in gold, and $\frac{1}{3}$ in money, or else all in money, in giuing $1\frac{1}{2}$ for the hundreth.

Changes at Naples and other Townes.

Item at Naples they giue or deliuer almost 112 duckets to receiue at Rome 100 duckets of the chamber at the old value.

Through Luques and Florence they deliuer 100 duckets Carlins to receiue there almost 86 ∆ of gold.

Through Palerme and Messine one ducket of 5 Tary to receiue there almost 164 grains.

Through Millan one ducket, to receiue there almost 90 soulz.

Through Geanes one ducket, to receiue there almost 65 soulz. The whole summe to be paide within 10 dayes after the sight of the bill of exchange.

Also at Naples they deliuer one ducket to receiue in Antwerpe almost 67 d or deniers de Gros, within 2 moneths. At London almost

most 60 ℔ sterling in three moneths. At Barselone almost 20 Soulx within two moneths. At Valence almost 18 soulx within two moneths. At Lisbone 333 Raies, within three moneths, & at the fayre at Castil, almost 340 Marueides at the same fayre.

Change of Venice to other places.

At Venice they deliuer 100 Duckats currant to receiue in Almaine almost 140 Florenes at 60 Krentzers the peece.

At Lucques and Florence almost 108 ∆ of gold in 10 daies.

Likewise at Venice they deliuer a Duckat currant to receiue at Palerme and Mesine almost 21 Carlins, at Milan almost 93 Soulx. At Geanes almost 62 Soulx, the whole at 10 daies end.

Of the Pair or Pari.

As touching the exchange, it is necessarie to vnderstand or know the Pair, which the Italians call Pari, which is no other thing than to make the money of the change of one Citie or Towne to or with the money of another, by meanes whereof they do find the gaines or losse vpon the exchange.

Example.

Item hauing receiued letters of credite of one of Antwerpe that the ∆ of the Sunne is there worth 7 Soulx. The question is what the same is worth at London when the Pair

or

or exchange goeth for 23 shillings.

Answere. Say if 23 giue but 20, what giueth 7? Worke & finde 6 shillings 1 $\frac{1}{23}$ d, and so much is the Δ of the Sunne worth at London.

The seuenteenth Chapter conteineth also a declaration of the diuersitie of the waights and measures of most places of Christendome for traffike. At the ende of which discourse are two Tables, the one for waight, & the other for measure, proportionate and reduced to an equalitie of our English measure and waight, by the aid wherof the ingenious may easily by the Rule of three, conuert the one into the other at their pleasure, &c.

AT London & so all England through are vsed two kindes of waights and measures, as the Troy waight & the Haberdepoise: from the Troy waight is deriued the proportion and quantitie of al kinde of dry & liquide measures, as Pecks, Bushels, Quarters, &c. wherewith is bought and solde all kinde of grayne and other commodities mette by the bushell. And in Liquid, Ale, Beere, Wine, Oyle, Butter, Honie, &c. vpon these groundes and statutes is bread made, and sold by the Troy waight. And so is Gold, Siluer, Pearle, Pretious stones and Iewels. The least quantitie of this Troy waight

waight is a graine: 24 of these graines make a penie waight, 20 penie waightes an ounce, and 12 ounces a pound: 2 pounds or 2 pintes of this waight maketh a quart. And so ascending into bigger quantities is produced the measures whereby are solde our other naturall sustenance, viz. Ale, or Beere, with also other necessarie commodities, as Butter, Honie, Herrings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contayned) bee framed and deriued from the Troy waight, yet are they in traffike, with diuerse commodities, as Lead, Tinne, Flaxe, Waxe, with all other commodities both of this Realme, and of other forrayne Countries whatsoeuer, bought and solde by the Haberdepoise waight, after 16 ounces to the pound, and 112 pounds to the C. waight. And vnto euery C. is allowed 12 ℔ waight at the common beame. From hence is also deriued the weigh of Suffolke Cheese, which cõteineth 32 Cloues, 8 poundes to a Cloue, and wayeth in all 256 poundes. And also the barrell of Suffolke butter is or should be of like weight with the weigh of cheese, viz. 256 poundes. Note 14 of these pounds make a stone, and 26 stone containeth a sacke of English wool. Forraine wools, to wit, French, Spanish, and Estrich, is also sold by the pound or C. waight, but moste commonly by the Roue, 25 pound to a Roue: other commodities of Tale are bought and solde by the C.

fiue

fiue score to the C. Except headed ware, to wit, cattell, nayles, and Fish, which are solde after fyre score to the C. There is also two other sortes of measures, to wit, the Elle, and the Yard. By the Elle is vsually mette Linnen Cloth, as Canuas, &c. And by the yarde, Silks, woollen Clothes, &c.

Antwerpe.

321

At Antwerpe are also 2 sortes of waights, their gold and siluer waight, and their common waight. Gold and siluer is weyed by the Marke, the Marke is 8 ounces, the ounce 20 Esterlings, and the Esterling 32, as our graines. The Goldsmithes diuide that into smaller, but not the Merchaunts: the proofe of gold is made by Caracts, whereof 24 maketh a Marke of fine gold: the Caract is 24 graines, the proofe of the money is made by Deniers, 12 Deniers is 1 s̃ fine, that is a Marke of fine siluer, the Denier also is diuided into 24 graines, and the graine into foure quarters.

Item, 100 Marks in Antwerpe, Troy waight, maketh at Lions 103 Marks, two ½ ounces, and 20 graines 23 p̃. At Noremburg 103 markes, 2 ½ ounces, 2 quints, 3 Deniers, at Frankford 105 Marks, at Ausburge 104 markes, three ounces, 1 quint. At Venice 103 markes, 1 ounce, 7 Deniers, 18 graines. At London 66 pounds.

The marke of Gold or siluer at Antwerpe Troy

Troy waight, which is 8 ounces, maketh 7 ½ ounces common waight, with which all other marchandise is weyed. So that ỳ Troy waight is greater than the common waight by sixe ¼ in the C. By this waight of Troy they also weigh Muske, Amber, Perle, &c.

All silkes are bought at Antwerpe, by the Bruges Elle, which is greater than the cõmon measure, by which they retaile, by two in the C. Their common Elle is ¾ of our yard, and ⅘ of our Elle.

Lyons.

At Lyons is vsed 3 sorts of waights, whereof the first is the common Towne waight, with which they weigh all kinde of Spicerie, and diuerse other merchandise. The second is called Geneua waight, which is eight in the 100 greater than the common waight, with which they weigh silkes, &c. The third is French waight, called commonly the Marke waight, and 100 pounds thereof maketh 106 ¼ pouds Geneua, & 114 ¾ of their cõmon waight: with which French waight is weyed all things that payeth custome or toll.

At Lyons is also vsed two sorts of Elles or Aulnes. The one wherewith they measure grosse Clothes, as Canuas, and such like. The other is called the French Elle or Aulne, with which they measure all other kinde of Merchandise, whereof seuen cõmon Towne Elles

maketh 11 ordinarie French Elles.

Roan.

At Roan 6½ Muydes of salt, being the measure of the place, make an hundred at Armuiden in Zeland, & the C. of Bronage measure of Armuiden maketh at Roan 11 Muides. 30 Mines make a Last of Corne, and 16 a last of Dates, 100 pounds waight there, maketh at London 114 ¼, and 109 ⅝ at Antwerpe, And 100 Elles make at London 115 ⅜.

Noremburge.

A 100 pounds waight of Noremburge, maketh at London 111 ¾: at Antwerpe 107 ½. And 100 Elles of Noremburge make at London 75 ⅖, at Antwerpe 95 ⅗, &c.

Lisbone.

The 100 waight of Lisbone maketh foure Roues, euery Roue 32 poũds, so that their C. waight is 128 pounds, and their pound containeth 14 ounces, & 100 pounds of their waight maketh at London 113 ⅛.

Their Silke, cloth of Gold and woollen is measured with a measure which they call a Cubite, containing about ¾ of a Uarre of Castile. Howbeit, their common measure is called a Uarre which maketh 5 Palmes, and cõtaineth 1 ¼ of a Uarre of Castile, our Elle of London is equall with the Uarre of Lisbone.

All

All kind of merchandise brought frō Flanders, Roan or Brittaine, paieth at Lisbone, as a ductie or custome to the king, 20 in the 100 which they call the tenth in merchandise, and the other tenth in money.

Note also that al kinde of merchandise comming to Lisbone by land, payeth lesse in custome than that that commeth by water.

Ciuill.

The Roue of Ciuill is 30 pounds, 4 Roues make their C. waight, which is 120 pounds. The 100 pounds of Ciuill maketh at Londō 102 pounds. Their other common measure is a Uarre, whereof 100 maketh at London 74 Elles, and at Rome 40 Canes, &c.

Venice.

At Venice be two sorts of waights, the one called La Grosse, the other La Suttle, with the grosse is weyed all kinde of great wares, and with the small, all kinde of spicerie, and such like: 96 pounds of grosse waight there, maketh at London 100 pounds, & 100 pounds of spicerie there without any tare or allowāce, make at London 64, and with tare 56.

Their other common measure are Braces, whereof 100 make at London 55 ½ Elles, at Antwerpe 92 ½, &c.

Florence.

At Florence the 100 ℔ waight maketh at

Aquila, for Saffrõ 110, & 145 pounds of Florence make at Roan 100 pounds, the waight of Florence, and that of Luke is all one.

Their other measures are Braces, whereof 100 maketh at Antwerpe Burges measure 81 ⅔ Elles, 100 Braces there make at London 49 Elles, &c.

Lucque.

The Lucque Sattens are commõly sold at Lions by waight & 133 ½ poũds, maketh at Lions 100 ℔, so that 1 ℔ ½ maketh at Lions but 1 ℔.

Their other measures are Braces, whereof 100 of them make at London 50 Elles, of Antwerpe 83 ⅓ Elles, &c.

Aquila.

At Aquila their 100 poũds maketh at London 71 ¼, their 136 ⅔ pounds of Saffron maketh at Geneua but 100, and 11 pound of Geneua maketh 15 pound at Aquila.

Valentia.

At Valentia be 2 sortes of waights, a great and a small. The C. waight of great waight containeth 4 Roues, the Roue 36 ℔, so the 100 great waight is 144 ℔, & the C. waight small cõtaineth but 120 pounds, & is also parted into 4 Roues, which is 30 poũds to a Roue. By the small is sold the Scarlet graine, with all other kinde of spicerie, and by the great is sold wooll with al such like grosse wares. The 1 ⅓ poũds

of

of silke at Valentia maketh at Lions 1 ℔ Geneua waight. The charge of great merchandise at Valentia containeth 432 ℔, and in smal wares 360 ℔. The waight here & at Barsellone is all one. Their 100 pound waight maketh at London 78 ℔, and at Antwerpe 75.

Dansicke.

At Dansicke or Spruce land the rule is, that whosoeuer buyeth any merchandise there, buyeth it by the Shippound, which is 320 ℔, 20 Lispounds make a Shippound, and the Lispound containeth 16 ℔, which Shippound of Dansick maketh at Antwerpe 266 ⅔ ℔. Their 100 ℔ waight maketh at London 86 ⅛, &c.

Their other common measures are Elles, whereof 100 make at London 72 ¼, and at Antwerpe 120 ½ Elles.

Toulouse.

At Toulouse 6 Cabes of Woad, maketh a Charge, 2 Cesternes of Cozne, and all kinde of graine maketh a Charge, the Cesterne waieth 160 ℔ waight of that place. Their 100 ℔ in waight maketh at London but 91 ¼ ℔.

Geanes.

At Genua or Geanes, a 100 ℔ of their waight maketh at London 71 ¼, and at Antwerpe 68 ⅜, 100 ℔ waight at Genua maketh at Venice, to wit, Suttle 106 ℔.

Their other cõmon measures are Palmes, whereof 100 make at London 20 4/5 Elles, and at Antwerpe 34 3/5.

The eighteenth Chapter treateth of Sportes and Pastimes, done by Number.

IF you would know the number that any man doth thinke or imagine in his mind, as though you could diuine, bid him triple it, or put twice so much more to it, as it is, which done, aske him whether it be euen or odde? if he say odde, bid him take 1 to it, to make it euen, and for that 1 keepe 1 in your minde. Now after he hath taken 1 to it to make it euen, bid him giue away halfe, and keepe the other halfe for himselfe, which when he hath doone, bid him triple that halfe, and againe after he hath tripled it, aske him whether it be euen or odde: if he say odde: then bid him take 1 to make it euen againe, and for that last 1 keepe 2 in your minde, now after he hath made his number euen, bid him cast away the one halfe, and keepe the other still, from which halfe that he keepeth, cause him subtilly to put away or giue you 9 out of his number, and for each 9 that he giueth you keepe 4 in your minde, and thereunto ioyne the 3 which I bad you keepe, and you shal haue your desire.

Ex-

Example.

Imagine he thought 7, the triple whereof is 21, and because it is odde, he is to take 1 to make it euen, which first 1 giuen is: for you to keepe in minde. Then the halfe of his 22 being cast away, he reserueth still 11, which after you haue bid him triple, it maketh 33, then in giuing of him 1 againe to make it euen: vpon that last 1 reserue 2 in your minde, then his halfe of 34 maketh 17, from whence he can giue you 9 but once, Therefore that yeelding to you 4, and the 3 that you keepe, make 7 your desire.

An other kinde of Diuination, to tell your friend how manie pence or single peeces, reckening them one with another, he hath in his purse, or should thinke in his minde.

Which to do, first bid him double the peeces he hath in his purse, or the number he thinketh, if he participate his number or secrecie vnto some one friend that sitteth by him, that can but multiplie, and adde neuer so little: if their number be great, then shall they worke as you bid them so much the surer.

Now after he hath doubled his number, bid him adde thereunto 5 more, which doone, bid him multiplie that his number by 5 also: which doone, bid him tell you the iust summe

of his last Multiplication, which summe the giuer thinking it nothing auaileable, because it is so great aboue his pretended imagination: yet thereby shall you presently with the help of Subtraction tell his proposed number.

The Rule is this.

Imagine he thought 17: double 17, and it maketh 34, whereunto if you adde 5, it maketh 39: which multiplied by 5, as here is practised in the margent, it yeeldeth 195, which 195 is the summe deliuered you in the worke: then for a generall rule, you shall euermore cut off the last figure towards your right hand, with a dash of your pen, as here is performed, as a figure nothing auaileable vnto your worke, and then rebate 2 from your first figure, after 5 is cut off, and the rest shall euermore be your desire, as by this example doth appeare.

17	
2	
34	
5	
39	
5	
19	5
2	
17	

If in any company you are disposed to make them merie by maner of Diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you will absent your selfe frō them & they to deuise after you are gone which of them shall haue the keeping thereof, and that you at your returne will tell them what person hath it, vpon what hand, vpon what finger, and what ioint.

Which

which to doe, cause the persons to sit downe all on a rowe, and to keepe likewise an order of their fingers: now, after yee are gone out from them to some other place, say vnto one of the lookers on, that he double the number of him that hath the Ring, and vnto the double bid him adde 5, and then cause him to multiplie that Addition by 5, and vnto the product bid him adde the number of the finger, of the person that hath the Ring. And lastly, to end the worke, beyond that number towardes his right hand, let him set downe a figure signifying vpon which of the ioynts he hath the ring, as if it be vpon the second ioynt, let him put downe 2. Then demaund of him what number he keepeth, from the which you shal abate 250. And you shall haue three figures remaining at the least. The first towards your left hand shal signifie the number of the person which hath the Ring, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shall betoken the number of the ioynt.

Example.

Imagine the seuenth person is determined to keepe the Ring vpon the fifth finger, and the third ioint: first double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplyed by 5, yeeldeth 95: vnto which 95, adde the number of the finger, and it maketh 100: and beyond

pond 100 toward the right hand, I set downe 3 the number of the ioynt, all maketh 1003, which is the number that is to be deliuered you, from which abating 250, there resteth 753, which prefigureth vnto you the seuenth person, the fifth finger, and the third ioynt.

But note, that when you haue made your Subtraction, if there doe remaine a 0: in the place of tennes, that is to say, in the second place, you must then abate 1, from that figure which is in the place of hundreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tēth figure, as if there should remaine 803, you must say, that the seuenth person vpon his tenth finger, and vpon his third ioynt, hath the Ring.

And after the same manner, if a man doe cast 3 Dice, you may know the points of euerie one of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same summe to multiplie by 5, and vnto the product adde the points of one of the other Dice. And behind the number towards the right hand to put the figure which signifieth the points of the last Die, and then to aske what number he keepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the points of euerie Die.

An

Another.

If three diuerse thinges are to be hidden of three diuerse persons, and you to diuine which of the three persons hath the three diuerse things, doe thus: imagine the three things to be represented by A, B, C. Then secondly keep well in your minde which of the persons you meane to be the first, second and third. Then take 24 Counters or stones, and your three thinges, and giue A to the partie whom you imagine to be your first man, and therewithall giue him one of your 24 counters in his hand. And B, vnto your second man, and therewithall 2 Counters. And C, vnto your third man, and therewithall 3 Counters. And leaue the rest which are 18 still among them, which doone, seperate your selfe from them, and afterwards bid them change the things among them as they shall thinke good: which done, after they are agreed, bid him that hath such a thing, as before you haue represented by A. for euery counter that he hath in his hande to take vp as many more. And for him that hath B, for euerie one in his hande to take vp two. And for him that hath C, for euerie one in his hande to take vp 4, and the rest of them to leaue still vpon the boord. These three things and the three persons beyng fully printed in your minde, come to the Table, and you shall euermore finde one of these fiue numbers, 1. 2. 3. 5. 6. or 7. If therefore one remaine still vp-

on

on the boord, then haue they made no exchange, but keepe them still as they were deliuered vnto them. So that the first man hath A, the second B, and the third C. But if 2 remaine, then the first man hath B, your second man A, and your third man C. The rest of the worke and the order thereof are here apparant by the Table following. John Tacke.

1	1	A	5	1	B
	2	B		2	C
	3	C		3	A
2	1	B	6	1	C
	2	A		2	A
	3	C		3	B
3	1	A	7	1	C
	2	C		2	B
	3	B		3	A

An

Another Diuination of a number vpon the casting of two Dice.

First let the caster cast both the Dice, and marke well the number: then let him take vp one of them, it maketh no matter which, and looke what number it hath in the bottom, and adde all together: then cast the die againe, and keepe in his mynde what altogether maketh: then let the Dice stand: bying seuen with you and thereunto adde the rest of the pits that you see vpon the vpper side of the Dice, and so many did the caster cast in all.

FINIS.

I. D. To the earneſt Arithmetician.

Fo: Fackſon,

MY louing friend to Seience bent,
Some thing thou haſt by this booke woone:
But if thou wilt be excellent,
Another race thou muſt yet runne.

Supplies thereto but (few doe) neede,
And none but ſuch as in our phraſe,
(By Records *pen) thou mayſt well reade,*
Proceede therefore. Be not ſtunt awaſe.

The ground moſt ſure, whereon this race
With ſpeedefull courage muſt be paſt,
Of late hath turnd his Greekiſh face,
By Engliſh tilth, which aye will laſt.

The famous Greeke of Platos *lore,*
Euclide *I meane Geometer:*
So true, ſo plaine, ſo fraught with ſtore,
(As in our ſpeach) is yet nowhere.

A treaſure ſtrange, that booke will proone,
With numbers skill, matcht in due ſort,
This I thee warne of ſyncere loue,
And to proceede doe thee exhort.

Plus oultre.



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